Introduction to Polar Coordinates

We are all comfortable using rectangular (i.e., Cartesian) coordinates to describe points on the plane. In Figure 1 let's plot the point $P = \left(\sqrt{3}, 1\right)$ on the rectangular coordinate plane:

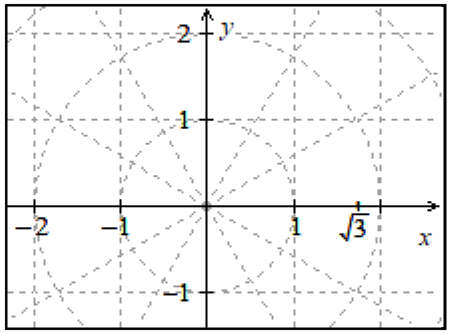
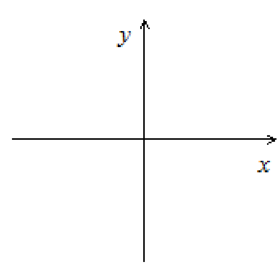


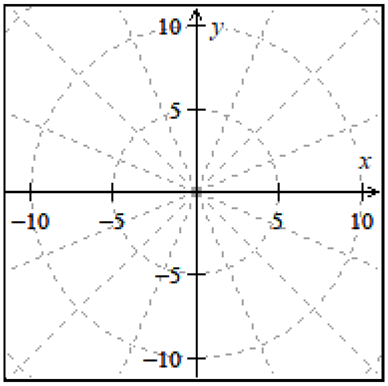
Figure 1

Instead of using these *rectangular* coordinates, we can use a *circular* coordinate system to describe points on the plane, i.e., we can use the **polar coordinate system**. Ordered pairs in polar coordinates have form (r, θ) where r represents the point's distance from the origin and θ represents the angular displacement of the point with respect to the positive x-axis. Let's find the polar coordinates that describe $P = \left(\sqrt{3}, 1\right)$.

The rectangular coordinates (x, y) are equivalent to the (r, θ) polar coordinates such that:

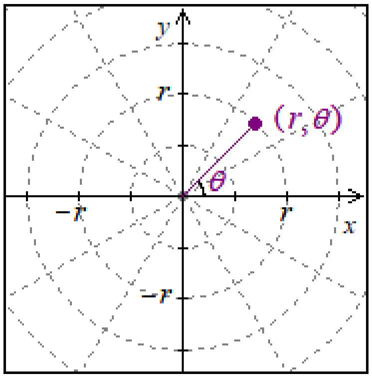


EXAMPLE: Plot the point $A = \left(10, \frac{5\pi}{4}\right)$ on the polar coordinate plane below and determine the rectangular coordinates of point A.



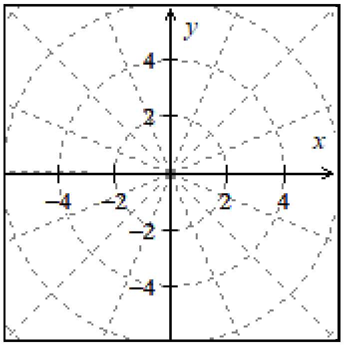
Plot $A = \left(10, \frac{5\pi}{4}\right)$.

The polar coordinates (r, θ) are equivalent to the following rectangular coordinates:



What happens if $r < 0$?

EXAMPLE: Plot the point $B = \left(-4, \frac{2\pi}{3}\right)$ on the polar coordinate plane below and list a few other ordered pairs that are plotted at the same location.

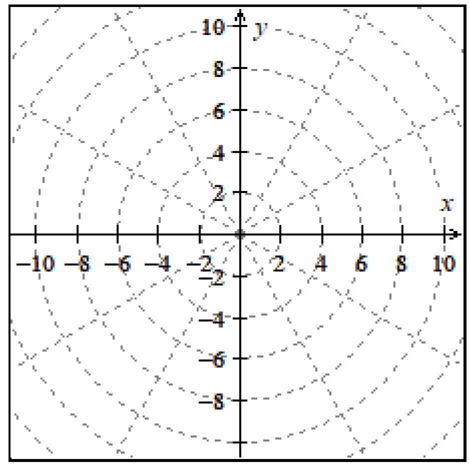


Plot $B = \left(-4, \frac{2\pi}{3}\right)$.

Graphing Polar Functions

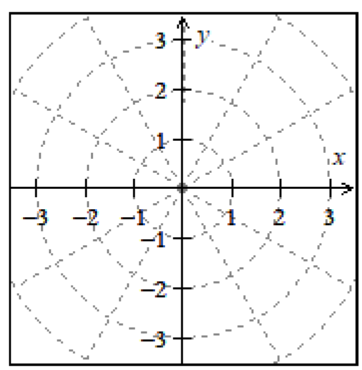
Now let's graph polar functions, i.e., functions that involve polar coordinates. These functions will have form $r = f(\theta)$, so the input, θ, is an angle and the output, r, is the distance from the origin. **Notice polar ordered pairs, (r, θ), have the *output* variable, r, first and the *input* variable, θ, second** which is a **different order** than rectangular ordered pairs of the form (x, y) which have the *input* variable first and the *output* variable second.

EXAMPLE: Sketch a graph of the polar function $r = \theta$.



Sketch a graph $r = \theta$.

EXAMPLE: Sketch a graph of the polar function $r = 3$.

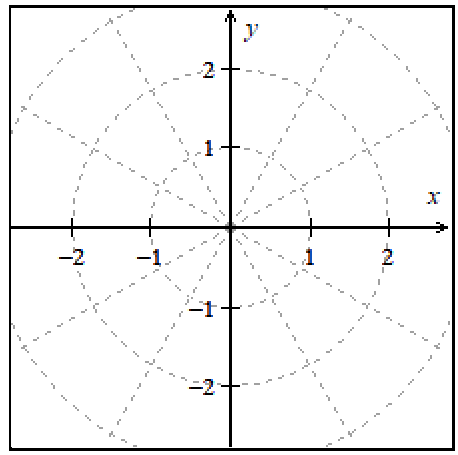


Sketch a graph $r = 3$.

EXAMPLE: Sketch a graph of the $r = 2\sin(2\theta)$ on the polar coordinate plane.

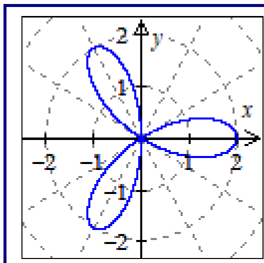
To sketch the graph of $r = 2\sin(2\theta)$, let's find some ordered pairs that satisfy the function.

θ	$r = 2\sin(2\theta)$	(r, θ)
0		
$\frac{\pi}{12}$		
$\frac{\pi}{6}$		
$\frac{\pi}{4}$		
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		
$\frac{2\pi}{3}$	$-\sqrt{3} \approx -1.7$	
$\frac{3\pi}{4}$	-2	
$\frac{5\pi}{6}$	$-\sqrt{3} \approx -1.7$	
π	0	
$\frac{7\pi}{6}$	$\sqrt{3} \approx 1.7$	
$\frac{5\pi}{4}$	2	
$\frac{4\pi}{3}$	$\sqrt{3} \approx 1.7$	
$\frac{3\pi}{2}$	0	
$\frac{5\pi}{3}$	$-\sqrt{3} \approx -1.7$	
$\frac{7\pi}{4}$	-2	
$\frac{11\pi}{6}$	$-\sqrt{3} \approx -1.7$	

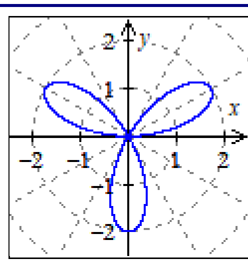


Sketch a graph $r = 2\sin(2\theta)$.

Below are the graphs of a few other functions defined via polar coordinates.

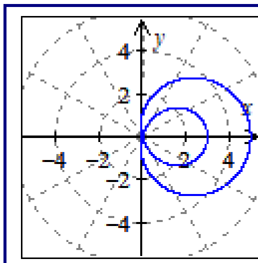


$r = 2\cos(3\theta)$

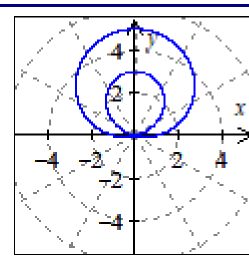


$r = 2\sin(3\theta)$

These graphs are called "**roses**".



$r = 1 + 4\cos(\theta)$



$r = 1 + 4\sin(\theta)$

These graphs are called "**limaçons**".