Sum and Difference Identities

Now let's look at identities involving expressions of the form $\sin(A \pm B)$ and $\cos(A \pm B)$. These identities allow us to calculate the sine and cosine of the sum and difference of two angles if we know the sine and cosine of the angles. (There are similar identities for tangent but we can use the sine and cosine identities along with the definition of tangent rather than studying another identity for tangent.)

THE SUM AND DIFFERENCE IDENTITIES

sine :
$$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$
$$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$

cosine :
$$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

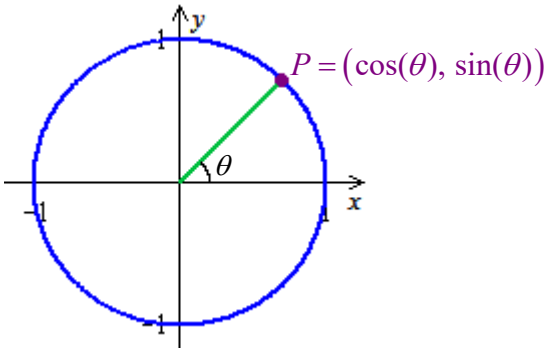
EXAMPLE 1: Use an appropriate identity to calculate $\cos\left(15^\circ\right)$.

EXAMPLE 2: Use an appropriate identity to calculate $\sin\left(\frac{13\pi}{12}\right)$.

Double- and Half-Angle Identities

In these notes, we will familiarize ourselves with identities that allow us to find $\sin(2\theta)$ and $\cos(2\theta)$ if we know the values of $\cos(\theta)$ and $\sin(\theta)$ (we call these "*double-angle identities*") and we will familiarize ourselves with identities that allow us to find $\sin\left(\frac{\theta}{2}\right)$ and $\cos\left(\frac{\theta}{2}\right)$ if we know the values of $\cos(\theta)$ and $\sin(\theta)$ (we call these "*half-angle identities*").

Let's start by deriving the **double-angle identity for sine** and then we'll derive the **double-angle identity for cosine**.



DOUBLE-ANGLE IDENTITIES

sine :

cosine :

EXAMPLE 1: If $\cos(A) = \frac{1}{3}$, where $\frac{3\pi}{2} < A < 2\pi$, find $\cos(2A)$, $\sin(2A)$, and $\tan(2A)$.

We can use the double-angle identities for cosine to derive **half-angle identities**.

Recall that $\cos(2\theta) = 1 - 2\sin^2(\theta)$ we can use this identity to find a half-angle identity for sine. Let $A = 2\theta$. Then $\theta = \frac{A}{2}$ and …

We can use $\cos(2\theta) = 2\cos^2(\theta) - 1$ to find a half-angle identity for cosine. Again, suppose that $A = 2\theta$. Then $\theta = \frac{A}{2}$ and ...

HALF-ANGLE IDENTITIES

sine :

cosine :

When using the half-angle identities, you need to decide which sign to use by determining which quadrant $\frac{\theta}{2}$ falls in.

EXAMPLE 2: If $\cos(A) = \frac{1}{3}$, where $\frac{3\pi}{2} < A < 2\pi$. Find $\cos\left(\frac{A}{2}\right)$ and $\sin\left(\frac{A}{2}\right)$.

EXAMPLE 3: Use a half-angle identity to find $\cos\left(15^\circ\right)$.