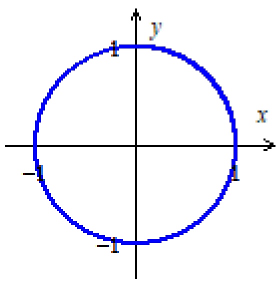
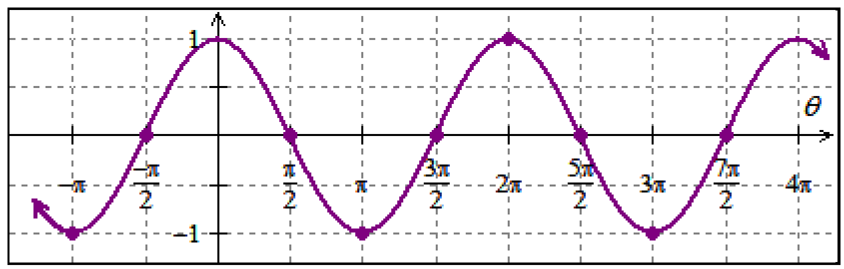
SOLUTIONS: Week 2 Practice Worksheet

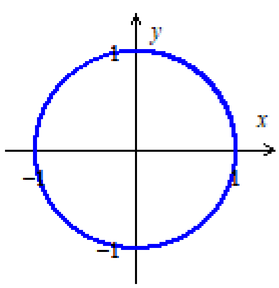
Intro to the Trig Functions

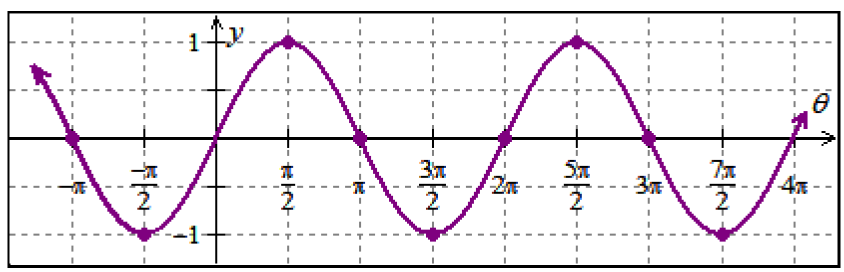
1. Draw an *accurate* graph of at least two periods of $y = \cos(\theta)$ and $y = \sin(\theta)$ on the coordinate planes below. To draw each graph, correctly label at least **two** of the unlabeled "tics" on the y-axis and all **eight** of the unlabeled "tics" on the θ-axis (two tics on the θ-axis have been labeled for you); then **plot a point** on your graph that corresponds to **each of the eight tics** that you've labeled on the θ-axis and the two pre-labeled tics, and then connect the points to create your graph.





A graph of $y = \cos(\theta)$.





A graph of $y = \sin(\theta)$.

2. If $\frac{3\pi}{2} < \theta < 2\pi$ and $\cos(\theta) = \frac{\sqrt{5}}{4}$, find the following. Be sure to use proper notation to communicate your answer, i.e., link the given expression and your answer with an equal sign. For example, your response to part (a) should have the form $\sin(\theta) =$ ____ since you're asked to tell me what $\sin(\theta)$ equals.)

a. $\sin(\theta)$

To find $\sin(\theta)$, we can use the Pythagorean Identity: $\sin^2(\theta) + \cos^2(\theta) = 1$:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\Rightarrow \sin^2(\theta) + \left(\frac{\sqrt{5}}{4}\right)^2 = 1$$

$$\Rightarrow \sin^2(\theta) + \frac{5}{16} = 1$$

$$\Rightarrow \sin^2(\theta) = 1 - \frac{5}{16}$$

$$\Rightarrow \sin(\theta) = -\sqrt{\frac{11}{16}}$$

(since $\frac{3\pi}{2} < \theta < 2\pi$, $\sin(\theta) < 0$ so we take the negative square root)

$$\Rightarrow \sin(\theta) = -\frac{\sqrt{11}}{4}$$

b. $\tan(\theta)$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$= \frac{-\sqrt{11}/4}{\sqrt{5}/4}$$

$$= -\frac{\sqrt{11}}{\sqrt{5}} = -\frac{\sqrt{55}}{5}$$

(you aren't required to rationalize the denominator)

c. $\sec(\theta)$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$= \frac{1}{\sqrt{5}/4}$$

$$= \frac{4}{\sqrt{5}} = \frac{4\sqrt{5}}{5}$$

(you aren't required to rationalize the denominator)

d. $\csc(\theta)$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{-\sqrt{11}/4} = -\frac{4}{\sqrt{11}} = -\frac{4\sqrt{11}}{11}$$

(you aren't required to rationalize the denominator)

3. If $\frac{\pi}{2} < \theta < \pi$ and $\sin(\theta) = \frac{6}{7}$, find the following. Be sure to use proper notation to communicate your answer, i.e., link the given expression and your answer with an equal sign. For example, your response to part (a) should have the form $\cos(\theta) =$ ____ since you're asked to tell me what $\cos(\theta)$ equals.)

a. $\cos(\theta)$

To find $\cos(\theta)$, we can use the Pythagorean Identity: $\sin^2(\theta) + \cos^2(\theta) = 1$:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$
$$\Rightarrow \left(\tfrac{6}{7}\right)^2 + \cos^2(\theta) = 1$$
$$\Rightarrow \tfrac{36}{49} + \cos^2(\theta) = 1$$
$$\Rightarrow \cos^2(\theta) = 1 - \tfrac{36}{49}$$
$$\Rightarrow \cos(\theta) = -\sqrt{\tfrac{13}{49}}$$

(since $\frac{\pi}{2} < \theta < \pi$, $\cos(\theta) < 0$ so we take the negative square root)

$$\Rightarrow \cos(\theta) = -\frac{\sqrt{13}}{7}$$

b. $\tan(\theta)$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{6/7}{-\sqrt{13}/7} = -\frac{6}{\sqrt{13}} = -\frac{6\sqrt{13}}{13}$$

(you aren't required to rationalize the denominator)

c. $\sec(\theta)$

$$\begin{aligned}\sec(\theta) &= \frac{1}{\cos(\theta)} \\ &= \frac{1}{-\sqrt{13}/7} \\ &= -\frac{7}{\sqrt{13}} = -\frac{7\sqrt{13}}{13}\end{aligned}$$

(you aren't required to rationalize the denominator)

d. $\csc(\theta)$

$$\begin{aligned}\csc(\theta) &= \frac{1}{\sin(\theta)} \\ &= \frac{1}{6/7} \\ &= \frac{7}{6}\end{aligned}$$

4. If $\frac{\pi}{2} < \theta < \pi$ and $\csc(\theta) = 3$, find the following. Be sure to use proper notation to communicate your answer, i.e., link the given expression and your answer with an equal sign. For example, your response to part (a) should have the form $\sin(\theta) =$ ____ since you're asked to tell me what $\sin(\theta)$ equals.)

a. $\sin(\theta)$

To find $\sin(\theta)$, we can use the fact that the definition of cosecant is $\csc(\theta) = \frac{1}{\sin(\theta)}$:

$\csc(\theta) = 3$ (this is the given information)

$\Rightarrow \frac{1}{\csc(\theta)} = \frac{1}{3}$ (taking the reciprocal of both sides of the equation)

$\Rightarrow \sin(\theta) = \frac{1}{3}$ (since $\csc(\theta) = \frac{1}{\sin(\theta)}$)

b. $\cos(\theta)$

To find $\cos(\theta)$, we can use the Pythagorean Identity: $\sin^2(\theta) + \cos^2(\theta) = 1$:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\Rightarrow \left(\tfrac{1}{3}\right)^2 + \cos^2(\theta) = 1$$ (since we've learned that $\sin(\theta) = \frac{1}{3}$)

$$\Rightarrow \tfrac{1}{9} + \cos^2(\theta) = 1$$

$$\Rightarrow \cos^2(\theta) = 1 - \tfrac{1}{9}$$

$$\Rightarrow \cos(\theta) = -\sqrt{\tfrac{8}{9}}$$ (since $\frac{\pi}{2} < \theta < \pi$, $\cos(\theta) < 0$ so we take the negative square root)

$$\Rightarrow \cos(\theta) = -\tfrac{2\sqrt{2}}{3}$$

c. $\tan(\theta)$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{1/3}{-2\sqrt{2}/3} = -\frac{1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

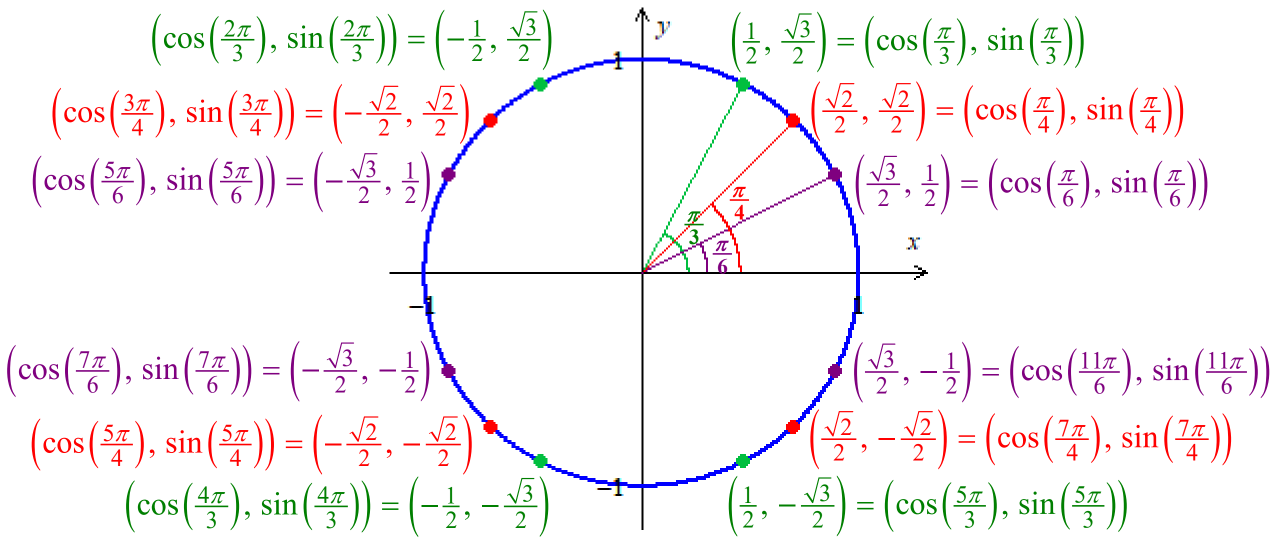
(you aren't required to rationalize the denominator)

d. $\sec(\theta)$

$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{-2\sqrt{2}/3} = -\frac{3}{2\sqrt{2}} = -\frac{3\sqrt{2}}{4}$$

(you aren't required to rationalize the denominator)

5. Label the **coordinates** of each of the 12 dots (three in each of the four quadrants) on the unit circle below. Fill-in the input for the sine and cosine functions in order to indicate the **angle** that specifies each point. Some of the information for the points in Quad 1 has been filled-in.



6. Find $\sin(\theta)$, $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$, and $\csc(\theta)$ if:

a. $\theta = \frac{\pi}{3}$

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$	$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$	$\tan\left(\frac{\pi}{3}\right) = \frac{\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{3}\right)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$
$\cot\left(\frac{\pi}{3}\right) = \frac{1}{\tan\left(\frac{\pi}{3}\right)} = \frac{1}{\sqrt{3}}$	$\sec\left(\frac{\pi}{3}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)} = \frac{1}{\frac{1}{2}} = 2$	$\csc\left(\frac{\pi}{3}\right) = \frac{1}{\sin\left(\frac{\pi}{3}\right)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

b. $\theta = \frac{\pi}{4}$

$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$	$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$	$\tan\left(\frac{\pi}{4}\right) = \frac{\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$
$\cot\left(\frac{\pi}{4}\right) = \frac{1}{\tan\left(\frac{\pi}{4}\right)} = \frac{1}{1} = 1$	$\sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}$	$\csc\left(\frac{\pi}{4}\right) = \frac{1}{\sin\left(\frac{\pi}{4}\right)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}$

7. Find $\sin(\theta)$, $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$, and $\csc(\theta)$ if:

a. $\theta = 150^\circ$

$\sin(150^\circ) = \sin(30^\circ) = \frac{1}{2}$	$\cos(150^\circ) = -\cos(30^\circ) = -\frac{\sqrt{3}}{2}$	$\tan(150^\circ) = \frac{\sin(150^\circ)}{\cos(150^\circ)} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$
$\cot(150^\circ) = \frac{1}{\tan(150^\circ)} = \frac{1}{-\frac{1}{\sqrt{3}}} = -\sqrt{3}$	$\sec(150^\circ) = \frac{1}{\cos(150^\circ)} = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$	$\csc(150^\circ) = \frac{1}{\sin(150^\circ)} = \frac{1}{\frac{1}{2}} = 2$

b. $\theta = -\frac{3\pi}{4}$

$\sin\left(-\frac{3\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$	$\cos\left(-\frac{3\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$	$\tan\left(-\frac{3\pi}{4}\right) = \frac{-\sin\left(\frac{\pi}{4}\right)}{-\cos\left(\frac{\pi}{4}\right)} = \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 1$
$\cot\left(-\frac{3\pi}{4}\right) = \frac{1}{\tan\left(-\frac{3\pi}{4}\right)} = \frac{1}{1} = 1$	$\sec\left(-\frac{3\pi}{4}\right) = \frac{1}{\cos\left(-\frac{3\pi}{4}\right)} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}}$	$\csc\left(-\frac{3\pi}{4}\right) = \frac{1}{\sin\left(-\frac{3\pi}{4}\right)} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}}$

c. $\theta = \frac{4\pi}{3}$

$\sin\left(\frac{4\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$	$\cos\left(\frac{4\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$	$\tan\left(\frac{4\pi}{3}\right) = \frac{-\sin\left(\frac{\pi}{3}\right)}{-\cos\left(\frac{\pi}{3}\right)} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$
$\cot\left(\frac{4\pi}{3}\right) = \frac{1}{\tan\left(\frac{4\pi}{3}\right)} = \frac{1}{\sqrt{3}}$	$\sec\left(\frac{4\pi}{3}\right) = \frac{1}{\cos\left(\frac{4\pi}{3}\right)} = \frac{1}{-\frac{1}{2}} = -2$	$\csc\left(\frac{4\pi}{3}\right) = \frac{1}{\sin\left(\frac{4\pi}{3}\right)} = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$

8. Find the ***exact*** value for each of the following expressions. Be sure to use proper notation to communicate your answer, i.e., link the given expression and your answer with an equal sign. If the given expression is undefined write, "*The expression is undefined*." An example has been provided to clarify how your response should look.

ex. $\sin(0)$

$\sin(0) = 0$

(Write all of this to communicate what "$\sin(0)$" *equals*.)

a. $\cos\left(\frac{\pi}{6}\right)$.

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

b. $\sin\left(\frac{\pi}{4}\right)$.

$$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

c. $\cos\left(60^\circ\right)$.

$$\cos\left(60^\circ\right) = \frac{1}{2}$$

d. $\sin\left(-\frac{5\pi}{3}\right)$.

$$\sin\left(-\frac{5\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

e. $\sin\left(\frac{17\pi}{6}\right)$.

$$\sin\left(\frac{17\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

f. $\cos\left(\frac{7\pi}{6}\right)$.

$$\cos\left(\frac{7\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

g. $\sin\left(240^\circ\right)$.

$$\sin\left(240^\circ\right) = -\sin\left(60^\circ\right) = -\frac{\sqrt{3}}{2}$$

h. $\cos\left(135^\circ\right)$.

$$\cos\left(135^\circ\right) = -\cos\left(45^\circ\right) = -\frac{\sqrt{2}}{2}$$

i. $\sec(2\pi)$.

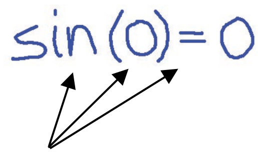
$$\sec(2\pi) = \frac{1}{\cos(2\pi)} = \frac{1}{1} = 1$$

j. $\tan\left(\frac{4\pi}{3}\right)$.

$$\tan\left(\frac{4\pi}{3}\right) = \frac{\sin\left(\frac{4\pi}{3}\right)}{\cos\left(\frac{4\pi}{3}\right)} = \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3}$$

9. Find the ***exact*** value for each of the following expressions. Be sure to use proper notation to communicate your answer, i.e., link the given expression and your answer with an equal sign. If the given expression is undefined write, "*The expression is undefined*." An example has been provided to clarify how your response should look.

ex. $\sin(0)$



(Write all of this to communicate what "$\sin(0)$" *equals*.)

a. $\sin\left(\frac{\pi}{2}\right)$.

$$\sin\left(\frac{\pi}{2}\right) = 1$$

b. $\cos\left(180^\circ\right)$.

$$\cos\left(180^\circ\right) = -1$$

c. $\cos\left(\frac{5\pi}{6}\right)$.

$$\cos\left(\frac{5\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

d. $\sin\left(\frac{7\pi}{6}\right)$.

$$\sin\left(\frac{7\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$

e. $\sin\left(120^\circ\right)$.

$$\sin\left(120^\circ\right) = \sin\left(60^\circ\right) = \frac{\sqrt{3}}{2}$$

f. $\cos\left(\frac{5\pi}{4}\right)$.

$$\cos\left(\frac{5\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

g. $\cos\left(\frac{5\pi}{3}\right)$.

$$\cos\left(\frac{5\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

h. $\sin\left(\frac{20\pi}{3}\right)$.

$$\sin\left(\frac{20\pi}{3}\right) = \sin\left(\frac{20\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

i. $\tan(2\pi)$.

$$\tan(2\pi) = \frac{\sin(2\pi)}{\cos(2\pi)} = \frac{0}{1} = 0$$

j. $\csc\left(\frac{11\pi}{6}\right)$.

$$\csc\left(\frac{11\pi}{6}\right) = \frac{1}{\sin\left(\frac{11\pi}{6}\right)} = \frac{1}{-\frac{1}{2}} = -2$$

10. A circle with a radius of 6 units is given in Figure 1. The point Q is specified by the angle $\frac{5\pi}{6}$. Use the sine and cosine function to find the exact coordinates of point Q.
[**Be sure to *show* your use of sine and cosine**.]

The point Q is specified by $\frac{5\pi}{6}$ on the circumference of a circle of radius 6 units. Therefore,

$$\begin{aligned} Q &= \left(6\cos\left(\tfrac{5\pi}{6}\right),\ 6\sin\left(\tfrac{5\pi}{6}\right)\right) \\ &= \left(6\cdot\left(-\tfrac{\sqrt{3}}{2}\right),\ 6\cdot\left(\tfrac{1}{2}\right)\right) \\ &= \left(-3\sqrt{3},\ 3\right) \end{aligned}$$

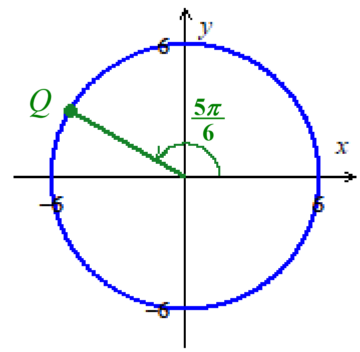


Figure 1

11. The point P in Figure 2 is specified by $\frac{7\pi}{4}$ on the circumference of a circle with a radius of 12 units. Use the sine and cosine function to find the **exact** coordinates of P.
[**Be sure to *show* your use of sine and cosine**.]

The point P is specified by $\frac{7\pi}{4}$ on the circumference of a circle of radius 12 units. Therefore,

$$\begin{aligned} P &= \left(12\cos\left(\tfrac{7\pi}{4}\right),\ 12\sin\left(\tfrac{7\pi}{4}\right)\right) \\ &= \left(12\cdot\left(\tfrac{\sqrt{2}}{2}\right),\ 12\cdot\left(-\tfrac{\sqrt{2}}{2}\right)\right) \\ &= \left(6\sqrt{2},\ -6\sqrt{2}\right) \end{aligned}$$

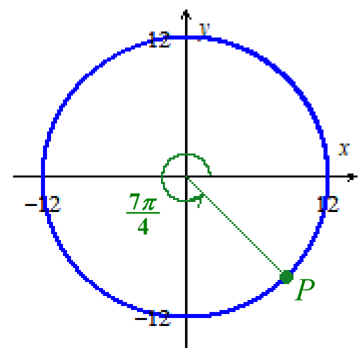


Figure 2

12. The point S in Figure 3 is specified by $\frac{4\pi}{3}$ on the circumference of a circle with a radius of 20 units. Use the sine and cosine function to find the **exact** coordinates of S.
[**Be sure to *show* your use of sine and cosine**.]

The point S is specified by $\frac{4\pi}{3}$ on the circumference of a circle of radius 20 units. Therefore,

$$\begin{aligned} S &= \left(20\cos\left(\tfrac{4\pi}{3}\right),\ 20\sin\left(\tfrac{4\pi}{3}\right)\right) \\ &= \left(20\cdot\left(-\tfrac{1}{2}\right),\ 20\cdot\left(-\tfrac{\sqrt{3}}{2}\right)\right) \\ &= \left(-10,\ -10\sqrt{3}\right) \end{aligned}$$

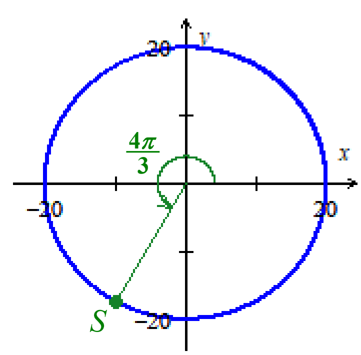


Figure 3