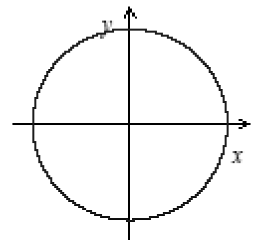
(This packet includes the MTH 112 Practice Worksheets.)

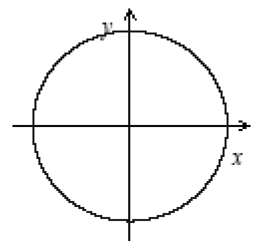
Week 1 Practice Worksheet

Angles, Arc-length, and Periodic Functions

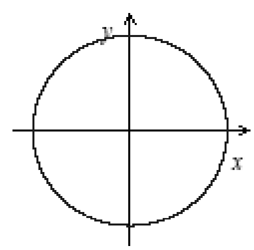
1. a. Draw 140° in standard position on the provided coordinate plane.



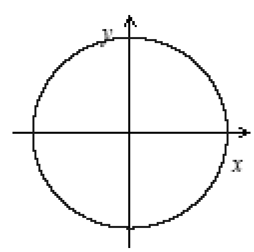
b. Draw -250° in standard position on the provided coordinate plane.



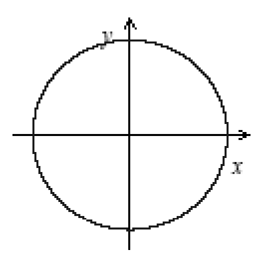
c. Draw $\frac{4\pi}{5}$ radians in standard position on the provided coordinate plane.



d. Draw 4 radians in standard position on the provided coordinate plane.



d. Draw -3 radians in standard position on the provided coordinate plane.



2. The Greek letters θ and ϕ are often used as variables in mathematics. They are ***different*** symbols so it's important to distinguish between them. Which of these symbols is pronounced "phi"? And which is pronounced "theta"?

3. **a.** Convert $\frac{\pi}{10}$ radians into degrees.

b. Convert 4 radians into degrees.

c. Convert 10° into radians.

d. Convert 140° into radians.

e. Convert 1200° into radians.

4. a. Find both a positive and a negative angle that are coterminal with 140°. (Answer in degrees.)

b. Find both a positive and a negative angle that are coterminal with $-\frac{5\pi}{8}$. (Answer in radians.)

c. Represent infinitely many different angles that are coterminal with 70°.

d. Represent infinitely many different angles that are coterminal with $\frac{3\pi}{5}$.

5. Complete the table below.

θ (degrees)	0°	30°		60°	90°		270°	360°	
θ (radians)			$\frac{\pi}{4}$			π			20π

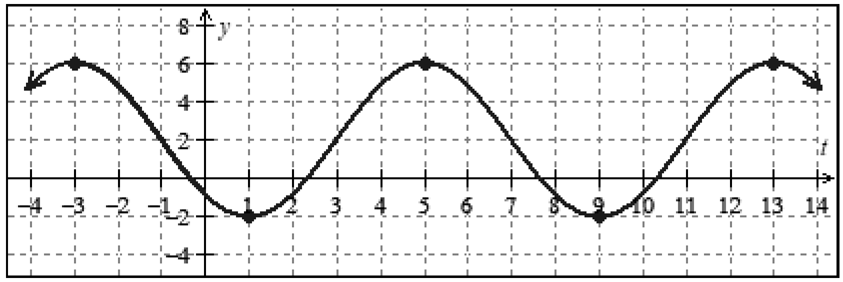
6. What is the length of the arc spanned by the angle 4 radians on a circle of radius 20 yards?

7. What is the length of the arc spanned by the angle 25° on a circle of radius 30 inches?

8. What is the length of the arc spanned by the angle $\frac{4\pi}{5}$ radians on a circle of radius 15 feet?

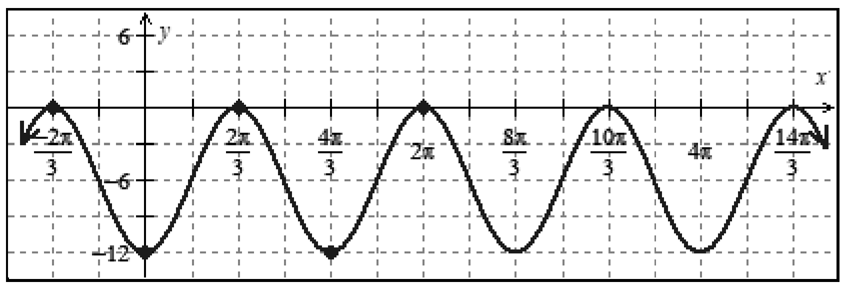
9. What is the length of the arc spanned by the angle 135° on a circle of radius 8 meters?

10. Determine the period, midline and amplitude of the function $y = f(t)$ graphed below. Note that the following points are on the graph: $(-3, 6)$, $(1, -2)$, $(5, 6)$, $(9, -2)$, and $(13, 6)$.



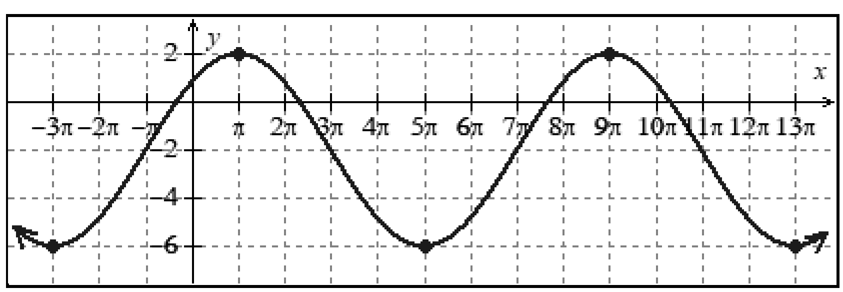
The graph of $y = f(t)$.

11. Determine the period, midline and amplitude of the function $y = g(t)$ graphed below. Note that the following points are on the graph: $\left(-\frac{2\pi}{3}, 0\right)$, $(0, -12)$, $\left(\frac{2\pi}{3}, 0\right)$, and $\left(\frac{4\pi}{3}, -12\right)$.



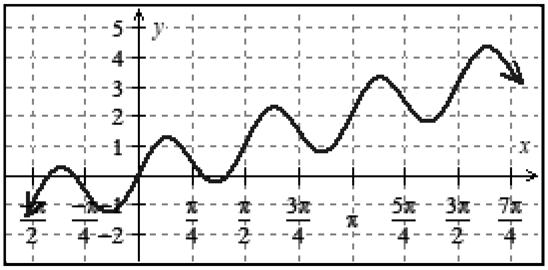
The graph of $y = g(t)$.

12. Determine the period, midline and amplitude of the function $y = p(x)$ graphed below. Note that the following points are on the graph: $(-3\pi, -6)$, $(\pi, 2)$, $(5\pi, -6)$, and $(9\pi, 2)$.

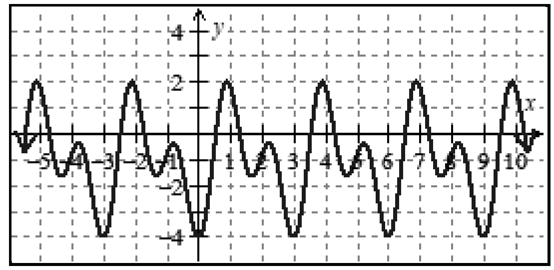


The graph of $y = p(x)$.

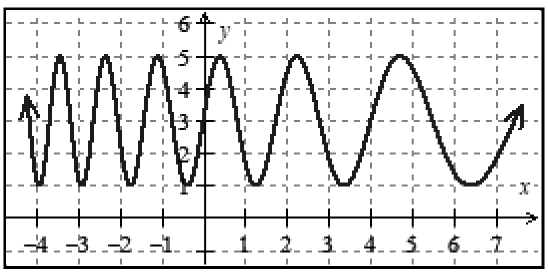
13. Determine which of the functions graphed below are periodic functions and find the period, midline, and amplitude of the periodic functions.



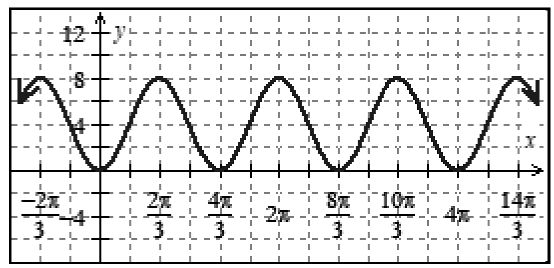
$y = A(x)$



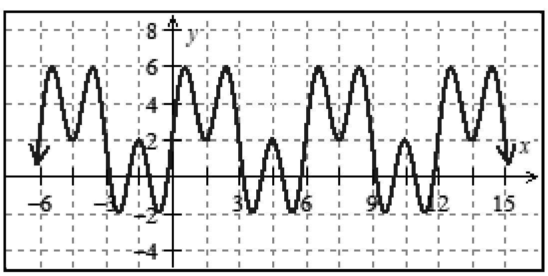
$y = B(x)$



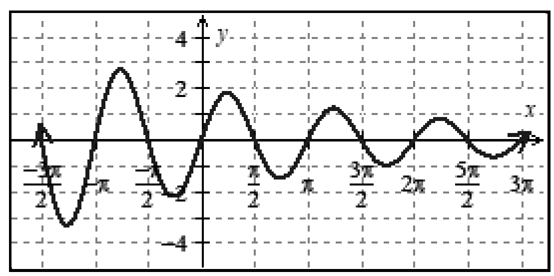
$y = C(x)$ is



$y = D(x)$



$y = E(x)$



$y = F(x)$

14. There's a Ferris wheel with a diameter of 80 feet. The wheel rotates at a constant rate, and completes a full rotation every 20 minutes. The wheel is lifted 10 feet above the ground level, and passengers load into carriages at the lowest point in the wheel's travel (so passengers start their trip 10 feet above the ground). Determine the period, midline and amplitude of the function that associates amount of time a passenger spends in a carriage travelling around the wheel with the height (in feet) above the ground of such a passenger.

15. The following verbally-described functions are approximately-periodic: assume that they are truly periodic functions and determine reasonable values for their periods, midlines, and amplitudes.

a. Determine the period, midline, and amplitude of the function that associates calendar date (so input-values for this function are dates like 01/01/2023, 01/02/2023, … , 12/31/2023, … and continue for a few years) with the high temperature (in degrees Fahrenheit) at PDX on that date.

b. Determine the period, midline, and amplitude of the function that associates calendar date (same input values as in part (a)) with the number of hours of daylight (i.e., time between sunrise and sunset) at PDX on that date.

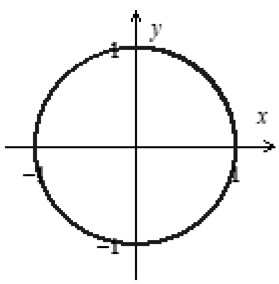
c. Suppose that the water depth in a harbor oscillates between 40 feet at high tide and 10 feet at low tide, and then back to 40 feet at the next high tide 12 hours after the previous high tide. Determine the period, midline, and amplitude of the function that associates time-of-day (e.g., 12am, 1am, …, 10pm, 11pm, … and continue for a few days) and the water depth in the harbor at that time-of-day.

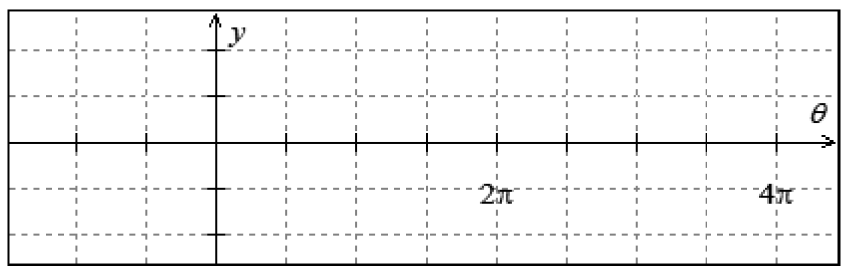
Week 2 Practice Worksheet

Intro to the Trig Functions

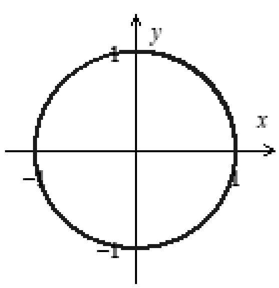
You should complete all of these problems ***without a calculator*** in order to prepare for the Midterm which is a no-calculator exam.

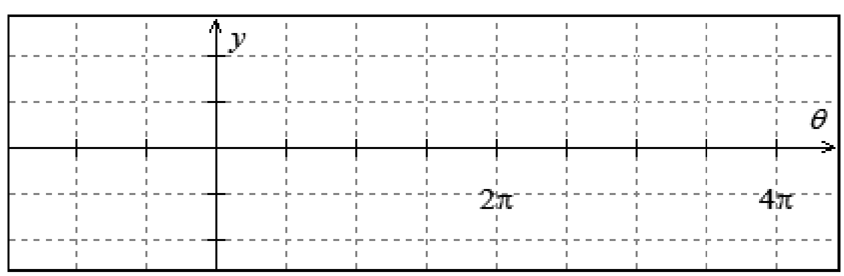
1. Draw an *accurate* graph of at least two periods of $y = \cos(\theta)$ and $y = \sin(\theta)$ on the coordinate planes below. To draw each graph, correctly label at least **two** of the unlabeled "tics" on the *y*-axis and all **eight** of the unlabeled "tics" on the θ-axis (two tics on the θ-axis have been labeled for you); then **plot a point** on your graph that corresponds to **each of the eight tics** that you've labeled on the θ-axis and the two pre-labeled tics, and then connect the points to create your graph.





Draw a graph of $y = \cos(\theta)$.
Label *all* axis "tics" and plot *all* key points.





Draw a graph of $y = \sin(\theta)$.
Label *all* axis "tics" and plot *all* key points.

2. If $\frac{3\pi}{2} < \theta < 2\pi$ and $\cos(\theta) = \frac{\sqrt{5}}{4}$, find the following. Be sure to use proper notation to communicate your answer, i.e., link the given expression and your answer with an equal sign. For example, your response to part (a) should have the form $\sin(\theta) =$ ____ since you're asked to tell me what $\sin(\theta)$ equals.)

a. $\sin(\theta)$ **b.** $\tan(\theta)$ **c.** $\sec(\theta)$ **d.** $\csc(\theta)$

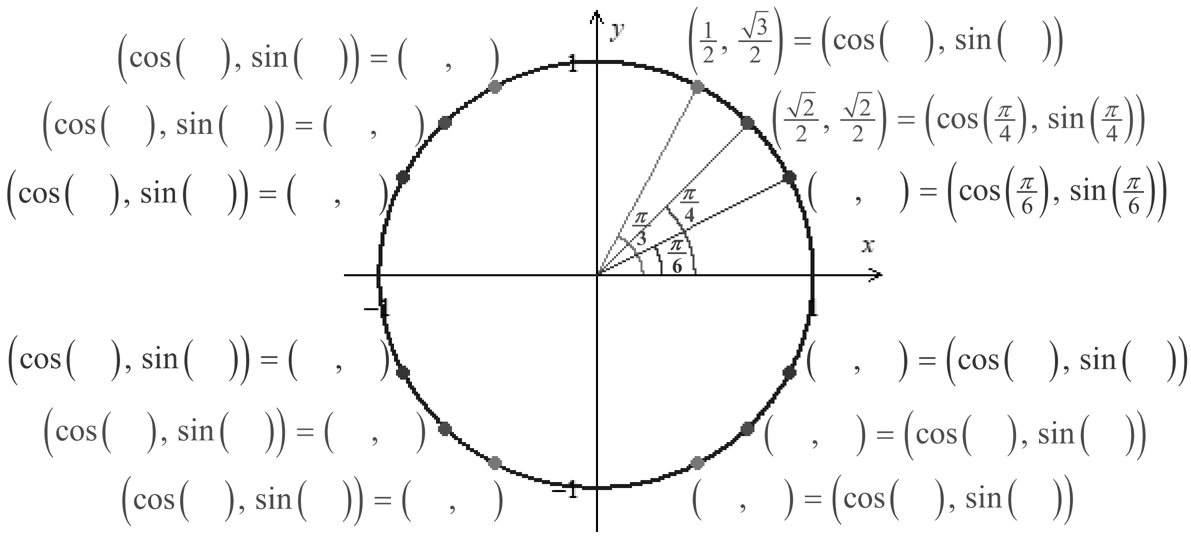
3. If $\frac{\pi}{2} < \theta < \pi$ and $\sin(\theta) = \frac{6}{7}$, find the following. Be sure to use proper notation to communicate your answer, i.e., link the given expression and your answer with an equal sign. For example, your response to part (a) should have the form $\cos(\theta) = \_\_\_\_$ since you're asked to tell me what $\cos(\theta)$ equals.)

a. $\cos(\theta)$ **b.** $\tan(\theta)$ **c.** $\sec(\theta)$ **d.** $\csc(\theta)$

4. If $\frac{\pi}{2} < \theta < \pi$ and $\csc(\theta) = 3$, find the following. Be sure to use proper notation to communicate your answer, i.e., link the given expression and your answer with an equal sign. For example, your response to part (a) should have the form $\sin(\theta) =$ ____ since you're asked to tell me what $\sin(\theta)$ equals.)

a. $\sin(\theta)$ **b.** $\cos(\theta)$ **c.** $\tan(\theta)$ **d.** $\sec(\theta)$

5. Label the **coordinates** of each of the 12 dots (three in each of the four quadrants) on the unit circle below. Fill-in the input for the sine and cosine functions in order to indicate the **angle** that specifies each point. Some of the information for the points in Quad 1 has been filled-in.



6. Find $\sin(\theta)$, $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$, and $\csc(\theta)$ if:

a. $\theta = \frac{\pi}{3}$

b. $\theta = \frac{\pi}{4}$

7. Find $\sin(\theta)$, $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$, and $\csc(\theta)$ if:

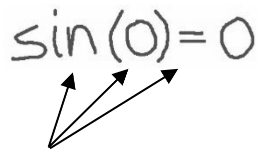
a. $\theta = 150^\circ$

b. $\theta = -\frac{3\pi}{4}$

c. $\theta = \frac{4\pi}{3}$

8. Find the ***exact*** value for each of the following expressions. Be sure to use proper notation to communicate your answer, i.e., link the given expression and your answer with an equal sign. If the given expression is undefined write, "*The expression is undefined*." An example has been provided to clarify how your response should look.

ex. $\sin(0)$

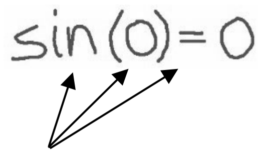


(Write all of this to communicate what "$\sin(0)$" *equals*.)

a. $\cos\left(\frac{\pi}{6}\right)$.	**b.** $\sin\left(\frac{\pi}{4}\right)$.
c. $\cos\left(60^\circ\right)$.	**d.** $\sin\left(-\frac{5\pi}{3}\right)$.
e. $\sin\left(\frac{17\pi}{6}\right)$.	**f.** $\cos\left(\frac{7\pi}{6}\right)$.
g. $\sin\left(240^\circ\right)$.	**h.** $\cos\left(135^\circ\right)$.
i. $\sec(2\pi)$.	**j.** $\tan\left(\frac{4\pi}{3}\right)$.

9. Find the ***exact*** value for each of the following expressions. Be sure to use proper notation to communicate your answer, i.e., link the given expression and your answer with an equal sign. If the given expression is undefined write, "*The expression is undefined*." An example has been provided to clarify how your response should look.

ex. $\sin(0)$



(Write all of this to communicate what "$\sin(0)$" *equals*.)

a. $\sin\left(\frac{\pi}{2}\right)$.

b. $\cos\left(180^\circ\right)$.

c. $\cos\left(\frac{5\pi}{6}\right)$.

d. $\sin\left(\frac{7\pi}{6}\right)$.

e. $\sin\left(120^\circ\right)$.

f. $\cos\left(\frac{5\pi}{4}\right)$.

g. $\cos\left(\frac{5\pi}{3}\right)$.

h. $\sin\left(\frac{20\pi}{3}\right)$.

i. $\tan(2\pi)$.

j. $\csc\left(\frac{11\pi}{6}\right)$.

10. A circle with a radius of 6 units is given in Figure 1. The point Q is specified by the angle $\frac{5\pi}{6}$. Use the sine and cosine function to find the exact coordinates of point Q.

[**Be sure to *show* your use of sine and cosine.**]

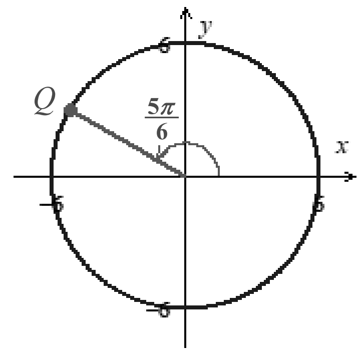


Figure 1

11. The point P in Figure 2 is specified by $\frac{7\pi}{4}$ on the circumference of a circle with a radius of 12 units. Use the sine and cosine function to find the **exact** coordinates of P.

[**Be sure to *show* your use of sine and cosine.**]

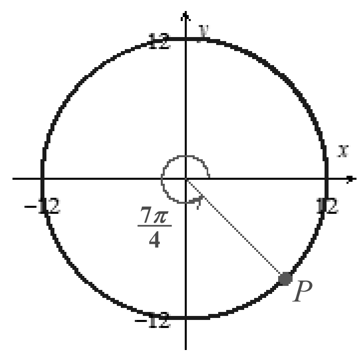


Figure 2

12. The point S in Figure 3 is specified by $\frac{4\pi}{3}$ on the circumference of a circle with a radius of 20 units. Use the sine and cosine function to find the **exact** coordinates of S.

[**Be sure to *show* your use of sine and cosine.**]

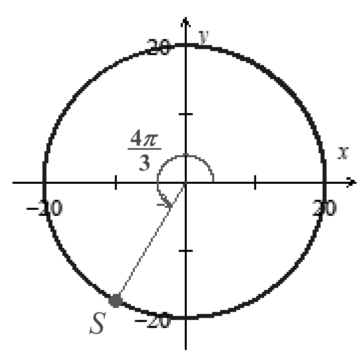


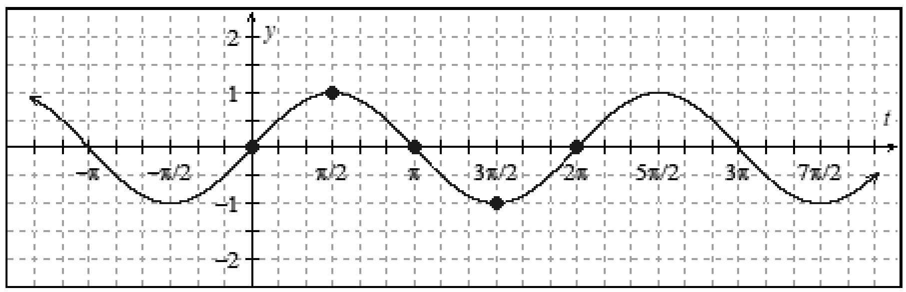
Figure 3

Week 3 Practice Worksheet

Graphs of Trig Functions

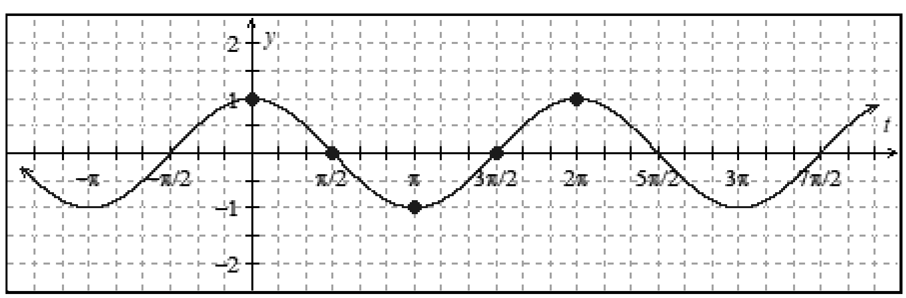
You should complete all of these problems ***without a calculator*** in order to prepare for the Midterm which is a no-calculator exam.

1. The graph of $y = \sin(t)$ is given below with **five key points** emphasized. As we know, the graph of $y = \sin\left(t - \frac{2\pi}{3}\right)$ is a *transformation* of $y = \sin(t)$. Use what we know about graph transformations to "transform" the **five** key points on $y = \sin(t)$ and then connect these points in order to construct a graph of $y = \sin\left(t - \frac{2\pi}{3}\right)$.



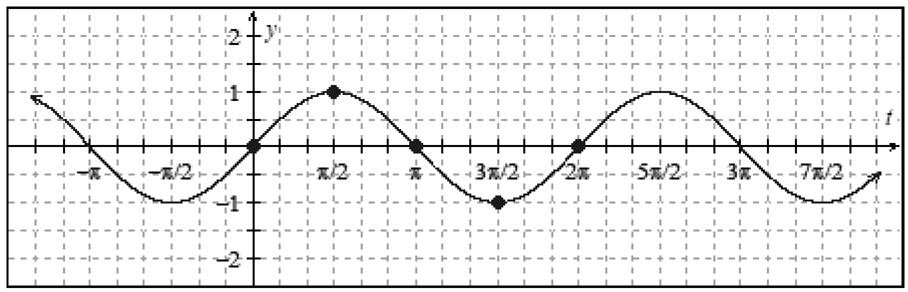
The graph of $y = \sin(t)$ is given; draw a graph of $y = \sin\left(t - \frac{2\pi}{3}\right)$.

2. The graph of $y = \cos(t)$ is given below with **five key points** emphasized. As we know, the graph of $y = \cos\left(t + \frac{5\pi}{6}\right)$ is a *transformation* of $y = \cos(t)$. Use what we know about graph transformations to "transform" the **five key points** on $y = \cos(t)$ and then connect these points in order to construct a graph of $y = \cos\left(t + \frac{5\pi}{6}\right)$.



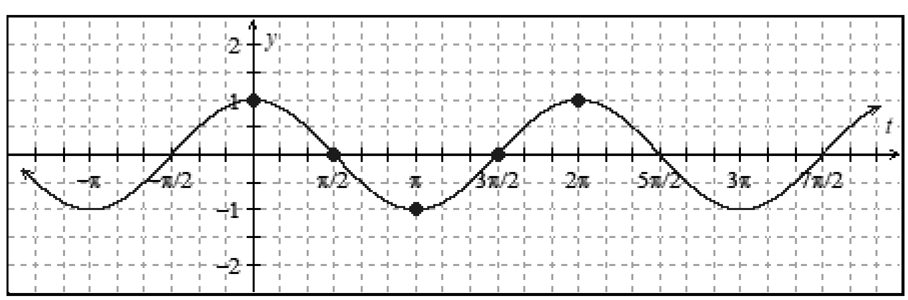
The graph of $y = \cos(t)$ is given; draw a graph of $y = \cos\left(t + \frac{5\pi}{6}\right)$.

3. The graph of $y = \sin(t)$ is given below with **five key points** emphasized. As we know, the graph of $y = \sin(t) - 1$ is a *transformation* of $y = \sin(t)$. Use what we know about graph transformations to "transform" the **five** key points on $y = \sin(t)$ and then connect these points in order to construct a graph of $y = \sin(t) - 1$.



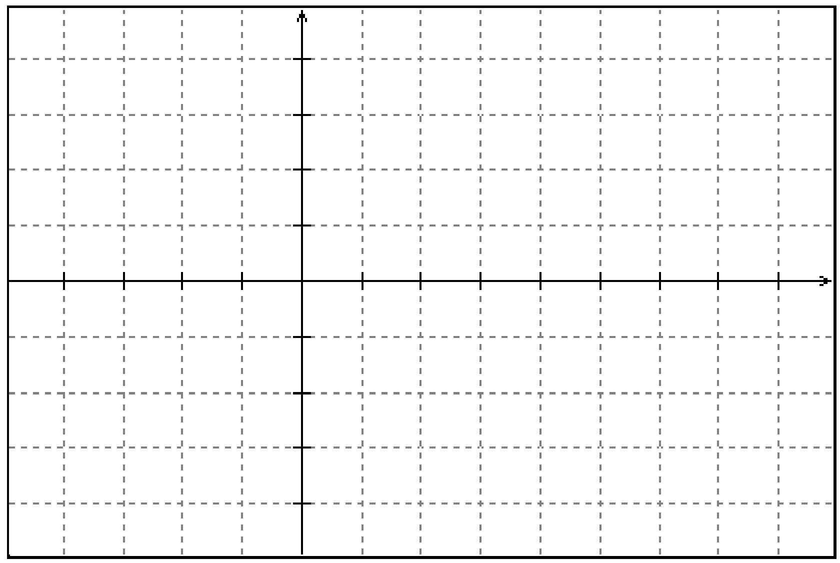
The graph of $y = \sin(t)$ is given; draw a graph of $y = \sin(t) - 1$.

4. The graph of $y = \cos(t)$ is given below with **five key points** emphasized. As we know, the graph of $y = 2\cos(t)$ is a *transformation* of $y = \cos(t)$. Use what we know about graph transformations to "transform" the **five key points** on $y = \cos(t)$ and then connect these points in order to construct a graph of $y = 2\cos(t)$.



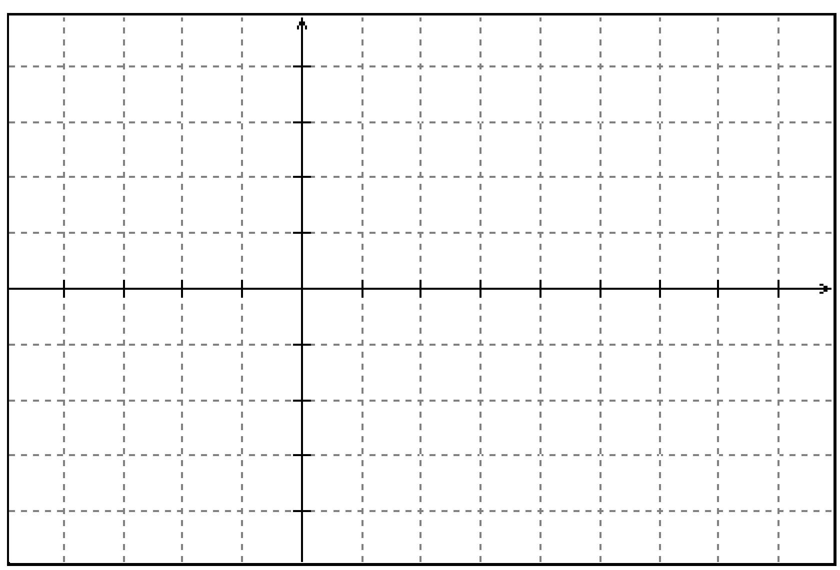
The graph of $y = \cos(t)$ is given; draw a graph of $y = 2\cos(t)$.

5. Scale the axes on the given coordinate plane an appropriately for a graph of $y = \sin(2t) + 3$ and then draw a graph of $y = \sin(2t) + 3$ by first plotting the points where the graph will intersect the midline and the points where the graph will reach maximum and minimum values, and then connect these points with an appropriately curved sinusoidal wave.



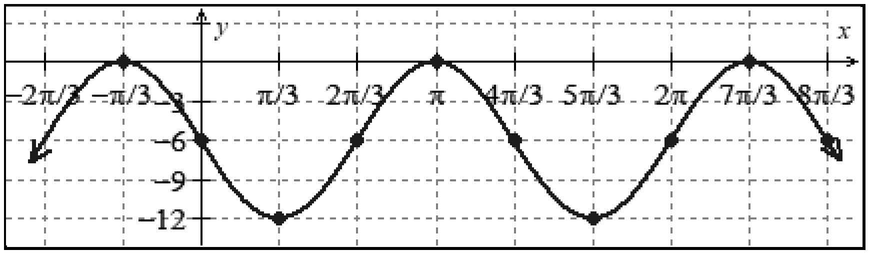
Draw a graph of $y = \sin(2t) + 3$.

6. Scale the axes on the given coordinate plane an appropriately for a graph of $y = 3\cos(\pi t)$ and then draw a graph of $y = 3\cos(\pi t)$ by first plotting the points where the graph will intersect the midline and the points where the graph will reach maximum and minimum values, and then connect these points with an appropriately curved sinusoidal wave.



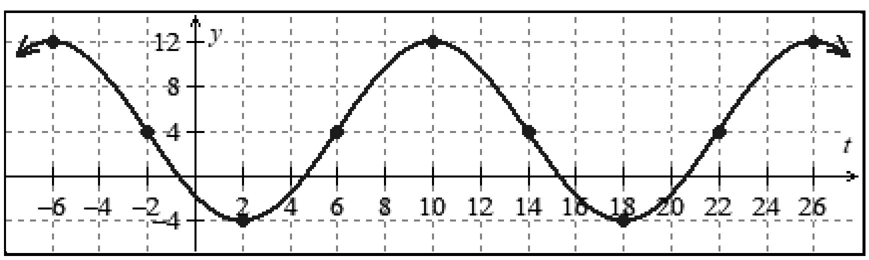
Draw a graph of $y = 3\cos(\pi t)$.

7. Find two different algebraic rules for the sinusoidal function $y = p(x)$ graphed below. One of your rules should involve sine and the other should involve cosine.



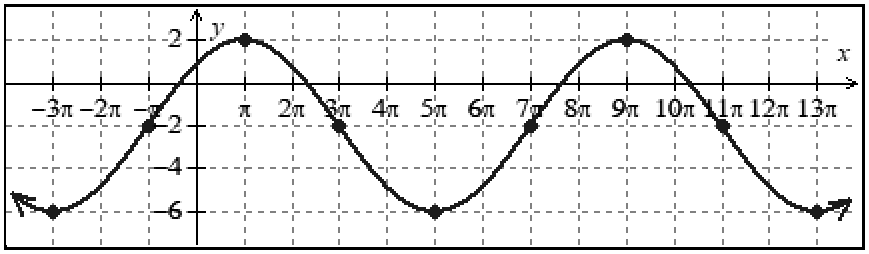
The graph of $y = p(x)$.

8. Find two different algebraic rules for the sinusoidal function $y = q(t)$ graphed below. One of your rules should involve sine and the other should involve cosine.



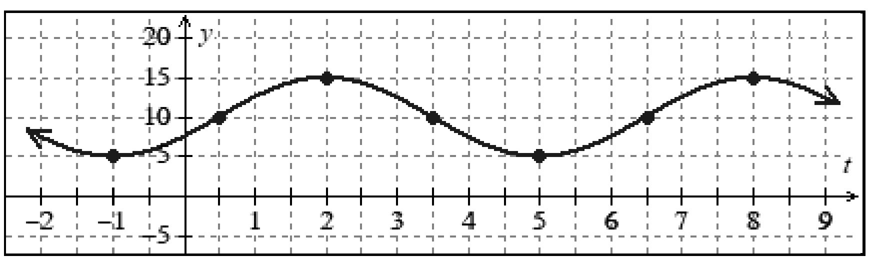
The graph of $y = q(t)$.

9. Find two different algebraic rules for the sinusoidal function $y = m(x)$ graphed below. One of your rules should involve sine and the other should involve cosine.



The graph of $y = m(x)$.

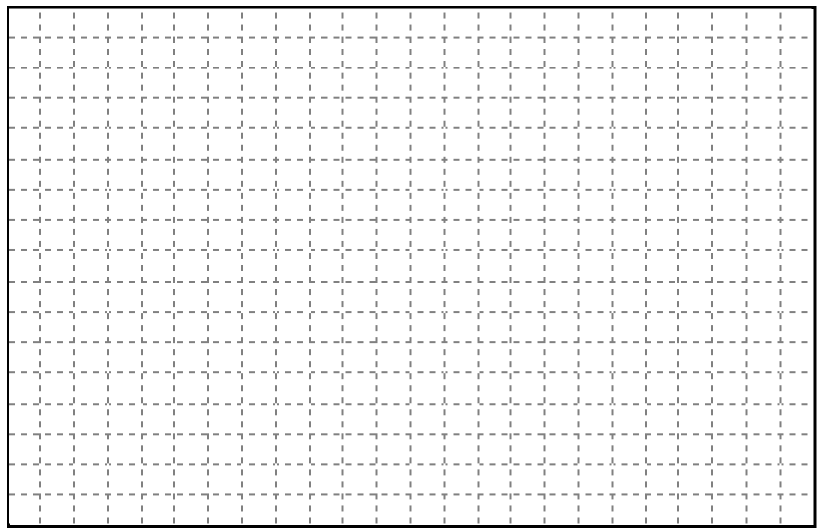
10. Find two different algebraic rules for the sinusoidal function $y = n(t)$ graphed below. One of your rules should involve sine and the other should involve cosine.



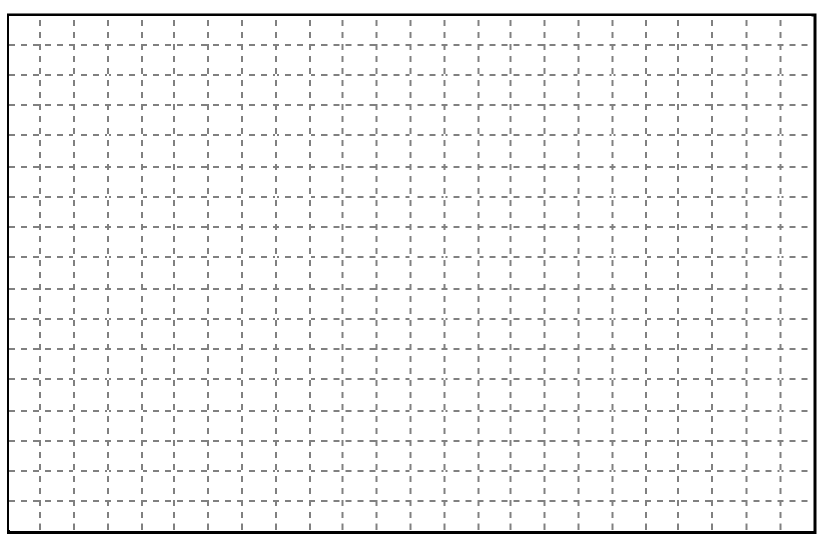
The graph of $y = n(t)$.

11. Draw a graph of at least two periods of the functions in **a–f** below by first *plotting the points* where the graph will intersect the midline and *plotting the points* where the graph will reach maximum and minimum values, and then *connect these points* with an appropriately curved sinusoidal wave. List the period, midline, and amplitude of each function. (Be sure to label the scale on the axes of your graph.)

a. $f(t) = -5\cos(4t) + 3$



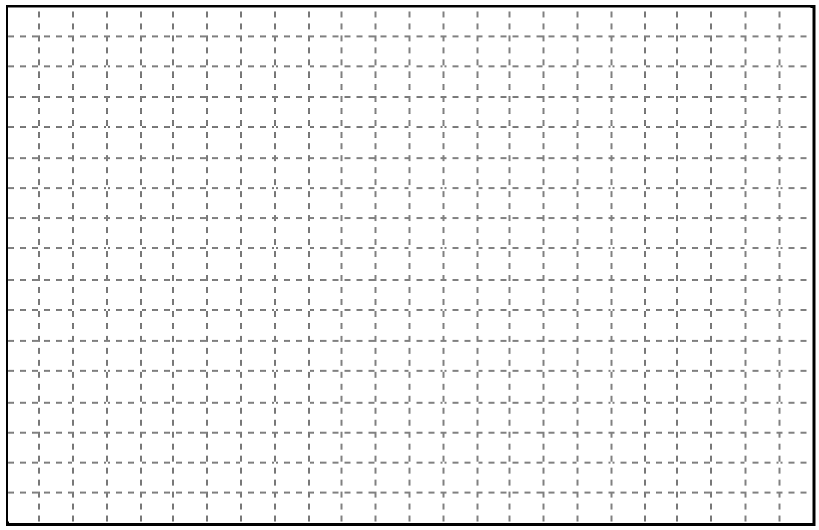
b. $g(x) = 4\sin\left(\pi\left(x - \frac{1}{4}\right)\right) - 2$



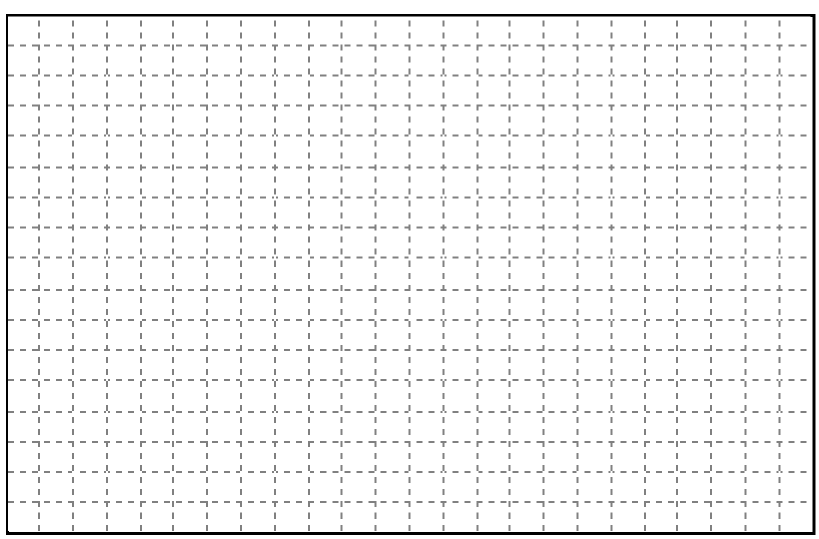
c. $G(x) = 3\cos\left(2x + \frac{\pi}{2}\right) + 4$

d. $F(t) = -2\sin\left(\frac{\pi}{3}t + \pi\right) - 1$

e. $p(x) = 10\cos\left(\dfrac{x + 2\pi}{4}\right) - 5$



f. $q(t) = -4\sin\left(\frac{\pi}{4}t - \frac{\pi}{4}\right) - 2$



12. Determine the domain of $y = \tan(t)$.

13. Determine the domain of $y = \sec(t)$.

14. Determine the domain of $y = \cot(t)$.

15. Determine the domain of $y = \csc(t)$.

Week 4 Practice Worksheet

Inverse Trig Functions and Solving Trig Equations

You should complete all of these problems ***without a calculator*** in order to prepare for the Midterm which is a no-calculator exam.

1. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to ***directly communicate*** what the given expressions equal.

a. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

b. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

c. $\sin^{-1}\left(-\frac{1}{2}\right)$

d. $\cos^{-1}(0)$

e. $\tan^{-1}\left(-\sqrt{3}\right)$

f. $\tan^{-1}(-1)$

2. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to ***directly communicate*** what the given expressions equal.

a. $\sin\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

b. $\cos\left(\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$

c. $\cos^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right)$

d. $\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right)$

e. $\sin\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

3. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to ***directly communicate*** what the given expressions equal.

a. $\sin^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right)$

b. $\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right)$

c. $\cos^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$

d. $\tan^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right)$

e. $\sin\left(\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$

4. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to ***directly communicate*** what the given expressions equal.

a. $\sin^{-1}\left(\sin\left(\frac{7\pi}{8}\right)\right)$

b. $\cos^{-1}\left(\cos\left(\frac{7\pi}{5}\right)\right)$

c. $\sin^{-1}\left(\sin\left(\frac{9\pi}{7}\right)\right)$

5. Find *all* of the solutions to the equations below; provide *exact* solutions.

a. $\sin(t) = -\frac{\sqrt{2}}{2}$

b. $2\cos(x) - \sqrt{3} = 0$

6. Find the solutions on the interval $[0, 2\pi)$ for the equations below; provide *exact* solutions.

a. $\cos(t) = -\frac{1}{2}$

b. $\dfrac{\sin(x)}{2} - \dfrac{\sqrt{3}}{4} = 0$

7. Find *all* of the solutions to the equations below; provide *exact* solutions.

a. $\sin(6t) = -\frac{\sqrt{3}}{2}$

b. $5 + 4\cos(2\theta) = 1$

c. $16\cos(4x) + 11 = 3$

d. $16 - 24\sin(8t) = 4$

8. Find the solutions on the interval $[0, 2\pi)$ to following equations.

a. $5 + 4\cos(2\theta) = 1$

b. $4 - 6\sin(2x) = 7$

c. $6\sqrt{2}\cos(3\alpha) + 10 = 4$

Week 5 Practice Worksheet

Some Additional Practice for the Midterm Exam

This isn't meant to be a “practice test” or “the only thing you need to study to be prepared for the exam.” It's just some additional practice problems. If you're comfortable with all of these problems, then you should be prepared for the exam but, in addition to the problems below, you should also study the ClassNotes/Videos, the Online Lecture Notes, the Weekly Practice Worksheets, the Weekly Graded Worksheets, and the suggested practice problems from the online textbook.

You should NOT use a calculator for ANY of the problems below since you won't be allowed to use a calculator on the Midterm.

1. a. Convert 24° into radians.

b. Convert $\frac{3}{2}$ radians into degrees.

2. Find the arc-length spanned by an angle measuring 75° on a circle of radius 30 feet.

3. Evaluate the following expressions:

a. $\sin\left(\frac{\pi}{3}\right)$.	**b.** $\cos(45^\circ)$.
c. $\sin(330^\circ)$.	**d.** $\cos\left(\frac{\pi}{2}\right)$.
e. $\cos\left(\frac{2\pi}{3}\right)$.	**f.** $\sin\left(\frac{5\pi}{4}\right)$.
g. $\sin\left(\frac{7\pi}{6}\right)$.	**h.** $\cos\left(\frac{11\pi}{6}\right)$.
i. $\cos\left(\frac{17\pi}{6}\right)$.	**k.** $\sin\left(\frac{16\pi}{3}\right)$.
l. $\tan\left(\frac{3\pi}{4}\right)$.	**m.** $\cot\left(\frac{4\pi}{3}\right)$.
n. $\csc(\pi)$.	**o.** $\sec(360^\circ)$.

4. a. Use the sine and cosine functions to find the coordinates of the point P in Figure 1a that is specified by $\frac{5\pi}{3}$ on the circumference of a circle of radius 10 units. **Clearly show your use of the sine and cosine functions**.

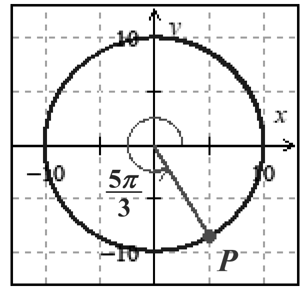


Figure 1a

b. Use the sine and cosine functions to find the coordinates of the point Q in Figure 1b that is specified by $\frac{7\pi}{6}$ on the circumference of a circle of radius 3 units. **Clearly show your use of the sine and cosine functions**.

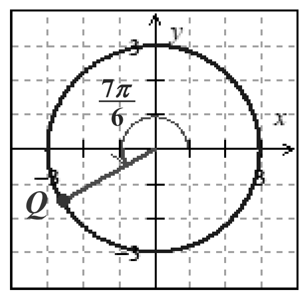


Figure 1b

5. If $\sin(\theta) = \frac{3}{4}$ and $\frac{\pi}{2} < \theta < \pi$, find the exact value of the following:

a. $\cos(\theta)$

b. $\sin(\theta + 2\pi)$

c. $\tan(\theta)$

d. $\sec(\theta)$

e. $\csc(\theta)$

6. Find *all* of the solutions to the following equations:

a. $3\sin(x) + 4 = 5$

b. $2\cos(\theta) = 1$

c. $2\sin(3\theta) + \sqrt{3} = 0$

d. $7 + 3\sqrt{2}\cos(4t) = 4$

7. Find all of the solutions to the following equations on the interval $[0, 2\pi)$.

a. $6\cos(2x) + 5 = 2$

b. $2 - 3\sin(4\theta) = 5$

8. Evaluate the following. Show your steps. (Do not use a calculator!)

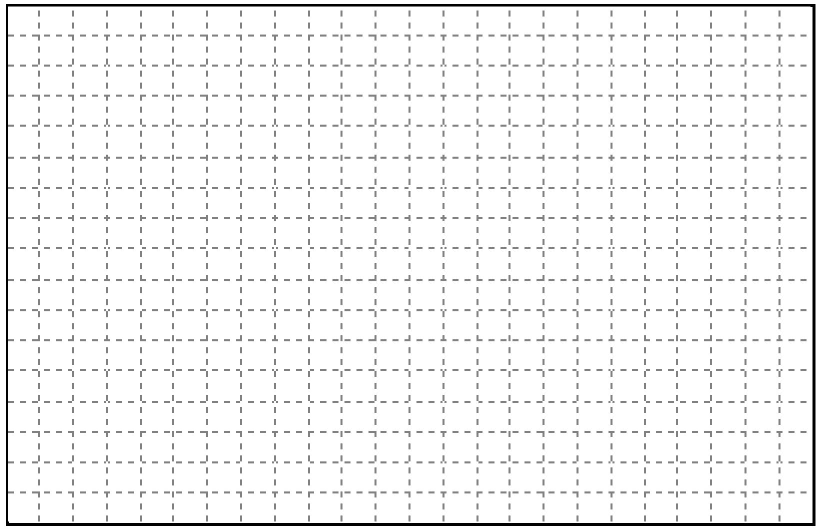
a. $\sin\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$

b. $\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right)$

c. $\cos^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right)$

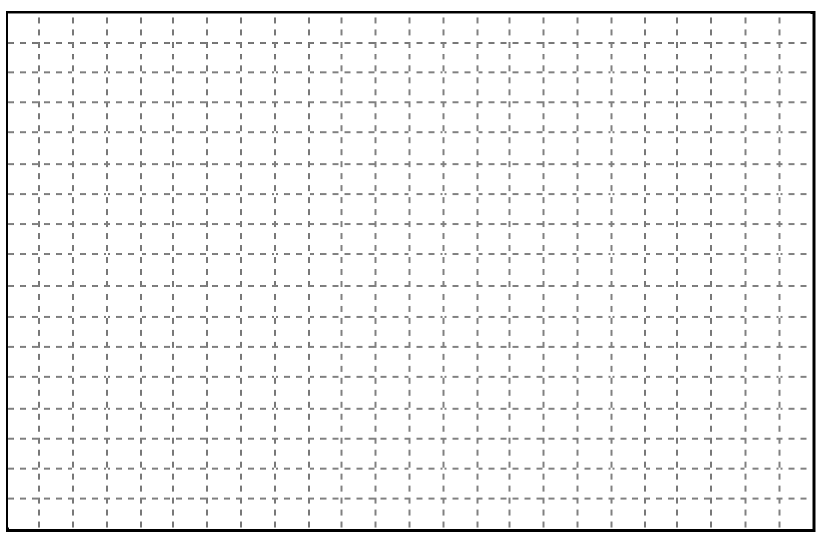
d. $\cos^{-1}\left(\cos\left(\frac{8\pi}{7}\right)\right)$

9. Sketch a graph of the function $g(t) = 3\sin\left(2t + \frac{\pi}{4}\right) - 2$. State the period, midline, and amplitude of g.



Draw a graph of $y = g(t)$.

10. Sketch a graph of the function $q(x) = 2\cos\left(\frac{\pi}{2}x + \frac{\pi}{4}\right) - 3$. State the period, midline, and amplitude of q.



Draw a graph of $y = q(x)$.

11. Find four algebraic rules (one using "positive sine", one using "negative (reflected) sine", one using "positive cosine", and one using "negative (reflected) cosine") for the function $y = p(x)$ graphed in Figure 2.

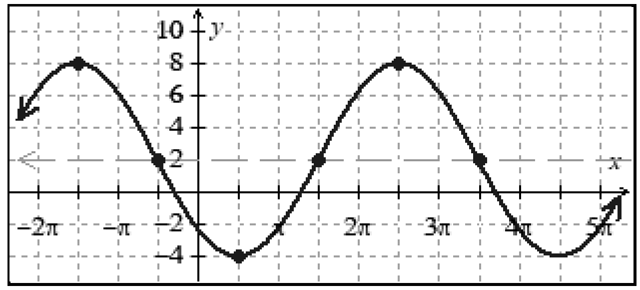


Figure 2: A graph of $y = p(x)$.

12. a. Find four algebraic rules (one using "positive sine", one using "negative (reflected) sine", one using "positive cosine", and one using "negative (reflected) cosine") for the function $y = f(t)$ graphed in Figure 3.

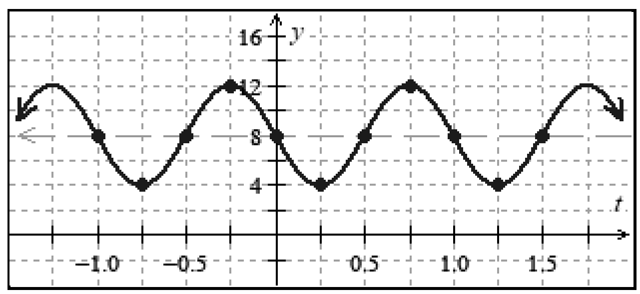


Figure 3: A graph of $y = f(t)$.

b. Use one of your answers to part **a** to find exact solutions to $f(t) = 10$.

Week 6 Practice Worksheet

Right Triangles and Non-right Triangles

1. a. Find the exact value of all six trig functions for the angles A and B in the triangle in Figure 1. (The triangle may not be drawn to scale.)

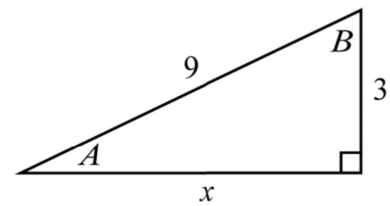


Figure 1

b. Solve the triangle in Figure 1 by finding approximate measurements (in degrees) of angles A and B and the exact length of the side x.

2. **a.** Find the exact value of all six trig functions for the angles θ and ϕ in the triangle in Figure 2. (The triangle may not be drawn to scale.)

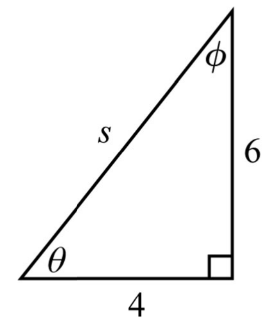


Figure 2

b. Solve the triangle in Figure 2 by finding approximate measurements (in degrees) of angles θ and ϕ and the exact length of the side s.

3. Find the values of c, A, and B in the triangle in Figure 3. You should approximate the values (in degrees for the angles) and denote your approximations correctly.
(The triangle may not be drawn to scale.)

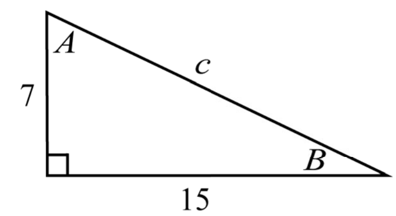


Figure 3

4. Find the values of p, q, and θ in the triangle in Figure 4. You should approximate the values (in degrees for the angles) and denote your approximations correctly.
(The triangle may not be drawn to scale.)

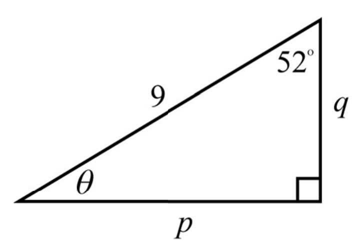


Figure 4

5. Find the values of z, X and Y in the triangle in Figure 5. You should approximate the values (in degrees for the angles) and denote your approximations correctly.
(The triangle may not be drawn to scale.)

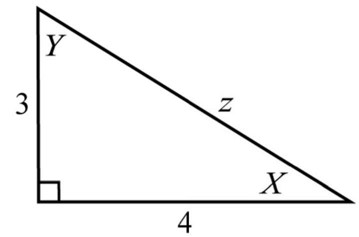


Figure 5

6. Find the values of b, A, and B in the triangle in Figure 6. You should approximate the values (in degrees for the angles) and denote your approximations correctly.
(The triangle may not be drawn to scale.)

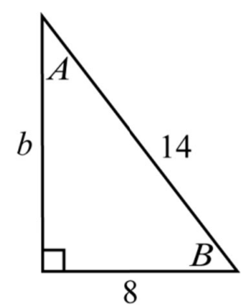


Figure 6

7. In parts (**a**) – (**i**), you are given some info about a non-right triangle that has sides a, b, and c and angles A, B, and C oriented as shown in Figure 7. Find the length of the missing side(s) and the measure of the missing angle(s). (You'll need to use the Laws of Sines and Cosines which are included on the Identities and Formulas Reference Sheet that will be provided to you during the Final Exam so you don't need to memorize them.) You should approximate the values (in degrees for the angles) and denote your approximations correctly. [You'll need your own paper to work these problems.]

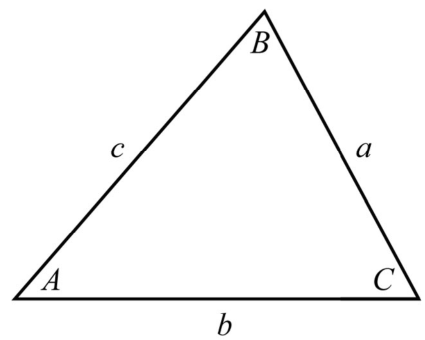


Figure 7

a. $a = 5$, $b = 6$, $c = 7$

b. $B = 76^\circ$, $a = 8$, $c = 6$

c. $B = 118^\circ$, $C = 37^\circ$, $a = 5$

d. $A = 62^\circ$, $B = 70^\circ$, $b = 10$

e. $A = 64^\circ$, $C = 76^\circ$, $b = 9$

f. $C = 40^\circ$, $a = 9$, $b = 13$

g. $A = 40^\circ$, $a = 11$, $c = 8$

h. $A = 28^\circ$, $a = 7$, $c = 12$ (this is an "ambiguous situation" where there are two possible triangles that satisfy the given information: try to find both of the solutions)

i. $C = 67^\circ$, $a = 8$, $c = 5$ (this is an "impossible situation" where there is no triangle that satisfies: try to figure out why it is impossible)

8. As you know, a triangle has three sides and three angles: these are the "six components of a triangle." Notice that in each part of the previous problem (#7) you are given the measurements of **three** of these six components. Sometimes (like in part (a)) you are given the lengths of all three sides; usually you are given a combination of sides and angles; but you aren't ever given all three angle measures: contemplate, discuss, and explain why.

Week 7 Practice Worksheet

1. Prove the following identities. (Be sure to organize your proof as shown in the Online Lecture Notes and class notes videos.) This means that you should start your proof by writing one side of the identity and then use equal signs between equivalent expressions until you obtain the other side of the identity. You should only include one step on each line and you should align your equal signs on the left of each step. Compare your proofs with those given in the solutions to make sure that you are using the correct organization and technique.)

a. $\tan(x)\sec(x) = \sin(x)\sec^2(x)$

b. $\csc(t) - \sin(t) = \cot(t)\cos(t)$

c. $\dfrac{\sec(\theta)}{\sin(\theta)} - \tan(\theta) = \cot(\theta)$

d. $\dfrac{1}{1 - \cos(x)} - \dfrac{1}{1 + \cos(x)} = 2\cot(x)\csc(x)$

e. $\sec(\theta) + \tan(\theta) = \dfrac{\cos(\theta)}{1 - \sin(\theta)}$

f. $\cot(A) = \csc(A)\sec(A) - \tan(A)$

2. Use a **sum-of-angles** or **difference-of-angles identity** to calculate the *exact value* of each of the following. (These identities are included on the Identities and Formulas Reference Sheet that will be provided to you during the Final Exam.)

a. $\sin(165^\circ)$

b. $\cos\left(\frac{13\pi}{12}\right)$

c. $\tan\left(\frac{17\pi}{12}\right)$

3. In order to get familiar with the **sum-of-angles**, **difference-of-angles**, **double-angle** and **half-angle identities**, we'll use these identities to calculate some "friendly" sine and cosine values – so we already know these sine and cosine values and we'll verify that the identities lead us to these values. To help explain the activity part (a) has been worked out for you, and part (b) has been started. (Theses identities are included on the Identities and Formulas Reference Sheet that will be provided to you during the Final Exam.)

a. Find $\sin\left(\frac{\pi}{2}\right)$ using the fact that $\frac{\pi}{2} = \frac{\pi}{6} + \frac{\pi}{3}$.

$$\begin{aligned}
\sin\left(\frac{\pi}{2}\right) &= \sin\left(\frac{\pi}{6} + \frac{\pi}{3}\right) \\
&= \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{3}\right) \\
&= \frac{1}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} \\
&= \frac{1}{4} + \frac{3}{4} \\
&= 1
\end{aligned}$$

We know that $\sin\left(\frac{\pi}{2}\right) = 1$ so the identity gave us the correct value.

b. Find $\cos(\pi)$ using the fact that $\pi = \frac{5\pi}{6} + \frac{\pi}{6}$.

c. Find $\sin\left(\frac{2\pi}{3}\right)$ using the fact that $\frac{2\pi}{3} = 2\cdot\frac{\pi}{3}$. (hint: use a double-angle identity)

d. Find $\cos\left(\frac{\pi}{3}\right)$ using the fact that $\frac{\pi}{3} = 2 \cdot \frac{\pi}{6}$. (hint: use a double-angle identity)

e. Find $\sin\left(\frac{\pi}{4}\right)$ using the fact that $\frac{\pi}{4} = \frac{\pi/2}{2}$. (Hint: use a half-angle identity)

f. Find $\cos(30^\circ)$ using the fact that $30^\circ = \frac{60^\circ}{2}$. (hint: use a half-angle identity)

4. Suppose that $\sin(\alpha) = -\frac{\sqrt{65}}{9}$ and $\pi < \alpha < \frac{3\pi}{2}$. Calculate the *exact value* of each of the following using an appropriate **double-angle** or **half-angle identity**. (These identities are included on the Identities and Formulas Reference Sheet.)

a. $\sin(2\alpha)$. [Hint: first find $\cos(\alpha)$]

b. $\cos(2\alpha)$.

c. $\sin\left(\frac{\alpha}{2}\right)$.

5. Suppose that $\cos(\theta) = \frac{7}{10}$ and $\frac{3\pi}{2} < \theta < 2\pi$. Calculate the *exact value* of each of the following using an appropriate **double-angle** or **half-angle identity**. (These identities are included on the Identities and Formulas Reference Sheet.)

a. $\sin(2\theta)$. [Hint: first find $\sin(\theta)$]

b. $\cos(2\theta)$.

c. $\cos\left(\frac{\theta}{2}\right)$.

6. Suppose that $\sin(x) = \frac{9}{11}$ and $\frac{\pi}{2} < x < \pi$. Calculate the *exact value* of each of the following using an appropriate **double-angle** or **half-angle identity**. (These identities are included on the Identities and Formulas Reference Sheet.)

a. $\sin(2x)$. [Hint: first find $\cos(x)$]

b. $\cos(2x)$.

c. $\sin\left(\frac{x}{2}\right)$.

7. Prove the following identities using the double-angle identities for sine and cosine included on the Identities and Formulas Reference Sheet. (Be sure to organize your proof as shown in the Online Lecture Notes and class notes videos.)

a. $\tan(2x) = \dfrac{2\tan(x)}{1 - \tan^2(x)}$ (this is known as the double-angle identity for tangent; to prove it, start with the left side and use the double-angle identities for sine and cosine; for cosine, use $\cos(2x) = \cos^2(x) - \sin^2(x)$; a "trick" here is to introduce a term that will create the denominator that you need)

b. $\dfrac{1 - \cos(2t)}{\sin(2t)} = \tan(t)$

Week 8 Practice Worksheet

1. Translate each of the following rectangular coordinates (x, y) into polar coordinates (r, θ). Your answers should be ordered pairs involving exact values, with θ in radians.

a. $(x, y) = (-4, -4)$

b. $(x, y) = \left(6, -6\sqrt{3}\right)$

2. Translate each of the following rectangular coordinates $(x,\ y)$ into polar coordinates $(r,\ \theta)$. Your answers should be ordered pairs involving approximations of θ in radians.

a. $(x,\ y) = (10,\ -2)$

b. $(x,\ y) = (-3,\ 7)$

3. Translate each of the following polar coordinates (r, θ) into rectangular coordinates (x, y). Your answers should be ordered pairs involving exact values.

a. $(r, \theta) = \left(5, \frac{2\pi}{3}\right)$

b. $(r, \theta) = \left(16, 210^\circ\right)$

4. Translate each of the following polar coordinates (r, θ) into rectangular coordinates (x, y). Your answers should be ordered pairs involving approximate values.

a. $(r, \theta) = \left(3, 80^\circ\right)$

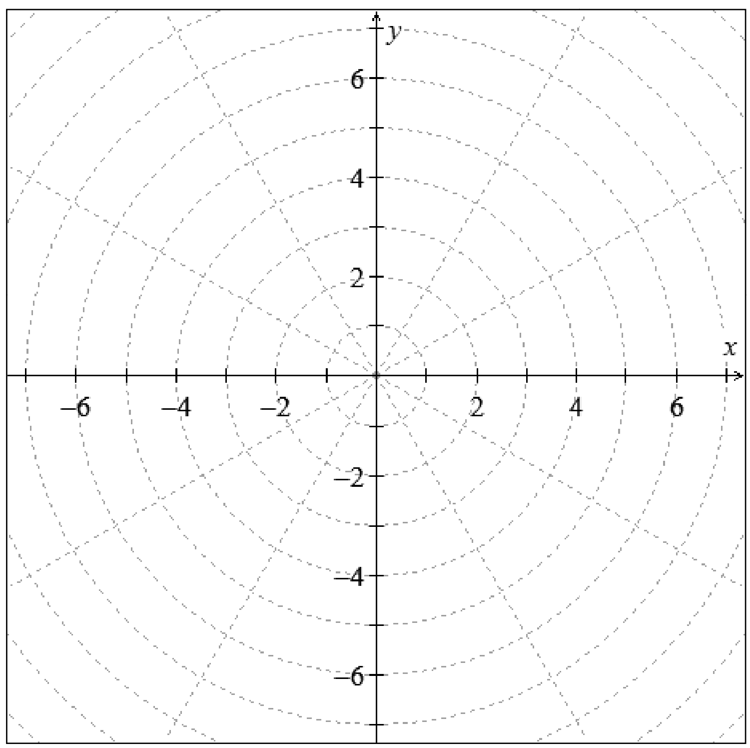
b. $(r, \theta) = \left(7, -\frac{\pi}{10}\right)$

5. The polar ordered pair $\left(5, \frac{\pi}{3}\right)$ can be plotted as a "dot" on the polar coordinate plane. List four other (different) polar ordered pairs that are plotted on the same "dot." Use "$r = -5$" for at least one of your ordered pairs.

6. Complete the 1st column of the table below with appropriate multiples of $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$; then determine the corresponding values of r if $r = 6\cos(2\theta)$ in order to complete the 2nd column of the table; then plot the points implied by each of the 16 rows of the table on the polar plane below and connect those points in order to draw a graph of $r = 6\cos(2\theta)$.

(HINT: Graph the function in Desmos so that you can predict its shape before you draw it.)

	θ	r
	0	
Quad. 1 angles		
	$\pi/2$	
Quad. 2 angles		
	π	
Quad. 3 angles		
	$3\pi/2$	
Quad. 4 angles		



Draw a graph of $r = 6\cos(2\theta)$.

7. Express the complex number $z = 10e^{i \cdot \frac{11\pi}{6}}$ in the form $z = a + bi$.

8. Express the complex number $z = 8e^{i \cdot \frac{2\pi}{3}}$ in the form $z = a + bi$.

9. Express the complex number $z = -7e^{i \cdot \frac{5\pi}{4}}$ in the form $z = a + bi$.

10. Find a polar form, $z = re^{i\theta}$, of the complex number $z = -3 - 3\sqrt{3}\,i$.

11. Find a polar form, $z = re^{i\theta}$, of the complex number $z = 2 - 2i$.

12. Find a polar form, $z = re^{i\theta}$, of the complex number $z = -4\sqrt{3} + 12i$.

13. Find three different a polar forms, $z = re^{i\theta}$, of the complex number $z = 4i$. (HINT: $i = 0 + 4i$ can be associated with the point $(0, 4)$ so find three different angles that can be used to represent the "direction of this point" and use each angle to create a polar form.)

Week __ Practice Worksheet

1. Suppose that $\vec{v} = \langle -3, 7 \rangle$ and $\vec{w} = \langle 2, 10 \rangle$.

a. Express $\vec{v}$ and $\vec{w}$ using unit vectors.

b. Find $\|\vec{v}\|$ and $\|\vec{w}\|$.

c. Find $2\vec{v} - 5\vec{w}$.

d. Find $4\vec{w} + 3\vec{v}$.

2. Suppose that $\vec{m} = 7\vec{i} - 4\vec{j}$ and $\vec{n} = -5\vec{i} - 2\vec{j}$.

a. Express $\vec{m}$ and $\vec{n}$ using “pointy vector brackets” (i.e., $\langle a, b \rangle$).

b. Find $\|\vec{m}\|$ and $\|\vec{n}\|$.

c. Find $\vec{m} + \vec{n}$.

d. Find $3\vec{m} - \vec{n}$.

3. Suppose that $\vec{p} = \langle -1, 4 \rangle$ and $\vec{q} = \langle 3, -5 \rangle$.

a. Express $\vec{p}$ and $\vec{q}$ using unit vectors.

b. Find $\|\vec{p}\|$ and $\|\vec{q}\|$.

c. Find $2\vec{p} + 3\vec{q}$.

d. Find $2\vec{q} - 3\vec{p}$.

4. Suppose that the tail (or initial point) of $\vec{b}$ is $(2, -3)$ and the tip (or terminal point) is $(-4, 7)$. Find the components of $\vec{b}$ in order to express $\vec{b}$ using both "pointy vector brackets" and unit vectors.

5. Suppose that the tail (or initial point) of $\vec{r}$ is $(-5, 1)$ and the tip (or terminal point) is $(3, 6)$. Find the components of $\vec{r}$ in order to express $\vec{r}$ using both "pointy vector brackets" and unit vectors.

6. Suppose that $\|\vec{a}\| = 34$ and that $\vec{a}$ makes an angle of 150° with the positive x-axis. Find the components of $\vec{a}$ in order to express $\vec{a}$ using both "pointy vector brackets" and unit vectors.

7. Suppose that $\|\vec{s}\| = 18$ and that $\vec{s}$ makes an angle of -45° with the positive x-axis. Find the components of $\vec{s}$ in order to express $\vec{s}$ using both "pointy vector brackets" and unit vectors.

8. Suppose that $\vec{v} = \langle -3, 7 \rangle$ and $\vec{w} = \langle 2, 10 \rangle$.

a. Find $\vec{v} \cdot \vec{w}$.

b. Find the angle between $\vec{v}$ and $\vec{w}$.

9. Suppose that $\vec{m} = 7\vec{i} - 4\vec{j}$ and $\vec{n} = -5\vec{i} - 2\vec{j}$.

a. Find $\vec{m} \cdot \vec{n}$.

b. Find the angle between $\vec{m}$ and $\vec{n}$.

10. Suppose that $\vec{p} = \langle -1, 4 \rangle$ and $\vec{q} = \langle 3, -5 \rangle$.

a. Find $\vec{p} \cdot \vec{q}$.

b. Find the angle between $\vec{p}$ and $\vec{q}$.

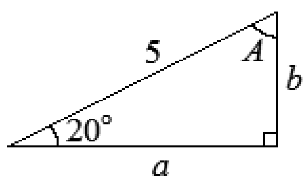
Week __ Practice Worksheet

Some Additional Practice for the Final Exam

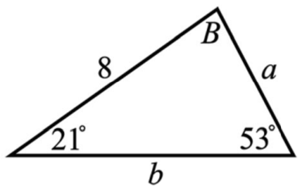
This isn't meant to be a "practice test" or "the only thing you need to study to be prepared for the exam." It's just some additional practice problems covering the material that we studied after the Midterm. In addition to the problems below, you should also study the Class Notes Videos, the Online Lecture Notes, the Weekly Practice Worksheets (especially "Week 5: Practice Worksheet"), the Weekly Graded Worksheets, and the suggested practice problems from the online textbook.

1. Find the missing side(s) and missing angle(s) for the triangles given below. (The triangles may not be drawn to scale.)

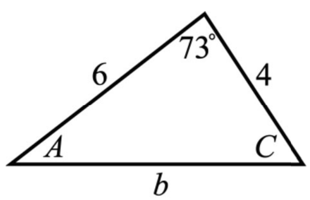
a.



b.



c.



2. Suppose that $\sin(\alpha) = \frac{5}{13}$ and $\cos(\beta) = \frac{3}{5}$, and where $0 < \alpha < \frac{\pi}{2}$ and $\frac{3\pi}{2} < \beta < 2\pi$. Find the exact value of:

a. $\sin(\alpha + \beta)$.

b. $\cos(\alpha - \beta)$.

c. $\sin(2\beta)$.

d. $\cos(2\beta)$.

e. $\sin\left(\frac{\beta}{2}\right)$.

f. $\cos\left(\frac{\beta}{2}\right)$.

3. Prove the following identities.

a. $\tan(x) + \cot(x) = \sec(x)\csc(x)$

b. $\tan^2(x) - \sin^2(x) = \tan^2(x)\sin^2(x)$

c. $\cos(2x) = \cos^4(x) - \sin^4(x)$

4. Convert the following polar ordered pairs into Cartesian (i.e., rectangular) coordinates.

a. $\left(3, \frac{\pi}{2}\right)$

b. $\left(\pi, \frac{5\pi}{3}\right)$

c. $(10, -10^{\circ})$

5. Convert the following Cartesian (i.e., rectangular) ordered pairs into polar coordinates.

a. $(10, -10)$

b. $(-3, 0)$

c. $\left(-8, -8\sqrt{3}\right)$

6. Translate the complex number $z = -3 + 3\sqrt{3} \cdot i$ into its polar form $z = re^{i\theta}$.

7. Translate the polar form of the complex number $z = 4e^{i \cdot \frac{5\pi}{6}}$ into its rectangular form $z = a + bi$.

8. Determine the magnitude and direction (with respect to the positive x-axis) of the vector $\vec{v} = \langle -3, -7 \rangle$.

9. a. Find the horizontal and vertical components of the vector $\vec{v}$ that starts at the point $P = (5, 6)$ and ends at the point $Q = (2, 2)$.

b. Find the magnitude, $\|\vec{v}\|$, and the direction (with respect to the positive x-axis) of the vector $\vec{v}$ that you found in part **a**?

10. Suppose $\vec{v} = \langle -4, 1 \rangle$ and $\vec{u} = \langle 3, -6 \rangle$.

a. Find $\vec{w} = \vec{v} - 2\vec{u}$.

b. Use the *dot product* to find the angle between $\vec{v} = \langle -4, 1 \rangle$ and $\vec{u} = \langle 3, -6 \rangle$?