

SOLUTIONS: Week 4 Practice Worksheet**Inverse Trig Functions and Solving Trig Equations**

1. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to **directly communicate** what the given expressions equal.

a. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} \quad (\text{since } \frac{\sqrt{3}}{2} = \sin\left(\frac{\pi}{3}\right) \text{ and } -\frac{\pi}{2} \leq \frac{\pi}{3} \leq \frac{\pi}{2})$$

b. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} \quad (\text{since } \frac{\sqrt{2}}{2} = \cos\left(\frac{\pi}{4}\right) \text{ and } 0 \leq \frac{\pi}{4} \leq \pi)$$

c. $\sin^{-1}\left(-\frac{1}{2}\right)$

$$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6} \quad (\text{since } -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right) \text{ and } -\frac{\pi}{2} \leq -\frac{\pi}{6} \leq \frac{\pi}{2})$$

d. $\cos^{-1}(0)$

$$\cos^{-1}(0) = \frac{\pi}{2} \quad (\text{since } 0 = \cos\left(\frac{\pi}{2}\right) \text{ and } 0 \leq \frac{\pi}{2} \leq \pi)$$

e. $\tan^{-1}(-\sqrt{3})$

$$\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3} \quad (\text{since } -\sqrt{3} = \tan\left(-\frac{\pi}{3}\right) \text{ and } -\frac{\pi}{2} < -\frac{\pi}{3} < \frac{\pi}{2})$$

f. $\tan^{-1}(-1)$

$$\tan^{-1}(-1) = -\frac{\pi}{4} \quad (\text{since } -1 = \tan\left(-\frac{\pi}{4}\right) \text{ and } -\frac{\pi}{2} < -\frac{\pi}{4} < \frac{\pi}{2})$$

2. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to **directly communicate** what the given expressions equal.

a. $\sin\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

$$\begin{aligned}\sin\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) &= \sin\left(\frac{\pi}{3}\right) && \text{(since } \frac{\sqrt{3}}{2} = \sin\left(\frac{\pi}{3}\right) \text{ and } -\frac{\pi}{2} \leq \frac{\pi}{3} \leq \frac{\pi}{2}\text{)} \\ &= \frac{\sqrt{3}}{2} && \text{(since } \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}\text{)}\end{aligned}$$

b. $\cos\left(\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$

$$\begin{aligned}\cos\left(\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right) &= \cos\left(\frac{3\pi}{4}\right) && \text{(since } -\frac{\sqrt{2}}{2} = \cos\left(\frac{3\pi}{4}\right) \text{ and } 0 \leq \frac{3\pi}{4} \leq \pi\text{)} \\ &= -\frac{\sqrt{2}}{2} && \text{(since } \cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}\text{)}\end{aligned}$$

c. $\cos^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right)$

$$\begin{aligned}\cos^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right) &= \cos^{-1}\left(\frac{1}{2}\right) && \text{(since } \cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}\text{)} \\ &= \frac{\pi}{3} && \text{(since } \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \text{ and } 0 \leq \frac{\pi}{3} \leq \pi\text{)}\end{aligned}$$

d. $\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right)$

$$\begin{aligned}\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right) &= \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) && \text{(since } \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}\text{)} \\ &= -\frac{\pi}{3} && \text{(since } \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \text{ and } -\frac{\pi}{2} \leq -\frac{\pi}{3} \leq \frac{\pi}{2}\text{)}\end{aligned}$$

e. $\sin\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

$$\begin{aligned}\sin\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) &= \sin\left(\frac{5\pi}{6}\right) && \text{(since } -\frac{\sqrt{3}}{2} = \cos\left(\frac{5\pi}{6}\right) \text{ and } 0 \leq \frac{5\pi}{6} \leq \pi\text{)} \\ &= \frac{1}{2} && \text{(since } \sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}\text{)}\end{aligned}$$

3. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to **directly communicate** what the given expressions equal.

a. $\sin^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right)$

$$\begin{aligned}\sin^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right) &= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) && \text{(since } \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}\text{)} \\ &= \frac{\pi}{3} && \text{(since } \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \text{ and } -\frac{\pi}{2} \leq \frac{\pi}{3} \leq \frac{\pi}{2}\text{)}\end{aligned}$$

b. $\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right)$

$$\begin{aligned}\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right) &= \tan^{-1}\left(-\sqrt{3}\right) && \text{(since } \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}\text{)} \\ &= -\frac{\pi}{3} && \text{(since } \tan\left(-\frac{\pi}{3}\right) = -\sqrt{3} \text{ and } -\frac{\pi}{2} < -\frac{\pi}{3} < \frac{\pi}{2}\text{)}\end{aligned}$$

c. $\cos^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$

$$\begin{aligned}\cos^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right) &= \cos^{-1}(-1) && \text{(since } \tan\left(\frac{3\pi}{4}\right) = -1\text{)} \\ &= \pi && \text{(since } \cos(\pi) = -1 \text{ and } 0 \leq \pi \leq \pi\text{)}\end{aligned}$$

d. $\tan^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right)$

$$\begin{aligned}\tan^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right) &= \tan^{-1}(1) && \text{(since } \sin\left(\frac{\pi}{2}\right) = 1\text{)} \\ &= \frac{\pi}{4} && \text{(since } \tan\left(\frac{\pi}{4}\right) = 1 \text{ and } -\frac{\pi}{2} \leq \frac{\pi}{4} \leq \frac{\pi}{2}\text{)}\end{aligned}$$

e. $\sin\left(\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$

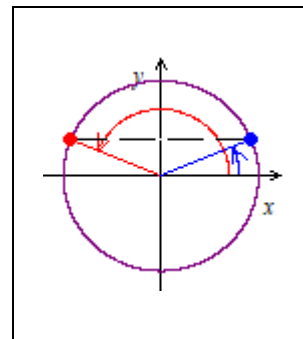
$$\begin{aligned}\sin\left(\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right) &= \sin\left(\frac{\pi}{6}\right) && \text{(since } \frac{1}{\sqrt{3}} = \tan\left(\frac{\pi}{6}\right) \text{ and } -\frac{\pi}{2} < \frac{\pi}{6} < \frac{\pi}{2}\text{)} \\ &= \frac{1}{2} && \text{(since } \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}\text{)}\end{aligned}$$

4. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to **directly communicate** what the given expressions equal.

a. $\sin^{-1}\left(\sin\left(\frac{7\pi}{8}\right)\right)$

$\frac{7\pi}{8}$ isn't a "friendly angle" so we aren't familiar with its sine so we need to rely on our conceptual understanding of the sine function along with the symmetry of a circle:

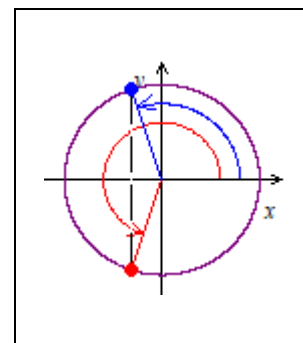
$$\begin{aligned} \sin^{-1}\left(\sin\left(\frac{7\pi}{8}\right)\right) &= \sin^{-1}\left(\sin\left(\frac{\pi}{8}\right)\right) && \text{(using the symmetry of a circle)} \\ &= \frac{\pi}{8} && \text{(since } -\frac{\pi}{2} \leq \frac{\pi}{8} \leq \frac{\pi}{2} \text{)} \end{aligned}$$



b. $\cos^{-1}\left(\cos\left(\frac{7\pi}{5}\right)\right)$

As in part (a) above, $\frac{7\pi}{5}$ isn't a "friendly angle" so we need to rely on our conceptual understanding of the cosine function along with the symmetry of a circle:

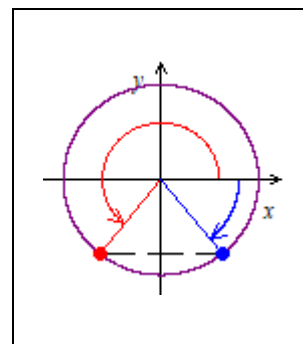
$$\begin{aligned} \cos^{-1}\left(\cos\left(\frac{7\pi}{5}\right)\right) &= \cos^{-1}\left(\cos\left(\frac{3\pi}{5}\right)\right) && \text{(using the symmetry of a circle)} \\ &= \frac{3\pi}{5} && \text{(since } 0 \leq \frac{3\pi}{5} \leq \pi \text{)} \end{aligned}$$



c. $\sin^{-1}\left(\sin\left(\frac{9\pi}{7}\right)\right)$

As in parts (a) and (b) above, $\frac{9\pi}{7}$ isn't a "friendly angle" so we need to rely on our conceptual understanding of the sine function along with the symmetry of a circle:

$$\begin{aligned} \sin^{-1}\left(\sin\left(\frac{9\pi}{7}\right)\right) &= \sin^{-1}\left(\sin\left(-\frac{2\pi}{7}\right)\right) && \text{(using the symmetry of a circle)} \\ &= -\frac{2\pi}{7} && \text{(since } -\frac{\pi}{2} \leq -\frac{2\pi}{7} \leq \frac{\pi}{2} \text{)} \end{aligned}$$



5. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to **directly communicate** what the given expressions equal.

a. $\sin(t) = -\frac{\sqrt{2}}{2}$

$$\sin(t) = -\frac{\sqrt{2}}{2}$$

$$\Rightarrow t = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) + 2k\pi \quad \text{or} \quad t = \pi - \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow t = -\frac{\pi}{4} + 2k\pi \quad \text{or} \quad t = \pi - \left(-\frac{\pi}{4}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow t = -\frac{\pi}{4} + 2k\pi \quad \text{or} \quad t = \frac{5\pi}{4} + 2k\pi, \quad k \in \mathbb{Z}$$

(In the step indicated by a red arrow (\Rightarrow) we've used the identity $\sin(t) = \sin(\pi - t)$ in order to generate the second family of solutions. Instead of using the inverse trig function in this step, you might choose to obtain the step indicated by a pink arrow (\Rightarrow) by utilizing your awareness of these two facts: $\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ and $\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ although using "awareness" might inspire you to use " $\frac{7\pi}{4}$ " instead of " $-\frac{\pi}{4}$ " for the first family of solutions, which will lead you to $t = \frac{7\pi}{4} + 2k\pi$ instead of $t = -\frac{\pi}{4} + 2k\pi$ for the second family of solutions)

b. $2 \cos(x) - \sqrt{3} = 0$

$$2 \cos(x) - \sqrt{3} = 0$$

$$\Rightarrow 2 \cos(x) = \sqrt{3}$$

$$\Rightarrow \cos(x) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + 2k\pi \quad \text{or} \quad x = -\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = -\frac{\pi}{6} + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = -\frac{\pi}{6} + 2k\pi, \quad k \in \mathbb{Z}$$

(In the step indicated by a red arrow (\Rightarrow) we've used the identity $\cos(x) = -\cos(x)$ in order to generate the second family of solutions. Instead of using the inverse trig function in this step, you might choose to obtain the step indicated by a pink arrow (\Rightarrow) by utilizing your awareness of these two facts: $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$, although using "awareness" might inspire you to use " $\frac{11\pi}{6}$ " instead of " $-\frac{\pi}{6}$ " for the second family of solutions, which will lead you to $x = \frac{11\pi}{6} + 2k\pi$ instead of $x = -\frac{\pi}{6} + 2k\pi$.)

6. Find the solutions on the interval $[0, 2\pi)$ for the equations below; provide *exact* solutions.

a. $\cos(t) = -\frac{1}{2}$

$$\cos(t) = -\frac{1}{2}$$

$$\Rightarrow t = \cos^{-1}\left(-\frac{1}{2}\right) + 2k\pi \quad \text{or} \quad x = -\cos^{-1}\left(-\frac{1}{2}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow t = \frac{2\pi}{3} + 2k\pi \quad \text{or} \quad x = -\frac{2\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$$

Now we need to substitute particular values of k in order to determine which solutions fall on the interval $[0, 2\pi)$:

$$\begin{aligned} k = -1: \quad x &= \frac{2\pi}{3} + 2(-1)\pi & \text{or} & \quad x = -\frac{2\pi}{3} + 2(-1)\pi \\ &= \frac{2\pi}{3} - \frac{6\pi}{3} & & \quad = -\frac{2\pi}{3} - \frac{6\pi}{3} \\ &= -\frac{4\pi}{3} & & \quad = -\frac{8\pi}{3} \end{aligned}$$

Since both of these values are negative, they aren't in the interval $[0, 2\pi)$. There's no reason to try smaller values of k since they'll produce yet smaller solutions which will certainly be outside the interval.

$$\begin{aligned} k = 0: \quad x &= \frac{2\pi}{3} + 2(0)\pi & \text{or} & \quad x = -\frac{2\pi}{3} + 2(0)\pi \\ &= \frac{2\pi}{3} & & \quad = -\frac{2\pi}{3} \end{aligned}$$

Only $\frac{2\pi}{3}$ is in the given interval.

$$\begin{aligned} k = 1: \quad x &= \frac{2\pi}{3} + 2(1)\pi & \text{or} & \quad x = -\frac{2\pi}{3} + 2(1)\pi \\ &= \frac{2\pi}{3} + 2\pi & & \quad = -\frac{2\pi}{3} + \frac{6\pi}{3} > 2\pi \\ &> 2\pi & & \quad = \frac{4\pi}{3} \end{aligned}$$

Only $\frac{2\pi}{3}$ is in the given interval. There's no reason to try larger values of k since they'll produce yet greater solutions which will certainly be outside the given interval.

Therefore, the solution set for $\cos(t) = -\frac{1}{2}$ on the interval $[0, 2\pi)$ is $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$.

$$\text{b. } \frac{\sin(x)}{2} - \frac{\sqrt{3}}{4} = 0$$

$$\frac{\sin(x)}{2} - \frac{\sqrt{3}}{4} = 0$$

$$2 \cdot \frac{\sin(x)}{2} = \frac{\sqrt{3}}{4} \cdot 2$$

$$\sin(x) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow t = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + 2k\pi \quad \text{or} \quad t = \pi - \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow t = \frac{\pi}{3} + 2k\pi \quad \text{or} \quad t = \pi - \left(\frac{\pi}{3}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow t = \frac{\pi}{3} + 2k\pi \quad \text{or} \quad t = \frac{2\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$$

Now we need to substitute particular values of k in order to determine which solutions fall on the interval $[0, 2\pi)$:

$$\begin{aligned} k = -1: \quad x &= \frac{\pi}{3} + 2(-1)\pi & \text{or} & \quad x = \frac{2\pi}{3} + 2(-1)\pi \\ &= \frac{\pi}{3} - \frac{6\pi}{3} & & \quad = \frac{2\pi}{3} - \frac{6\pi}{3} \\ &= -\frac{5\pi}{3} & & \quad = -\frac{4\pi}{3} \end{aligned}$$

Since both of these values are negative, they aren't in the interval $[0, 2\pi)$. There's no reason to try smaller values of k since they'll produce yet smaller solutions which will certainly be outside the interval.

$$\begin{aligned} k = 0: \quad x &= \frac{\pi}{3} + 2(0)\pi & \text{or} & \quad x = \frac{2\pi}{3} + 2(0)\pi \\ &= \frac{\pi}{3} & & \quad = \frac{2\pi}{3} \end{aligned}$$

Both $\frac{\pi}{3}$ and $\frac{2\pi}{3}$ are in the given interval.

$$\begin{aligned} k = 1: \quad x &= \frac{\pi}{3} + 2(1)\pi & \text{or} & \quad x = \frac{2\pi}{3} + 2(1)\pi \\ &= \frac{\pi}{3} + 2\pi > 2\pi & & \quad = \frac{2\pi}{3} + 2\pi > 2\pi \end{aligned}$$

Both of these solutions are outside the given interval. There's no reason to try larger values of k since they'll produce yet greater solutions which will certainly be outside the given interval.

Therefore, the solution set for $\frac{\sin(x)}{2} - \frac{\sqrt{3}}{4} = 0$ on the interval $[0, 2\pi)$ is $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$.

7. Find *all* of the solutions to the equations below; provide *exact* solutions.

a. $\sin(6t) = -\frac{\sqrt{3}}{2}$

$$\sin(6t) = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow 6t = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + 2k\pi \quad \text{or} \quad 6t = \pi - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow 6t = -\frac{\pi}{3} + 2k\pi \quad \text{or} \quad 6t = \pi - \left(-\frac{\pi}{3}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \frac{6t}{6} = \frac{-\frac{\pi}{3}}{6} + \frac{2k\pi}{6} \quad \text{or} \quad \frac{6t}{6} = \frac{\frac{4\pi}{3}}{6} + \frac{2k\pi}{6}, \quad k \in \mathbb{Z}$$

$$\Rightarrow t = -\frac{\pi}{18} + \frac{k\pi}{3} \quad \text{or} \quad t = \frac{2\pi}{9} + \frac{k\pi}{3}, \quad k \in \mathbb{Z}$$

b. $5 + 4\cos(2\theta) = 1$

$$5 + 4\cos(2\theta) = 1$$

$$\Rightarrow 4\cos(2\theta) = -4$$

$$\Rightarrow \cos(2\theta) = -1$$

$$\Rightarrow 2\theta = \cos^{-1}(-1) + 2k\pi, \quad k \in \mathbb{Z}$$

(there's only one "family" of solutions since the cosine function achieves the output -1 only once in each period)

$$\Rightarrow 2\theta = \pi + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \frac{2\theta}{2} = \frac{\pi}{2} + \frac{2k\pi}{2}, \quad k \in \mathbb{Z}$$

$$\Rightarrow \theta = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}$$

c. $16 \cos(4x) + 11 = 3$

$$16 \cos(4x) + 11 = 3$$

$$\Rightarrow 16 \cos(4x) = -8$$

$$\Rightarrow \cos(4x) = \frac{-8}{16} = -\frac{1}{2}$$

$$\Rightarrow 4x = \cos^{-1}\left(-\frac{1}{2}\right) + 2k\pi \quad \text{or} \quad 4x = -\cos^{-1}\left(-\frac{1}{2}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow 4x = \frac{2\pi}{3} + 2k\pi \quad \text{or} \quad 4x = -\frac{2\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \frac{4x}{4} = \frac{\frac{2\pi}{3}}{4} + \frac{2k\pi}{4} \quad \text{or} \quad \frac{4x}{4} = \frac{-\frac{2\pi}{3}}{4} + \frac{2k\pi}{4}, \quad k \in \mathbb{Z}$$

$$\Rightarrow x = \frac{2\pi}{3 \cdot 4} + \frac{2k\pi}{4} \quad \text{or} \quad x = -\frac{2\pi}{3 \cdot 4} + \frac{k\pi}{2}, \quad k \in \mathbb{Z}$$

$$\Rightarrow x = \frac{\pi}{6} + \frac{k\pi}{2} \quad \text{or} \quad x = -\frac{\pi}{6} + \frac{k\pi}{2}, \quad k \in \mathbb{Z}$$

d. $16 - 24 \sin(8t) = 4$

$$16 - 24 \sin(8t) = 4$$

$$\Rightarrow -24 \sin(8t) = -12$$

$$\Rightarrow \sin(8t) = \frac{-12}{-24} = \frac{1}{2}$$

$$\Rightarrow 8t = \sin^{-1}\left(\frac{1}{2}\right) + 2k\pi \quad \text{or} \quad 8t = \pi - \sin^{-1}\left(\frac{1}{2}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow 8t = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad 8t = \pi - \frac{\pi}{6} + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \frac{8t}{8} = \frac{\frac{\pi}{6}}{8} + \frac{2k\pi}{8} \quad \text{or} \quad \frac{8t}{8} = \frac{\frac{5\pi}{6}}{8} + \frac{2k\pi}{8}, \quad k \in \mathbb{Z}$$

$$\Rightarrow t = \frac{\pi}{48} + \frac{k\pi}{4} \quad \text{or} \quad t = \frac{5\pi}{48} + \frac{k\pi}{4}, \quad k \in \mathbb{Z}$$

8. Find the solutions on the interval $[0, 2\pi)$ to following equations.

a. $5 + 4\cos(2\theta) = 1$

$$5 + 4\cos(2\theta) = 1$$

$$\Rightarrow 4\cos(2\theta) = -4$$

$$\Rightarrow \cos(2\theta) = -1$$

$$\Rightarrow 2\theta = \cos^{-1}(-1) + 2k\pi, \quad k \in \mathbb{Z}$$

(there's only one "family" of solutions since the cosine function achieves the output -1 only once in each period)

$$\Rightarrow 2\theta = \pi + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \frac{2\theta}{2} = \frac{\pi}{2} + \frac{2k\pi}{2}, \quad k \in \mathbb{Z}$$

$$\Rightarrow \theta = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}$$

Now we need to substitute particular values of k in order to determine which solutions fall on the interval $[0, 2\pi)$:

$$k = -1: \quad \theta = \frac{\pi}{2} + (-1) \cdot \pi = -\frac{\pi}{2} < 0 \text{ so } -\frac{\pi}{2} \notin [0, 2\pi).$$

We could try smaller values of k but it should be clear that since $k = -1$ produced a value of θ that's too small, smaller values of k will produce even smaller values of θ so they won't produce solutions in the given interval so there's no need in trying smaller values of k .

$$k = 0: \quad \theta = \frac{\pi}{2} + 0 \cdot \pi = \frac{\pi}{2} \in [0, 2\pi), \text{ so } \frac{\pi}{2} \text{ is a solution in the given interval.}$$

$$k = 1: \quad \theta = \frac{\pi}{2} + 1 \cdot \pi = \frac{3\pi}{2} \in [0, 2\pi), \text{ so } \frac{3\pi}{2} \text{ is a solution in the given interval.}$$

$$k = 2: \quad \theta = \frac{\pi}{2} + 2 \cdot \pi = \frac{5\pi}{2} > 2\pi \text{ so } \frac{5\pi}{2} \notin [0, 2\pi).$$

We could try larger values of k but it should be clear that since $k = 2$ produced a value θ that's too large, larger values of k will produce even larger values of θ , so they won't produce solutions in the given interval so there's no need in trying larger values of k .

Therefore, the solution set to $5 + 4\cos(2\theta) = 1$ on the interval $[0, 2\pi)$ is $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$.

b. $4 - 6\sin(2x) = 7$

$$\begin{aligned}
 &4 - 6\sin(2x) = 7 \\
 \Rightarrow & -6\sin(2x) = 3 \\
 \Rightarrow & \sin(2x) = \frac{3}{-6} = -\frac{1}{2} \\
 \Rightarrow & 2x = \sin^{-1}\left(-\frac{1}{2}\right) + 2k\pi \quad \text{or} \quad 2x = \pi - \sin^{-1}\left(-\frac{1}{2}\right) + 2k\pi, \quad k \in \mathbb{Z} \\
 \Rightarrow & 2x = -\frac{\pi}{6} + 2k\pi \quad \text{or} \quad 2x = \pi - \left(-\frac{\pi}{6}\right) + 2k\pi, \quad k \in \mathbb{Z} \\
 \Rightarrow & \frac{2x}{2} = \frac{-\frac{\pi}{6}}{2} + \frac{2k\pi}{2} \quad \text{or} \quad \frac{2x}{2} = \frac{\frac{7\pi}{6}}{2} + \frac{2k\pi}{2}, \quad k \in \mathbb{Z} \\
 \Rightarrow & x = -\frac{\pi}{12} + k\pi \quad \text{or} \quad x = \frac{7\pi}{12} + k\pi, \quad k \in \mathbb{Z}
 \end{aligned}$$

Now we need to substitute particular values of k into these equations to determine which solutions fall on the interval $[0, 2\pi)$:

$$\begin{aligned}
 k = -1: \quad x &= -\frac{\pi}{12} + (-1)\pi \quad \text{or} \quad x = \frac{7\pi}{12} + (-1)\pi \\
 &= -\frac{\pi}{12} - \frac{12\pi}{12} \quad \quad \quad = \frac{7\pi}{12} - \frac{12\pi}{12} \\
 &= -\frac{13\pi}{12} < 0 \quad \quad \quad = -\frac{5\pi}{12} < 0
 \end{aligned}$$

Since both of these values are negative, they aren't in the interval $[0, 2\pi)$. We could try smaller values of k but it should be clear that since $k = -1$ resulted in a values of α that are too small, smaller values of k will produces even smaller values of α so they won't produce solutions in the given interval so there's no need in trying smaller values of k .

$$\begin{aligned}
 k = 0: \quad x &= -\frac{\pi}{12} + (0)\pi \quad \text{or} \quad x = \frac{7\pi}{12} + (0)\pi \\
 &= -\frac{\pi}{12} < 0 \quad \quad \quad = \frac{7\pi}{12}
 \end{aligned}$$

Only $\frac{7\pi}{12}$ is in the given interval.

$$\begin{aligned}
 k = 1: \quad x &= -\frac{\pi}{12} + (1)\pi \quad \text{or} \quad x = \frac{7\pi}{12} + (1)\pi \\
 &= -\frac{\pi}{12} + \frac{12\pi}{12} \quad \quad \quad = \frac{7\pi}{12} + \frac{12\pi}{12} \\
 &= \frac{11\pi}{12} \quad \quad \quad = \frac{19\pi}{12}
 \end{aligned}$$

Both $\frac{11\pi}{12}$ and $\frac{19\pi}{12}$ are in the given interval.

$$\begin{aligned}k = 2: \quad x &= -\frac{\pi}{12} + (2)\pi & \text{or} & \quad x = \frac{7\pi}{12} + (2)\pi \\ &= -\frac{\pi}{12} + \frac{24\pi}{12} & & \quad = \frac{7\pi}{12} + \frac{24\pi}{12} \\ &= \frac{23\pi}{12} & & \quad = \frac{31\pi}{12} > 0\end{aligned}$$

Only $\frac{23\pi}{12}$ is in the given interval.

$$\begin{aligned}k = 3: \quad x &= -\frac{\pi}{12} + (3)\pi & \text{or} & \quad x = \frac{7\pi}{12} + (3)\pi \\ &= -\frac{\pi}{12} + \frac{36\pi}{12} & & \quad = \frac{7\pi}{12} + \frac{36\pi}{12} \\ &= \frac{35\pi}{12} > 2\pi & & \quad = \frac{43\pi}{12} > 2\pi\end{aligned}$$

Since both of these values are greater than 2π , they aren't in the interval $[0, 2\pi)$. We could try larger values of k but it should be clear that since $k = 3$ resulted in a values of x that are too larger, larger values of k will produces even larger values of x so they won't produce solutions in the given interval so there's no need in trying larger values of k .

Thus, the solution set to $4 - 6\sin(2x) = 7$ on the interval $[0, 2\pi)$ is

$$\left\{ \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12} \right\}.$$

c. $6\sqrt{2} \cos(3\alpha) + 10 = 4$

$$6\sqrt{2} \cos(3\alpha) + 10 = 4$$

$$\Rightarrow 6\sqrt{2} \cos(3\alpha) = -6$$

$$\Rightarrow \cos(3\alpha) = \frac{-6}{6\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow 3\alpha = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) + 2k\pi \quad \text{or} \quad 3\alpha = -\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow 3\alpha = \frac{3\pi}{4} + 2k\pi \quad \text{or} \quad 3\alpha = -\frac{3\pi}{4} + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \frac{3\alpha}{3} = \frac{\frac{3\pi}{4} + 2k\pi}{3} \quad \text{or} \quad \frac{3\alpha}{3} = \frac{-\frac{3\pi}{4} + 2k\pi}{3}, \quad k \in \mathbb{Z}$$

$$\Rightarrow \alpha = \frac{3\pi}{4 \cdot 3} + \frac{2k\pi}{3} \quad \text{or} \quad \alpha = -\frac{3\pi}{4 \cdot 3} + \frac{2k\pi}{3}, \quad k \in \mathbb{Z}$$

$$\Rightarrow \alpha = \frac{\pi}{4} + \frac{2k\pi}{3} \quad \text{or} \quad \alpha = -\frac{\pi}{4} + \frac{2k\pi}{3}, \quad k \in \mathbb{Z}$$

Now we need to substitute particular values of k into these equations to determine which solutions fall on the interval $[0, 2\pi)$:

$$\begin{aligned} k = -1: \quad \alpha &= \frac{\pi}{4} + \frac{2(-1)\pi}{3} & \text{or} & \quad \alpha = -\frac{\pi}{4} + \frac{2(-1)\pi}{3} \\ &= \frac{3\pi}{12} - \frac{8\pi}{12} & & \quad = -\frac{3\pi}{12} - \frac{8\pi}{12} \\ &= -\frac{5\pi}{12} & & \quad = -\frac{11\pi}{12} \end{aligned}$$

Since both of these values are negative, they aren't in the interval $[0, 2\pi)$.

We could try smaller values of k but it should be clear that since $k = -1$ resulted in a values of α that are too small, smaller values of k will produces even smaller values of α so they won't produce solutions in the given interval so there's no need in trying smaller values of k .

$$\begin{aligned} k = 0: \quad \alpha &= \frac{\pi}{4} + \frac{2 \cdot 0 \cdot \pi}{3} & \text{or} & \quad \alpha = -\frac{\pi}{4} + \frac{2 \cdot 0 \cdot \pi}{3} \\ &= \frac{\pi}{4} & & \quad = -\frac{\pi}{4} \end{aligned}$$

Only $\frac{\pi}{4}$ is in the given interval.

$$\begin{aligned} k = 1: \quad \alpha &= \frac{\pi}{4} + \frac{2 \cdot 1 \cdot \pi}{3} & \text{or} & \quad \alpha = -\frac{\pi}{4} + \frac{2 \cdot 1 \cdot \pi}{3} \\ &= \frac{3\pi}{12} + \frac{8\pi}{12} & & \quad = -\frac{3\pi}{12} + \frac{8\pi}{12} \\ &= \frac{11\pi}{12} & & \quad = \frac{5\pi}{12} \end{aligned}$$

Both $\frac{11\pi}{12}$ and $\frac{5\pi}{12}$ are in the given interval.

$$\begin{aligned}
 k = 2: \quad \alpha &= \frac{\pi}{4} + \frac{2 \cdot 2 \cdot \pi}{3} & \text{or} & \quad \alpha = -\frac{\pi}{4} + \frac{2 \cdot 2 \cdot \pi}{3} \\
 &= \frac{3\pi}{12} + \frac{16\pi}{12} & & \quad = -\frac{3\pi}{12} + \frac{16\pi}{12} \\
 &= \frac{19\pi}{12} & & \quad = \frac{13\pi}{12}
 \end{aligned}$$

Both $\frac{19\pi}{12}$ and $\frac{13\pi}{12}$ are in the given interval.

$$\begin{aligned}
 k = 3: \quad \alpha &= \frac{\pi}{4} + \frac{2 \cdot 3 \cdot \pi}{3} & \text{or} & \quad \alpha = -\frac{\pi}{4} + \frac{2 \cdot 3 \cdot \pi}{3} \\
 &= \frac{\pi}{4} + \frac{8\pi}{4} & & \quad = -\frac{\pi}{4} + \frac{8\pi}{4} \\
 &= \frac{9\pi}{4} > 2\pi & & \quad = \frac{7\pi}{4}
 \end{aligned}$$

Only $\frac{7\pi}{4}$ is in the given interval.

$$\begin{aligned}
 k = 4: \quad \alpha &= \frac{\pi}{4} + \frac{2 \cdot 4 \cdot \pi}{3} & \text{or} & \quad \alpha = -\frac{\pi}{4} + \frac{2 \cdot 4 \cdot \pi}{3} \\
 &= \frac{3\pi}{12} + \frac{32\pi}{12} & & \quad = -\frac{3\pi}{12} + \frac{32\pi}{12} \\
 &= \frac{35\pi}{12} > 2\pi & & \quad = \frac{29\pi}{12} > 2\pi
 \end{aligned}$$

Since both of these values are greater than 2π , they aren't in the interval $[0, 2\pi)$. We could try larger values of k but it should be clear that since $k = 4$ resulted in a values of α that are too larger, larger values of k will produces even larger values of α so they won't produce solutions in the given interval so there's no need in trying larger values of k .

Thus, the solution set to $6\sqrt{2} \cos(3\alpha) + 10 = 4$ on the interval $[0, 2\pi)$ is $\left\{ \frac{\pi}{4}, \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}, \frac{7\pi}{4} \right\}$.