Practice Worksheet: Proving Trig Identities

1. Prove the following identities. (Be sure to organize your proof as shown in the Online Lecture Notes and class notes videos.) This means that you should start your proof by writing one side of the identity and then use equal signs between equivalent expressions until you obtain the other side of the identity. You should only include one step on each line and you should align your equal signs on the left of each step. Compare your proofs with those given in the solutions to make sure that you are using the correct organization and technique.)

a. $\tan(x)\sec(x) = \sin(x)\sec^2(x)$

b. $\csc(t) - \sin(t) = \cot(t)\cos(t)$

c. $\dfrac{\sec(\theta)}{\sin(\theta)} - \tan(\theta) = \cot(\theta)$

d. $\tan(\theta) = \dfrac{\csc(\theta)}{\cos(\theta)} - \cot(\theta)$

e. $2\sec^2(x) = \dfrac{1}{1 - \sin(x)} - \dfrac{1}{1 + \sin(x)}$

f. $\dfrac{1}{1 - \cos(x)} - \dfrac{1}{1 + \cos(x)} = 2\cot(x)\csc(x)$

g. $\sec(\theta) + \tan(\theta) = \dfrac{\cos(\theta)}{1 - \sin(\theta)}$

h. $\cot(A) = \csc(A)\sec(A) - \tan(A)$

Practice Worksheet: Some Important Identities

1. Use a **sum-of-angles** or **difference-of-angles identity** to calculate the *exact value* of each of the following. (These identities are included on the Identities and Formulas Reference Sheet that will be provided to you during the Final Exam.)

a. $\sin(165^\circ)$

b. $\cos\left(\frac{13\pi}{12}\right)$

c. $\tan\left(\frac{17\pi}{12}\right)$

2. In order to get familiar with the **sum-of-angles**, **difference-of-angles**, **double-angle** and **half-angle identities**, we'll use these identities to calculate some "friendly" sine and cosine values – so we already know these sine and cosine values and we'll verify that the identities lead us to these values. To help explain the activity part (a) has been worked out for you, and part (b) has been started. (Theses identities are included on the Identities and Formulas Reference Sheet that will be provided to you during the Final Exam.)

a. Find $\sin\left(\frac{\pi}{2}\right)$ using the fact that $\frac{\pi}{2} = \frac{\pi}{6} + \frac{\pi}{3}$.

$$\begin{aligned}\sin\left(\frac{\pi}{2}\right) &= \sin\left(\frac{\pi}{6} + \frac{\pi}{3}\right) \\ &= \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{3}\right) \\ &= \frac{1}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} \\ &= \frac{1}{4} + \frac{3}{4} \\ &= 1\end{aligned}$$

We know that $\sin\left(\frac{\pi}{2}\right) = 1$ so the identity gave us the correct value.

b. Find $\cos(\pi)$ using the fact that $\pi = \frac{5\pi}{6} + \frac{\pi}{6}$.

c. Find $\sin\left(\frac{2\pi}{3}\right)$ using the fact that $\frac{2\pi}{3} = 2\cdot\frac{\pi}{3}$. (hint: use a double-angle identity)

d. Find $\cos\left(\frac{\pi}{3}\right)$ using the fact that $\frac{\pi}{3} = 2 \cdot \frac{\pi}{6}$. (hint: use a double-angle identity)

e. Find $\sin\left(\frac{\pi}{4}\right)$ using the fact that $\frac{\pi}{4} = \frac{\pi/2}{2}$. (Hint: use a half-angle identity)

f. Find $\cos(30^\circ)$ using the fact that $30^\circ = \frac{60^\circ}{2}$. (hint: use a half-angle identity)

3. Suppose that $\sin(\alpha) = -\frac{\sqrt{65}}{9}$ and $\pi < \alpha < \frac{3\pi}{2}$. Calculate the *exact value* of each of the following using an appropriate **double-angle** or **half-angle identity**. (These identities are included on the Identities and Formulas Reference Sheet.)

a. $\sin(2\alpha)$. [Hint: first find $\cos(\alpha)$]

b. $\cos(2\alpha)$.	**c.** $\sin\left(\frac{\alpha}{2}\right)$.

4. Suppose that $\cos(\theta) = \frac{7}{10}$ and $\frac{3\pi}{2} < \theta < 2\pi$. Calculate the *exact value* of each of the following using an appropriate **double-angle** or **half-angle identity**. (These identities are included on the Identities and Formulas Reference Sheet.)

a. $\sin(2\theta)$. [Hint: first find $\sin(\theta)$]

b. $\cos(2\theta)$.

c. $\cos\left(\frac{\theta}{2}\right)$.

5. Suppose that $\sin(x) = \frac{9}{11}$ and $\frac{\pi}{2} < x < \pi$. Calculate the *exact value* of each of the following using an appropriate **double-angle** or **half-angle identity**. (These identities are included on the Identities and Formulas Reference Sheet.)

a. $\sin(2x)$. [Hint: first find $\cos(x)$]

b. $\cos(2x)$.

c. $\sin\left(\frac{x}{2}\right)$.

6. Prove the following identities using the double-angle identities for sine and cosine included on the Identities and Formulas Reference Sheet. (Be sure to organize your proof as shown in the Online Lecture Notes and class notes videos.)

a. $\tan(2x) = \dfrac{2\tan(x)}{1-\tan^2(x)}$ (this is known as the double-angle identity for tangent; to prove it, start with the left side and use the double-angle identities for sine and cosine; for cosine, use $\cos(2x) = \cos^2(x) - \sin^2(x)$; a “trick” here is to introduce a term that will create the denominator that you need)

b. $\dfrac{1-\cos(2t)}{\sin(2t)} = \tan(t)$