Week 6 Practice Worksheet

Other Trig Identities and Polar Coordinates

1. Use a **sum-of-angles** or **difference-of-angles identity** to calculate the *exact value* of each of the following. (These identities are included on the Identities and Formulas Reference Sheet that will be provided to you during the Final Exam.)

a. $\sin(165^\circ)$

b. $\cos\left(\frac{13\pi}{12}\right)$

c. $\tan\left(\frac{17\pi}{12}\right)$

2. In order to get familiar with the **sum-of-angles**, **difference-of-angles**, **double-angle** and **half-angle identities**, we'll use these identities to calculate some "friendly" sine and cosine values – so we already know these sine and cosine values and we'll verify that the identities lead us to these values. To help explain the activity part (a) has been worked out for you, and part (b) has been started. (Theses identities are included on the Identities and Formulas Reference Sheet that will be provided to you during the Final Exam.)

a. Find $\sin\left(\frac{\pi}{2}\right)$ using the fact that $\frac{\pi}{2} = \frac{\pi}{6} + \frac{\pi}{3}$.

$$\begin{aligned}\sin\left(\frac{\pi}{2}\right) &= \sin\left(\frac{\pi}{6} + \frac{\pi}{3}\right)\\ &= \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{3}\right)\\ &= \frac{1}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}\\ &= \frac{1}{4} + \frac{3}{4}\\ &= 1\end{aligned}$$

We know that $\sin\left(\frac{\pi}{2}\right) = 1$ so the identity gave us the correct value.

b. Find $\cos(\pi)$ using the fact that $\pi = \frac{5\pi}{6} + \frac{\pi}{6}$.

c. Find $\sin\left(\frac{2\pi}{3}\right)$ using the fact that $\frac{2\pi}{3} = 2\cdot\frac{\pi}{3}$. (hint: use a double-angle identity)

d. Find $\cos\left(\frac{\pi}{3}\right)$ using the fact that $\frac{\pi}{3} = 2 \cdot \frac{\pi}{6}$. (hint: use a double-angle identity)

e. Find $\sin\left(\frac{\pi}{4}\right)$ using the fact that $\frac{\pi}{4} = \frac{\pi/2}{2}$. (Hint: use a half-angle identity)

f. Find $\cos(30^\circ)$ using the fact that $30^\circ = \frac{60^\circ}{2}$. (hint: use a half-angle identity)

3. Suppose that $\sin(\alpha) = -\frac{\sqrt{65}}{9}$ and $\pi < \alpha < \frac{3\pi}{2}$. Calculate the *exact value* of each of the following using an appropriate **double-angle** or **half-angle identity**. (These identities are included on the Identities and Formulas Reference Sheet.)

a. $\sin(2\alpha)$. [Hint: first find $\cos(\alpha)$]

b. $\cos(2\alpha)$.

c. $\sin\left(\frac{\alpha}{2}\right)$.

4. Suppose that $\cos(\theta) = \frac{7}{10}$ and $\frac{3\pi}{2} < \theta < 2\pi$. Calculate the *exact value* of each of the following using an appropriate **double-angle** or **half-angle identity**. (These identities are included on the Identities and Formulas Reference Sheet.)

a. $\sin(2\theta)$. [Hint: first find $\sin(\theta)$]

b. $\cos(2\theta)$.

c. $\cos\left(\frac{\theta}{2}\right)$.

5. Suppose that $\sin(x) = \frac{9}{11}$ and $\frac{\pi}{2} < x < \pi$. Calculate the *exact value* of each of the following using an appropriate **double-angle** or **half-angle identity**. (These identities are included on the Identities and Formulas Reference Sheet.)

a. $\sin(2x)$. [Hint: first find $\cos(x)$]

b. $\cos(2x)$.

c. $\sin\left(\frac{x}{2}\right)$.

6. Prove the following identities using the double-angle identities for sine and cosine included on the Identities and Formulas Reference Sheet. (Be sure to organize your proof as shown in the Online Lecture Notes and class notes videos.)

a. $\tan(2x) = \dfrac{2\tan(x)}{1 - \tan^2(x)}$

(this is known as the double-angle identity for tangent; to prove it, start with the left side and use the double-angle identities for sine and cosine; for cosine, use $\cos(2x) = \cos^2(x) - \sin^2(x)$; a "trick" here is to introduce a term that will create the denominator that you need)

b. $\dfrac{1 - \cos(2t)}{\sin(2t)} = \tan(t)$

7. Translate each of the following rectangular coordinates (x, y) into polar coordinates (r, θ). Your answers should be ordered pairs involving exact values, with θ in radians.

a. $(x, y) = (-4, -4)$

b. $(x, y) = \left(6, -6\sqrt{3}\right)$

8. Translate each of the following rectangular coordinates (x, y) into polar coordinates (r, θ). Your answers should be ordered pairs involving approximations of θ in radians.

a. $(x, y) = (10, -2)$

b. $(x, y) = (-3, 7)$

9. Translate each of the following polar coordinates (r, θ) into rectangular coordinates (x, y). Your answers should be ordered pairs involving exact values.

a. $(r, \theta) = \left(5, \frac{2\pi}{3}\right)$

b. $(r, \theta) = \left(16, 210^{\circ}\right)$

10. Translate each of the following polar coordinates (r, θ) into rectangular coordinates (x, y). Your answers should be ordered pairs involving approximate values.

a. $(r, \theta) = \left(3, 80^\circ\right)$

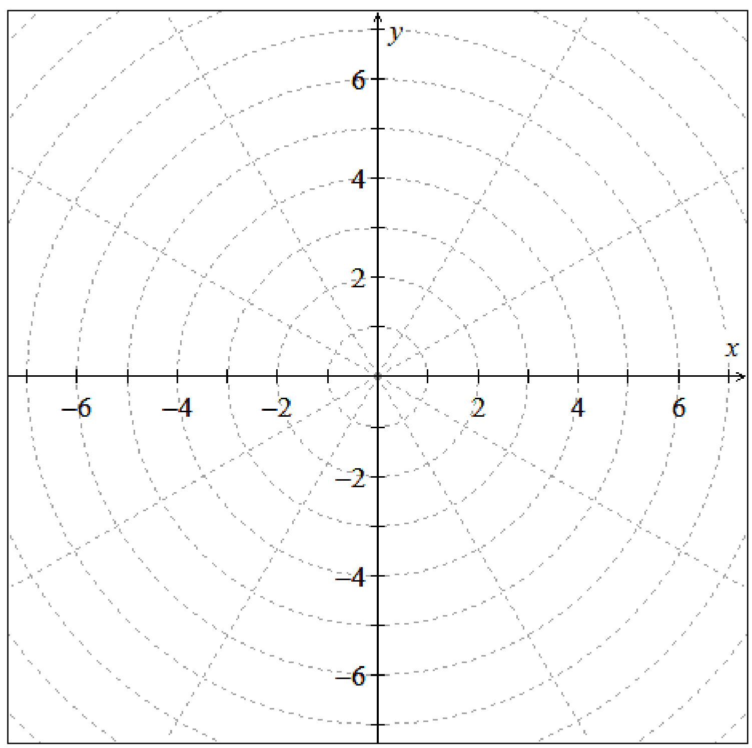
b. $(r, \theta) = \left(7, -\frac{\pi}{10}\right)$

11. The polar ordered pair $\left(5, \frac{\pi}{3}\right)$ can be plotted as a "dot" on the polar coordinate plane. List four other (different) polar ordered pairs that are plotted on the same "dot." Use "$r = -5$" for at least one of your ordered pairs.

12. Complete the 1st column of the table below with appropriate multiples of $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$; then determine the corresponding values of r if $r = 6\cos(2\theta)$ in order to complete the 2nd column of the table; then plot the points implied by each of the 16 rows of the table on the polar plane below and connect those points in order to draw a graph of $r = 6\cos(2\theta)$.

(HINT: Graph the function in Desmos so that you can predict its shape before you draw it.)

	θ	r
	0	
Quad. 1 angles		
	$\pi/2$	
Quad. 2 angles		
	π	
Quad. 3 angles		
	$3\pi/2$	
Quad. 4 angles		



Draw a graph of $r = 6\cos(2\theta)$.

13. Express the complex number $z = 10e^{i \cdot \frac{11\pi}{6}}$ in the form $z = a + bi$.

14. Express the complex number $z = 8e^{i \cdot \frac{2\pi}{3}}$ in the form $z = a + bi$.

15. Express the complex number $z = -7e^{i \cdot \frac{5\pi}{4}}$ in the form $z = a + bi$.

16. Find a polar form, $z = re^{i\theta}$, of the complex number $z = -3 - 3\sqrt{3}\,i$.

17. Find a polar form, $z = re^{i\theta}$, of the complex number $z = 2 - 2i$.

18. Find a polar form, $z = re^{i\theta}$, of the complex number $z = -4\sqrt{3} + 12i$.

19. Find three different a polar forms, $z = re^{i\theta}$, of the complex number $z = 4i$. (HINT: $i = 0 + 4i$ can be associated with the point $(0, 4)$ so find three different angles that can be used to represent the "direction of this point" and use each angle to create a polar form.)