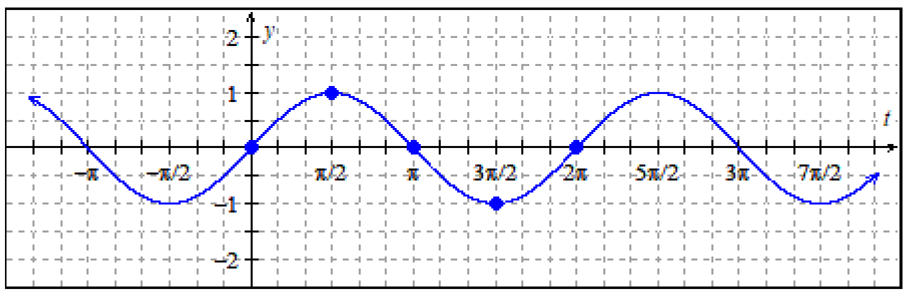
Week 3 Practice Worksheet

Graphs of Trig Functions, Inverse Trig Functions, and Solving Trig Equations

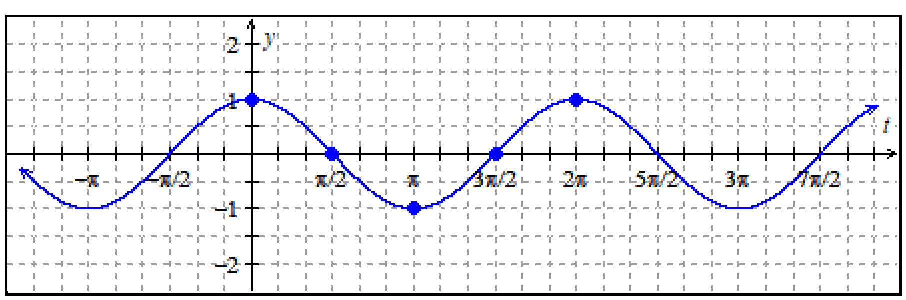
You should complete all of these problems ***without a calculator*** in order to prepare for the Midterm which is a no-calculator exam.

1. The graph of $y = \sin(t)$ is given below with **five key points** emphasized. As we know, the graph of $y = \sin\left(t - \frac{2\pi}{3}\right)$ is a *transformation* of $y = \sin(t)$. Use what we know about graph transformations to "transform" the **five** key points on $y = \sin(t)$ and then connect these points in order to construct a graph of $y = \sin\left(t - \frac{2\pi}{3}\right)$.



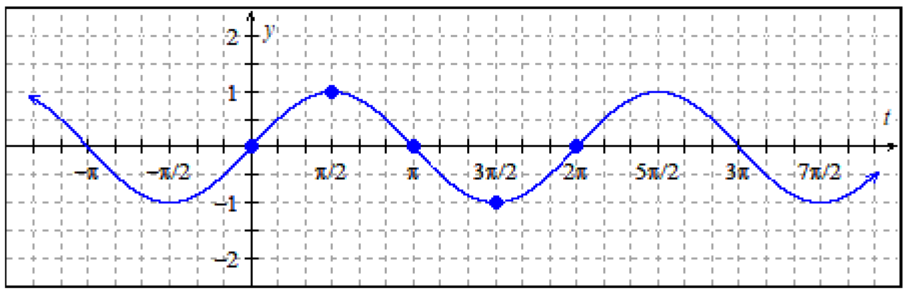
The graph of $y = \sin(t)$ is given; draw a graph of $y = \sin\left(t - \frac{2\pi}{3}\right)$.

2. The graph of $y = \cos(t)$ is given below with **five key points** emphasized. As we know, the graph of $y = \cos\left(t + \frac{5\pi}{6}\right)$ is a *transformation* of $y = \cos(t)$. Use what we know about graph transformations to "transform" the **five key points** on $y = \cos(t)$ and then connect these points in order to construct a graph of $y = \cos\left(t + \frac{5\pi}{6}\right)$.



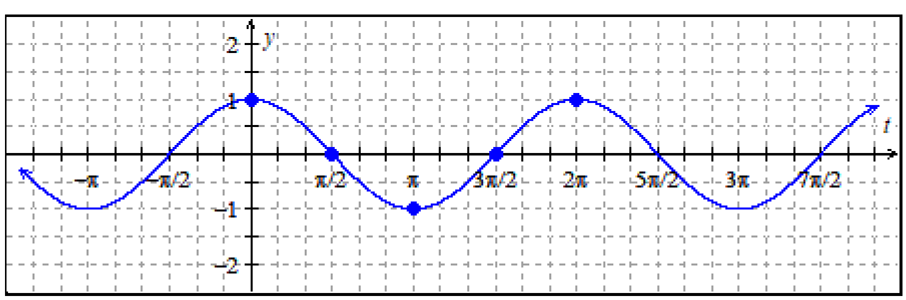
The graph of $y = \cos(t)$ is given; draw a graph of $y = \cos\left(t + \frac{5\pi}{6}\right)$.

3. The graph of $y = \sin(t)$ is given below with **five key points** emphasized. As we know, the graph of $y = \sin(t) - 1$ is a *transformation* of $y = \sin(t)$. Use what we know about graph transformations to "transform" the **five** key points on $y = \sin(t)$ and then connect these points in order to construct a graph of $y = \sin(t) - 1$.



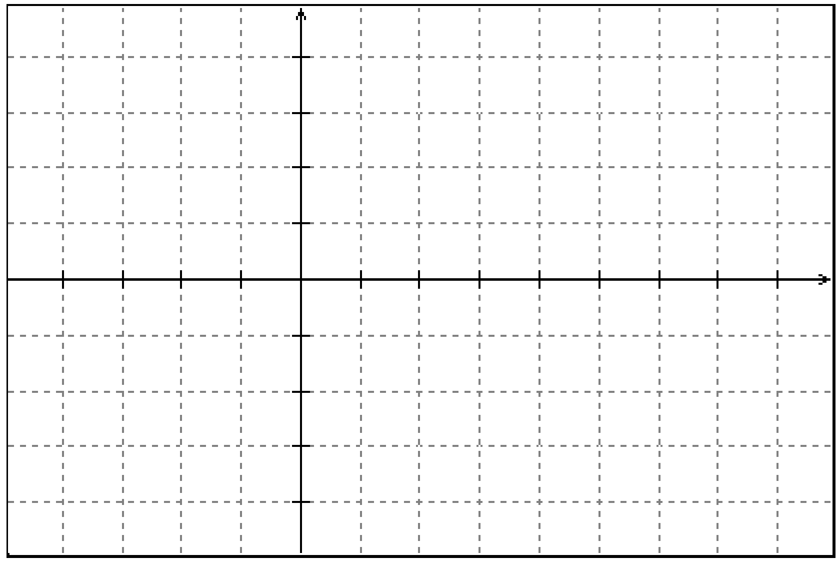
The graph of $y = \sin(t)$ is given; draw a graph of $y = \sin(t) - 1$.

4. The graph of $y = \cos(t)$ is given below with **five key points** emphasized. As we know, the graph of $y = 2\cos(t)$ is a *transformation* of $y = \cos(t)$. Use what we know about graph transformations to "transform" the **five key points** on $y = \cos(t)$ and then connect these points in order to construct a graph of $y = 2\cos(t)$.



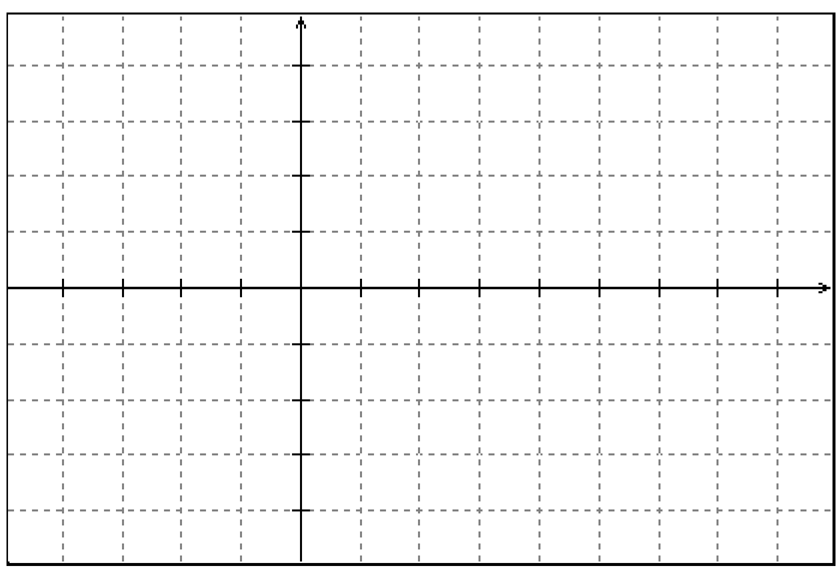
The graph of $y = \cos(t)$ is given; draw a graph of $y = 2\cos(t)$.

5. Scale the axes on the given coordinate plane an appropriately for a graph of $y = \sin(2t) + 3$ and then draw a graph of $y = \sin(2t) + 3$ by first plotting the points where the graph will intersect the midline and the points where the graph will reach maximum and minimum values, and then connect these points with an appropriately curved sinusoidal wave.



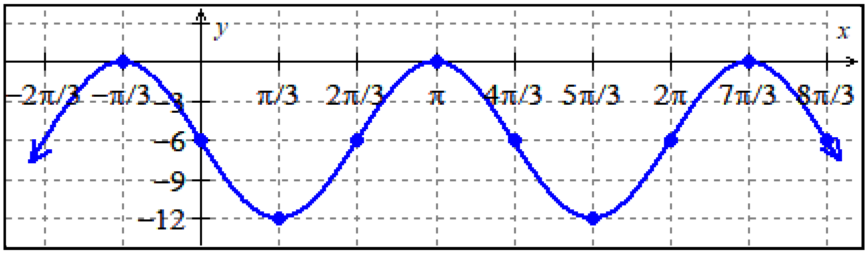
Draw a graph of $y = \sin(2t) + 3$.

6. Scale the axes on the given coordinate plane an appropriately for a graph of $y = 3\cos(\pi t)$ and then draw a graph of $y = 3\cos(\pi t)$ by first plotting the points where the graph will intersect the midline and the points where the graph will reach maximum and minimum values, and then connect these points with an appropriately curved sinusoidal wave.



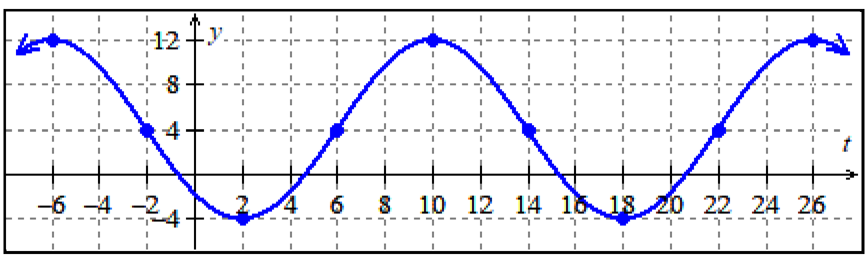
Draw a graph of $y = 3\cos(\pi t)$.

7. Find two different algebraic rules for the sinusoidal function $y = p(x)$ graphed below. One of your rules should involve sine and the other should involve cosine.



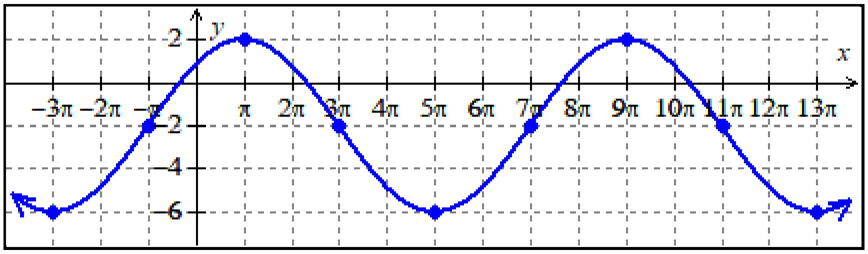
The graph of $y = p(x)$.

8. Find two different algebraic rules for the sinusoidal function $y = q(t)$ graphed below. One of your rules should involve sine and the other should involve cosine.



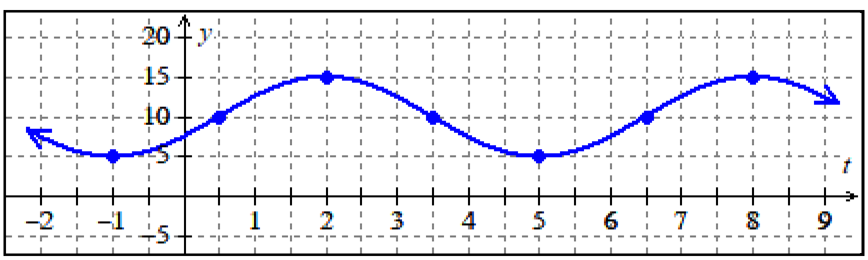
The graph of $y = q(t)$.

9. Find two different algebraic rules for the sinusoidal function $y = m(x)$ graphed below. One of your rules should involve sine and the other should involve cosine.



The graph of $y = m(x)$.

10. Find two different algebraic rules for the sinusoidal function $y = n(t)$ graphed below. One of your rules should involve sine and the other should involve cosine.



The graph of $y = n(t)$.

11. Draw a graph of at least two periods of the functions in **a–f** below by first *plotting the points* where the graph will intersect the midline and *plotting the points* where the graph will reach maximum and minimum values, and then *connect these points* with an appropriately curved sinusoidal wave. List the period, midline, and amplitude of each function. (Be sure to label the scale on the axes of your graph.)

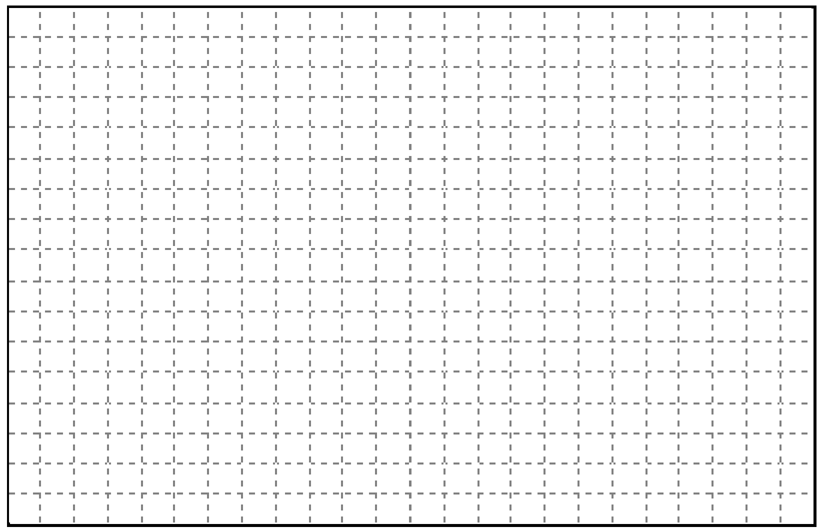
a. $f(t) = -5\cos\left(4t\right) + 3$

b. $g(x) = 4\sin\left(\pi\left(x - \frac{1}{4}\right)\right) - 2$

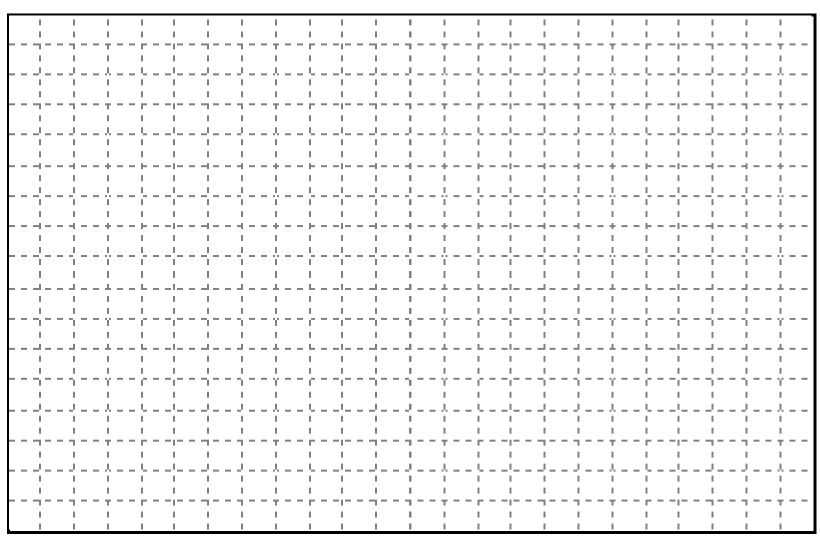
c. $G(x) = 3\cos\left(2x + \frac{\pi}{2}\right) + 4$

d. $F(t) = -2\sin\left(\frac{\pi}{3}t + \pi\right) - 1$

e. $p(x) = 10\cos\left(\dfrac{x + 2\pi}{4}\right) - 5$



f. $q(t) = -4\sin\left(\frac{\pi}{4}t - \frac{\pi}{4}\right) - 2$



12. Determine the domain of $y = \tan(t)$.

13. Determine the domain of $y = \sec(t)$.

14. Determine the domain of $y = \cot(t)$.

15. Determine the domain of $y = \csc(t)$.

16. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to ***directly communicate*** what the given expressions equal.

a. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

b. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

c. $\sin^{-1}\left(-\frac{1}{2}\right)$

d. $\cos^{-1}(0)$

e. $\tan^{-1}\left(-\sqrt{3}\right)$

f. $\tan^{-1}(-1)$

17. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to ***directly communicate*** what the given expressions equal.

a. $\sin\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

b. $\cos\left(\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$

c. $\cos^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right)$

d. $\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right)$

e. $\sin\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

18. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to ***directly communicate*** what the given expressions equal.

a. $\sin^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right)$

b. $\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right)$

c. $\cos^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$

d. $\tan^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right)$

e. $\sin\left(\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$

19. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to ***directly communicate*** what the given expressions equal.

a. $\sin^{-1}\left(\sin\left(\frac{7\pi}{8}\right)\right)$

b. $\cos^{-1}\left(\cos\left(\frac{7\pi}{5}\right)\right)$

c. $\sin^{-1}\left(\sin\left(\frac{9\pi}{7}\right)\right)$

20. Find *all* of the solutions to the equations below; provide *exact* solutions.

a. $\sin(t) = -\frac{\sqrt{2}}{2}$

b. $2\cos(x) - \sqrt{3} = 0$

21. Find the solutions on the interval $[0, 2\pi)$ for the equations below; provide *exact* solutions.

a. $\cos(t) = -\frac{1}{2}$

b. $\frac{\sin(x)}{2} - \frac{\sqrt{3}}{4} = 0$

22. Find *all* of the solutions to the equations below; provide *exact* solutions.

a. $\sin(6t) = -\frac{\sqrt{3}}{2}$

b. $5 + 4\cos(2\theta) = 1$

c. $16\cos(4x) + 11 = 3$

d. $16 - 24\sin(8t) = 4$

23. Find the solutions on the interval $[0, 2\pi)$ to following equations.

a. $5 + 4\cos(2\theta) = 1$ **b.** $4 - 6\sin(2x) = 7$ **c.** $6\sqrt{2}\cos(3\alpha) + 10 = 4$