SOLUTIONS: Week 5 Practice Worksheet

Some Additional Practice for the Midterm Exam

1. a. Convert 24° into radians.

$$24^\circ = 24^\circ \cdot \frac{2\pi \text{ rad.}}{360^\circ} = \frac{2\pi}{15} \text{ rad.}$$

b. Convert $\frac{3}{2}$ radians into degrees.

$$\frac{3}{2}\text{ rad.} = \frac{3}{2}\text{ rad.} \cdot \frac{360^\circ}{2\pi \text{ rad.}} = \frac{540^\circ}{2\pi} = \frac{270^\circ}{\pi}$$

2. Find the arc-length spanned by an angle measuring 75° on a circle of radius 30 feet.

Before we can use the formula $s = r|\theta|$, we need to convert the angle 75° into radians:

$$75^{\not\circ} \cdot \frac{\pi \text{ rad}}{180^{\not\circ}} = \frac{75\pi}{180}\text{ rad} = \frac{5\pi}{12}\text{ rad}$$

Now we can find the desired arc-length:

$$s = r\theta = (30\text{ feet}) \cdot \frac{5\pi}{12} = \frac{150\pi}{12}\text{ feet} = \frac{25\pi}{2}\text{ feet}$$

and we see that the arc-length spanned by an angle measuring 75° on a circle of radius 30 feet is $\frac{25\pi}{2}$ feet.

3. Evaluate the following expressions:

a. $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

b. $\cos(45^\circ) = \frac{\sqrt{2}}{2}$

c. $\sin(330^\circ) = -\sin(30^\circ)$ (since 30° is the reference angle for 330° and sine is negative in the 4^{th} quadrant)

$= -\frac{1}{2}$

d. $\cos\left(\frac{\pi}{2}\right) = 0$

e. $\cos\left(\frac{2\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right)$ (since $\frac{\pi}{3}$ is the reference angle for $\frac{2\pi}{3}$ and cosine is negative in the 2^{nd} quadrant)

$= -\frac{1}{2}$

f. $\sin\left(\frac{5\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right)$ (since $\frac{\pi}{4}$ is the reference angle for $\frac{5\pi}{4}$ and sine is negative in the 3^{rd} quadrant)

$= -\frac{\sqrt{2}}{2}$

g. $\sin\left(\frac{7\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right)$ (since $\frac{\pi}{6}$ is the reference angle for $\frac{7\pi}{6}$ and sine is negative in the 3^{rd} quadrant)

$= -\frac{1}{2}$

h. $\cos\left(\frac{11\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right)$ (since $\frac{\pi}{6}$ is the reference angle for $\frac{11\pi}{6}$ and cosine is positive in the 4^{th} quadrant)

$= \frac{\sqrt{3}}{2}$

i. $\cos\left(\frac{17\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right)$

$= -\frac{\sqrt{3}}{2}$

(since $\frac{\pi}{6}$ is the reference angle for $\frac{17\pi}{6}$ and cosine is negative in the 2^{nd} quadrant)

k. $\sin\left(\frac{16\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right)$

$= -\frac{\sqrt{3}}{2}$

(since $\frac{\pi}{3}$ is the reference angle for $\frac{16\pi}{3}$ and only sine is negative in the 3^{rd} quadrant)

l. $\tan\left(\frac{3\pi}{4}\right) = \frac{\sin\left(\frac{\pi}{4}\right)}{-\cos\left(\frac{\pi}{4}\right)}$

$= \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$

$= -1$

(since $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$, since $\frac{\pi}{4}$ is the reference angle for $\frac{3\pi}{4}$, and since only cosine is negative in the 2^{nd} quadrant)

m. $\cot\left(\frac{4\pi}{3}\right) = \frac{-\cos\left(\frac{\pi}{3}\right)}{-\sin\left(\frac{\pi}{3}\right)}$

$= \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$= \frac{1}{\sqrt{3}}$

(since $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$, since $\frac{\pi}{3}$ is the reference angle for $\frac{4\pi}{3}$, and since and both sine and cosine are negative in the 3^{rd} quad.)

n. $\csc(\pi) = \frac{1}{\sin(\pi)}$

Since $\sin(\pi) = 0$, $\csc(\pi)$ involves division by 0 but division by 0 is undefined so **$\csc(\pi)$ is undefined**.

o. $\sec\left(360^\circ\right) = \frac{1}{\cos\left(360^\circ\right)}$

$= \frac{1}{1}$

$= 1$

4. a. Use the sine and cosine functions to find the coordinates of the point P in Figure 1a that is specified by $\frac{5\pi}{3}$ on the circumference of a circle of radius 10 units. **Clearly show your use of the sine and cosine functions**.

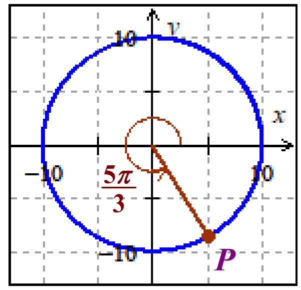


Figure 1a

Coordinates of P:

$$\left(10\cos\left(\tfrac{5\pi}{3}\right),\ 10\sin\left(\tfrac{5\pi}{3}\right)\right) = \left(10\cdot\left(\tfrac{1}{2}\right),\ 10\cdot\left(-\tfrac{\sqrt{3}}{2}\right)\right)$$
$$= \left(5,\ -5\sqrt{3}\right)$$

b. Use the sine and cosine functions to find the coordinates of the point Q in Figure 1b that is specified by $\frac{7\pi}{6}$ on the circumference of a circle of radius 3 units. **Clearly show your use of the sine and cosine functions**.

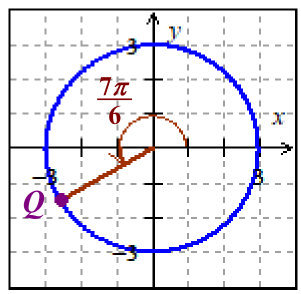


Figure 1b

Coordinates of Q:

$$\left(3\cos\left(\tfrac{7\pi}{6}\right),\ 3\sin\left(\tfrac{7\pi}{6}\right)\right) = \left(3\cdot\left(-\tfrac{\sqrt{3}}{2}\right),\ 3\cdot\left(-\tfrac{1}{2}\right)\right)$$
$$= \left(-\tfrac{3\sqrt{3}}{2},\ -\tfrac{3}{2}\right)$$

5. If $\sin(\theta) = \frac{3}{4}$ and $\frac{\pi}{2} < \theta < \pi$, find the exact value of the following expressions.

a. $\cos(\theta)$

We can use the Pythagorean identity to find $\cos(\theta)$:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\Rightarrow \left(\tfrac{3}{4}\right)^2 + \cos^2(\theta) = 1$$

$$\Rightarrow \cos^2(\theta) = 1 - \tfrac{9}{16}$$

$$\Rightarrow \cos^2(\theta) = \tfrac{7}{16}$$

$$\Rightarrow \cos(\theta) = -\tfrac{\sqrt{7}}{4}$$

(note that we take the negative square root since cosine is negative in the second quadrant)

b. $\sin(\theta + 2\pi) = \sin(\theta)$ (since the period of $y = \sin(\theta)$ is 2π)

$$= \tfrac{3}{4}$$

c. $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

$$= \frac{3/4}{-\sqrt{7}/4}$$

$$= -\frac{3}{\sqrt{7}}$$

d. $\sec(\theta) = \frac{1}{\cos(\theta)}$

$$= \frac{1}{-\sqrt{7}/4}$$

$$= -\frac{4}{\sqrt{7}}$$

e. $\csc(\theta) = \frac{1}{\sin(\theta)}$

$$= \frac{1}{3/4}$$

$$= \frac{4}{3}$$

6. Find *all* of the solutions to the following equations:

a. $3\sin(x) + 4 = 5$

$$3\sin(x) + 4 = 5$$

$$\Rightarrow \quad 3\sin(x) = 1$$

$$\Rightarrow \quad \sin(x) = \tfrac{1}{3}$$

(Since this sine value isn't one of the "friendly" values that we've studied, we'll need to use the inverse-sine function)

$$\Rightarrow \; x = \sin^{-1}\left(\tfrac{1}{3}\right) + 2k\pi \quad \text{or} \quad x = \pi - \sin^{-1}\left(\tfrac{1}{3}\right) + 2k\pi, \text{ for all } k \in \mathbb{Z}$$

(We found the 2nd family of solutions by using the identity $\sin(\theta) = \sin(\pi - \theta)$.)

b. $2\cos(\theta) = 1$

$$2\cos(\theta) = 1$$

$$\Rightarrow \quad \cos(\theta) = \tfrac{1}{2}$$

(Since this cosine value is "friendly," we can use our experience/knowledge, and don't need inverse-cosine)

$$\Rightarrow \; \theta = \tfrac{\pi}{3} + 2k\pi \quad \text{or} \quad \theta = -\tfrac{\pi}{3} + 2k\pi, \text{ for all } k \in \mathbb{Z}$$

(We started with the solution $\frac{\pi}{3}$ since we are familiar with the fact that $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$, and then we obtain $-\frac{\pi}{3}$ by using the identity $\cos(\theta) = \cos(-\theta)$ which assures us that $\cos\left(\frac{\pi}{3}\right) = \cos\left(-\frac{\pi}{3}\right)$.)

c. $2\sin(3\theta) + \sqrt{3} = 0$

$$
\begin{aligned}
& 2\sin(3\theta) + \sqrt{3} = 0 \\
\Rightarrow\quad & 2\sin(3\theta) = -\sqrt{3} \\
\Rightarrow\quad & \sin(3\theta) = \frac{-\sqrt{3}}{2} \\
\Rightarrow\quad & 3\theta = -\tfrac{\pi}{3} + 2k\pi \quad \text{or} \quad 3\theta = \pi - \left(-\tfrac{\pi}{3}\right) + 2k\pi,\ \ k \in \mathbb{Z} \\
\Rightarrow\quad & \frac{3\theta}{3} = \frac{-\pi/3}{3} + \frac{2k\pi}{3} \quad \text{or} \quad \frac{3\theta}{3} = \frac{4\pi/3}{3} + \frac{2k\pi}{3},\ \ k \in \mathbb{Z} \\
\Rightarrow\quad & \theta = -\frac{\pi}{9} + \frac{2k\pi}{3} \quad \text{or} \quad \theta = \frac{4\pi}{9} + \frac{2k\pi}{3},\ \ k \in \mathbb{Z}
\end{aligned}
$$

d. $7 + 3\sqrt{2}\cos(4t) = 4$

$$
\begin{aligned}
& 7 + 3\sqrt{2}\cos(4t) = 4 \\
\Rightarrow\quad & 3\sqrt{2}\cos(4t) = -3 \\
\Rightarrow\quad & \cos(4t) = \frac{-3}{3\sqrt{2}} \\
\Rightarrow\quad & \cos(4t) = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \\
\Rightarrow\quad & 4t = \frac{3\pi}{4} + 2k\pi \quad \text{or} \quad 4t = -\frac{3\pi}{4} + 2k\pi,\ \ k \in \mathbb{Z} \\
\Rightarrow\quad & \frac{4t}{4} = \frac{3\pi/4}{4} + \frac{2k\pi}{4} \quad \text{or} \quad \frac{4t}{4} = \frac{-3\pi/4}{4} + \frac{2k\pi}{4},\ \ k \in \mathbb{Z} \\
\Rightarrow\quad & t = \frac{3\pi}{16} + \frac{k\pi}{2} \quad \text{or} \quad t = -\frac{3\pi}{16} + \frac{k\pi}{2},\ \ k \in \mathbb{Z}
\end{aligned}
$$

7. Find all of the solutions to the following equations on the interval $[0, 2\pi)$:

a. $6\cos(2x) + 5 = 2$

First we'll find the general solution:

$$6\cos(2x) + 5 = 2$$
$$\Rightarrow \quad 6\cos(2x) = -3$$
$$\Rightarrow \quad \cos(2x) = \frac{-3}{6}$$
$$\Rightarrow \quad \cos(2x) = -\frac{1}{2}$$
$$\Rightarrow \quad 2x = \frac{2\pi}{3} + 2k\pi \quad \text{or} \quad 2x = -\frac{2\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$$
$$\Rightarrow \quad x = \frac{\pi}{3} + k\pi \quad \text{or} \quad x = -\frac{\pi}{3} + k\pi, \quad k \in \mathbb{Z}$$

Now we can find specific solutions on the interval $[0, 2\pi)$ but substituting specific values for k in the general solution:

$k = -1$: $x = \frac{\pi}{3} + (-1)\pi = \frac{\pi}{3} - \frac{3\pi}{3} = -\frac{2\pi}{3}$ or $x = -\frac{\pi}{3} + (-1)\pi = -\frac{\pi}{3} - \frac{3\pi}{3} = -\frac{4\pi}{3}$

Both of these values are negative, so they aren't in the interval $[0, 2\pi)$.

$k = 0$: $x = \frac{\pi}{3} + (0)\pi = \frac{\pi}{3}$ or $x = -\frac{\pi}{3} + (0)\pi = -\frac{\pi}{3}$

Only $\frac{\pi}{3}$ is in the given interval.

$k = 1$: $x = \frac{\pi}{3} + (1)\pi = \frac{\pi}{3} + \frac{3\pi}{3} = \frac{4\pi}{3}$ or $x = -\frac{\pi}{3} + (1)\pi = -\frac{\pi}{3} + \frac{3\pi}{3} = \frac{2\pi}{3}$

Both $\frac{4\pi}{3}$ and $\frac{2\pi}{3}$ are in the given interval.

$k = 2$: $x = \frac{\pi}{3} + (2)\pi = \frac{\pi}{3} + \frac{6\pi}{3} = \frac{7\pi}{3} > 2\pi$ or $x = -\frac{\pi}{3} + (2)\pi = -\frac{\pi}{3} + \frac{6\pi}{3} = \frac{5\pi}{3}$

Only $\frac{5\pi}{3}$ is in the given interval.

So the solution set for $6\cos(2x) + 5 = 2$ on the interval $[0, 2\pi)$ is $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$.

b. $2 - 3\sin(4\theta) = 5$

$$2 - 3\sin(4\theta) = 5$$

$$\Rightarrow \quad -3\sin(4\theta) = 3$$

$$\Rightarrow \quad \sin(4\theta) = -1$$

$$\Rightarrow \quad 4\theta = \sin^{-1}(-1) + 2k\pi, \quad k \in \mathbb{Z}$$

(there's only one "family" of solutions since the sine function achieves the output -1 only once in each period)

$$\Rightarrow \quad 4\theta = -\tfrac{\pi}{2} + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad \frac{4\theta}{4} = \frac{-\pi/2}{4} + \frac{2k\pi}{4}, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad \theta = -\tfrac{\pi}{8} + \tfrac{k\pi}{2}, \quad k \in \mathbb{Z}$$

Now we need to substitute particular values of k in order to determine which solutions fall on the interval $[0, 2\pi)$:

$k = -1$: $\theta = -\tfrac{\pi}{8} + \tfrac{(-1)\cdot\pi}{2} = -\tfrac{5\pi}{8} < 0$ so $-\tfrac{5\pi}{8}$ isn't in the given interval

$k = 0$: $\theta = -\tfrac{\pi}{8} + \tfrac{(0)\cdot\pi}{2} = -\tfrac{\pi}{8} < 0$, so $-\tfrac{\pi}{8}$ isn't in the given interval

$k = 1$: $\theta = -\tfrac{\pi}{8} + \tfrac{(1)\cdot\pi}{2} = \tfrac{3\pi}{8} \in [0, 2\pi)$, so $\tfrac{3\pi}{8}$ is a solution in the given interval

$k = 2$: $\theta = -\tfrac{\pi}{8} + \tfrac{(2)\cdot\pi}{2} = \tfrac{7\pi}{8} \in [0, 2\pi)$ so $\tfrac{7\pi}{8}$ is a solution in the given interval.

$k = 3$: $\theta = -\tfrac{\pi}{8} + \tfrac{(3)\cdot\pi}{2} = \tfrac{11\pi}{8} \in [0, 2\pi)$ so $\tfrac{11\pi}{8}$ is a solution in the given interval.

$k = 4$: $\theta = -\tfrac{\pi}{8} + \tfrac{(4)\cdot\pi}{2} = \tfrac{15\pi}{8} \in [0, 2\pi)$ so $\tfrac{15\pi}{8}$ is a solution in the given interval.

$k = 5$: $\theta = -\tfrac{\pi}{8} + \tfrac{(5)\cdot\pi}{2} = \tfrac{17\pi}{8} > 2\pi$ so $\tfrac{17\pi}{8}$ isn't in the given interval.

Therefore, the solution set to $2 - 3\sin(4\theta) = 5$ on the interval $[0, 2\pi)$ is $\left\{\tfrac{3\pi}{8}, \tfrac{7\pi}{8}, \tfrac{11\pi}{8}, \tfrac{15\pi}{8}\right\}$.

8. Evaluate the following:

a. $\sin\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$

$$\sin\left(\cos^{-1}\left(-\frac{1}{2}\right)\right) = \sin\left(\frac{2\pi}{3}\right)$$ (since the range of $y=\cos^{-1}(x)$ is $[0, \pi]$)

$$= \frac{\sqrt{3}}{2}$$

b. $\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right)$

$$\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right) = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$$

$$= -\frac{\pi}{4}$$ (since the range of $y=\sin^{-1}(x)$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$)

c. $\cos^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right)$

$$\cos^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right) = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$= \frac{5\pi}{6}$$ (since the range of $y=\cos^{-1}(x)$ is $[0, \pi]$)

d. $\cos^{-1}\left(\cos\left(\frac{8\pi}{7}\right)\right)$

Since $\frac{8\pi}{7}$ isn't "friendly" (i.e., since we don't know what $\cos\left(\frac{8\pi}{7}\right)$ equals) we can't use the method we used for the previous expressions. Instead, we'll need to rely on the definition of cosine and our familiarity with the symmetry of a circle.

$$\cos^{-1}\left(\cos\left(\frac{8\pi}{7}\right)\right) = \cos^{-1}\left(\cos\left(\frac{6\pi}{7}\right)\right)$$ ($\frac{8\pi}{7}$ and $\frac{6\pi}{7}$ share the same refernce angle and both angles have negative cosine values)

$$= \frac{6\pi}{7}$$ ($\frac{6\pi}{7}$ is in the range of $y=\cos^{-1}(x)$, $[0, \pi]$)

9. Sketch a graph of $g(t) = 3\sin\left(2t + \frac{\pi}{4}\right) - 2$. State the period, midline, and amplitude.

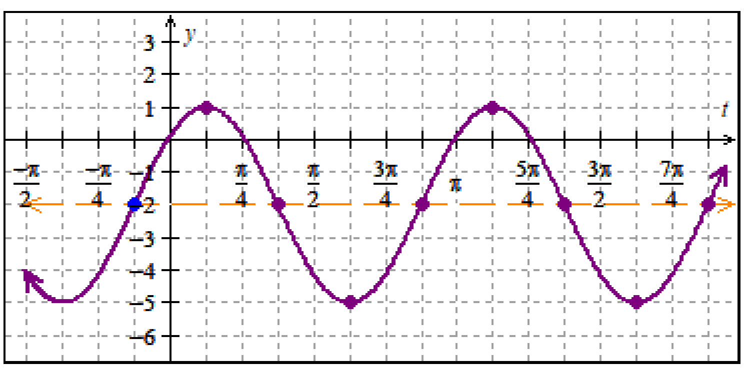
First, let's write the function in "standard form":

$$\begin{aligned} g(t) &= 3\sin\left(2t + \tfrac{\pi}{4}\right) - 2 \\ &= 3\sin\left(2\left(t + \tfrac{\pi}{8}\right)\right) - 2 \end{aligned}$$

Therefore, this is a function of the form $g(t) = A\sin\left(\omega(t - h)\right) + k$ where $A = 3$, $\omega = 2$, $h = -\frac{\pi}{8}$, and $k = -2$. Since $A = 3$ the function has **amplitude 3 units**. Using the fact that $\omega = 2$, we can find the period:

$$\begin{aligned} P &= \frac{2\pi}{\omega} \\ &= \frac{2\pi}{2}. \\ &= \pi \end{aligned}$$

So the **period is π units**. Since $k = -2$, the **midline is $y = -2$**. Since $h = -\frac{\pi}{8}$, we need to "start" our sine wave at $t = -\frac{\pi}{8}$, i.e., shift the wave left $\frac{\pi}{8}$ units. Below is a graph of $g(t) = 3\sin\left(2\left(t + \frac{\pi}{8}\right)\right) - 2$.



A graph of $g(t) = 3\sin\left(2t + \frac{\pi}{4}\right) - 2 = 3\sin\left(2\left(t + \frac{\pi}{8}\right)\right) - 2$.

10. Sketch a graph of $q(x) = 2\cos\left(\frac{\pi}{2}x + \frac{\pi}{4}\right) - 3$. State the period, midline, and amplitude.

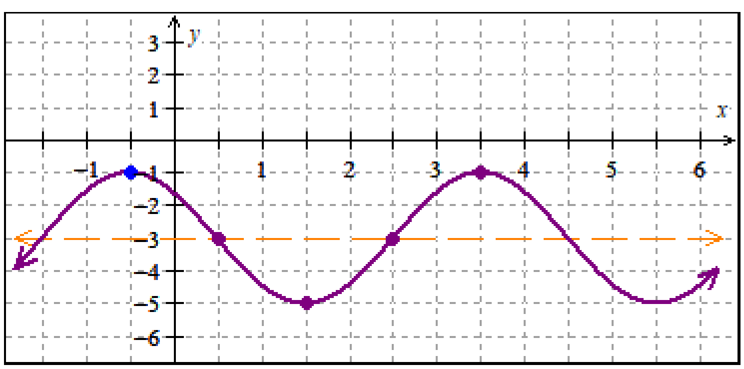
First, let's write the function in "standard form":

$$\begin{aligned} q(x) &= 2\cos\left(\frac{\pi}{2}x + \frac{\pi}{4}\right) - 3 \\ &= 2\cos\left(\frac{\pi}{2}\left(x + \frac{1}{2}\right)\right) - 3 \end{aligned}$$

Therefore, this is a function of the form $q(t) = A\sin\left(\omega(t-h)\right) + k$ where $A = 2$, $\omega = \frac{\pi}{2}$, $h = -\frac{1}{2}$, and $k = -3$. Since $A = 2$ the function has **amplitude 2 units**. Using the fact that $\omega = \frac{\pi}{2}$, we can find the period:

$$\begin{aligned} P &= \frac{2\pi}{\omega} \\ &= \frac{2\pi}{\pi/2}. \\ &= 4 \end{aligned}$$

So the **period is 4 units**. Since $k = -3$, the **midline is $y = -3$**. Since $h = -\frac{1}{2}$, we need to "start" our cosine wave at $x = -\frac{1}{2}$, i.e., shift the wave left $\frac{1}{2}$ units. Below is a graph of $q(x) = 2\cos\left(\frac{\pi}{2}x + \frac{\pi}{4}\right) - 3$.



A graph of $q(x) = 2\cos\left(\frac{\pi}{2}x + \frac{\pi}{4}\right) - 3 = 2\cos\left(\frac{\pi}{2}\left(x + \frac{1}{2}\right)\right) - 3$.

11. Find four algebraic rules (one using "positive sine", one using "negative (reflected) sine", one using "positive cosine", and one using "negative (reflected) cosine") for the function $y = p(x)$ graphed in Figure 2.

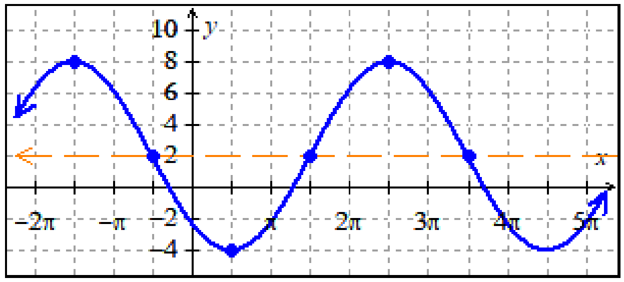


Figure 2: A graph of $y = p(x)$.

Whether we use sine or cosine for our rule, it will have form $p(x) = \pm|A|\sin\big(\omega(x-h)\big) + k$ or $p(x) = \pm|A|\cos\big(\omega(x-h)\big) + k$ so we can start by determining the values of $|A|$, ω, and k.

- the amplitude is 6 units so we know that $|A| = 6$
- the midline is $y = 2$ so we know that $k = 2$
- the period is 4π units so we know that ω must satisfy $4\pi = 2\pi \cdot \frac{1}{\omega}$. Thus,

$$4\pi = 2\pi \cdot \frac{1}{\omega}$$
$$\Rightarrow \omega = \frac{2\pi}{4\pi} = \frac{1}{2}$$

Since a positive sine wave "starts" at $x = \frac{3\pi}{2}$, we can use $h = \frac{3\pi}{2}$ to construct a rule:

$$g(x) = 6\sin\left(\frac{1}{2}\left(x - \frac{3\pi}{2}\right)\right) + 2$$

Since a negative (reflected) sine wave "starts" at $x = -\frac{\pi}{2}$, we can use $h = -\frac{\pi}{2}$:

$$g(x) = -6\sin\left(\frac{1}{2}\left(x + \frac{\pi}{2}\right)\right) + 2$$

Since a positive cosine wave "starts" at $x = -\frac{3\pi}{2}$, we can use $h = -\frac{3\pi}{2}$:

$$g(x) = 6\cos\left(\frac{1}{2}\left(x + \frac{3\pi}{2}\right)\right) + 2$$

Since a negative (reflected) cosine wave "starts" at $x = \frac{\pi}{2}$, we can use $h = \frac{\pi}{2}$:

$$g(x) = -6\cos\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right) + 2$$

(Note that there are many other possible answers.)

12. a. Find four algebraic rules (one using "positive sine", one using "negative (reflected) sine", one using "positive cosine", and one using "negative (reflected) cosine") for the function $y = f(t)$ graphed in Figure 3.

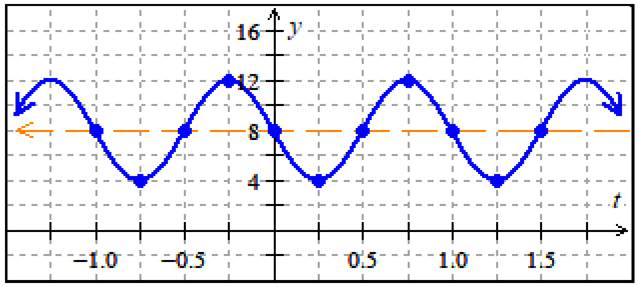


Figure 3: Graph of $y = f(t)$.

Whether we use sine or cosine for our rule, it will have form $f(t) = \pm|A|\sin\left(\omega(t-h)\right) + k$ or $f(t) = \pm|A|\cos\left(\omega(t-h)\right) + k$ so we can start by determining the values of $|A|$, ω, and k.

- the amplitude is 4 units so we know that $|A| = 4$
- the midline is $y = 8$ so we know that $k = 8$
- the period is 1 unit so we know that ω must satisfy $1 = 2\pi \cdot \frac{1}{\omega}$. Thus,

$$1 = 2\pi \cdot \frac{1}{\omega}$$
$$\Rightarrow \omega = \frac{2\pi}{1} = 2\pi$$

Since a positive sine wave "starts" at $t = 0.5$, we can use $h = 0.5$ to construct a rule:

$$f(t) = 4\sin\left(2\pi(t-0.5)\right) + 8$$

Since a negative (reflected) sine wave "starts" at $t = 0$, we can use $h = 0$:

$$f(t) = -4\sin\left(2\pi \cdot t\right) + 8$$

Since a positive cosine wave "starts" at $t = -0.25$, we can use $h = -0.25$:

$$f(t) = 4\cos\left(2\pi(t+0.25)\right) + 8$$

Since a negative (reflected) cosine wave "starts" at $t = 0.25$, we can use $h = 0.25$:

$$f(t) = -4\cos\left(2\pi(t-0.25)\right) + 8$$

(Note that there are many other possible answers.)

b. Use one of your answers to part **a** to find exact solutions to $f(t) = 10$.

We'll use $f(t) = -4\sin(2\pi \cdot t) + 8$ since it doesn't involve a horizontal shift so the equation will be simpler:

$$
\begin{aligned}
& f(t) = 10 \\
\Rightarrow \quad & -4\sin(2\pi t) + 8 = 10 \\
\Rightarrow \quad & -4\sin(2\pi t) = 2 \\
\Rightarrow \quad & \sin(2\pi t) = -\tfrac{1}{2} \\
\Rightarrow \quad & 2\pi t = -\tfrac{\pi}{6} + 2k\pi \quad \text{or} \quad 2\pi t = \tfrac{7\pi}{6} + 2k\pi, \quad k \in \mathbb{Z} \\
\Rightarrow \quad & t = \frac{-\frac{\pi}{6} + 2k\pi}{2\pi} \quad \text{or} \quad t = \frac{\frac{7\pi}{6} + 2k\pi}{2\pi}, \quad k \in \mathbb{Z} \\
\Rightarrow \quad & t = -\tfrac{1}{12} + k \quad \text{or} \quad t = \tfrac{7}{12} + k, \quad k \in \mathbb{Z}
\end{aligned}
$$