SOLUTIONS: Practice Worksheet: Right Triangles

1. a. Find the exact value of all six trig functions for the angles A and B in the triangle in Fig. 1. (The triangle may not be drawn to scale.)

First let's use the Pythagorean Theorem to find x

$$\begin{aligned} x^2 + 3^2 &= 9^2 \\ \Rightarrow \quad x^2 + 9 &= 81 \\ \Rightarrow \quad x^2 &= 72 \\ \Rightarrow \quad x &= \sqrt{72} \\ \Rightarrow \quad x &= 6\sqrt{2} \end{aligned}$$

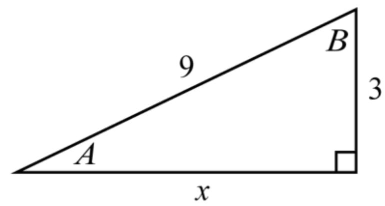

Figure 1

Now, let's find the value of all six trig functions for the angle A:

$\sin(A) = \frac{\text{OPP}}{\text{HYP}} = \frac{3}{9} = \frac{1}{3}$	$\cos(A) = \frac{\text{ADJ}}{\text{HYP}} = \frac{6\sqrt{2}}{9} = \frac{2\sqrt{2}}{3}$	$\tan(A) = \frac{\text{OPP}}{\text{ADJ}} = \frac{3}{6\sqrt{2}} = \frac{1}{2\sqrt{2}}$
$\csc(A) = \frac{1}{\sin(A)} = \frac{1}{\frac{1}{3}} = 3$	$\sec(A) = \frac{1}{\cos(A)} = \frac{1}{\frac{2\sqrt{2}}{3}} = \frac{3}{2\sqrt{2}}$	$\cot(A) = \frac{1}{\tan(A)} = \frac{1}{\frac{1}{2\sqrt{2}}} = 2\sqrt{2}$

Finally, let's find the value of all six trig functions for the angle B:

$\sin(B) = \frac{\text{OPP}}{\text{HYP}} = \frac{6\sqrt{2}}{9} = \frac{2\sqrt{2}}{3}$	$\cos(B) = \frac{\text{ADJ}}{\text{HYP}} = \frac{3}{9} = \frac{1}{3}$	$\tan(B) = \frac{\text{OPP}}{\text{ADJ}} = \frac{6\sqrt{2}}{3} = 2\sqrt{2}$
$\csc(B) = \frac{1}{\sin(B)} = \frac{1}{\frac{2\sqrt{2}}{3}} = \frac{3}{2\sqrt{2}}$	$\sec(B) = \frac{1}{\cos(B)} = \frac{1}{\frac{1}{3}} = 3$	$\cot(B) = \frac{1}{\tan(B)} = \frac{1}{2\sqrt{2}}$

b. Solve the triangle in Figure 1 by finding approximate measurements (in degrees) of angles A and B and the exact length of the side x.

In part (**a**) we discovered that $x = 6\sqrt{2}$.

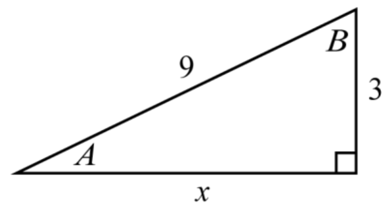

Figure 1 (again)

To find angle A we can use the given information and the sine function (I'm choosing to use sine instead of cosine or tangent since then I can use the given information, rather than information that I've derived; this way, if I've made a mistake on a previous portion of the problem, it won't impact this part of the problem):

$$\begin{aligned}
& \sin(A) = \frac{\text{OPP}}{\text{HYP}} \\
\Rightarrow\quad & \sin(A) = \frac{3}{9} = \frac{1}{3} \\
\Rightarrow\quad & \sin^{-1}(\sin(A)) = \sin^{-1}\left(\frac{1}{3}\right) \\
\Rightarrow\quad & A = \sin^{-1}\left(\frac{1}{3}\right) \approx 19.47^\circ
\end{aligned}$$

Now we can find angle B we can use the fact that the total angle measure in a triangle is always 180°:

$$\begin{aligned}
& A + B + 90^\circ = 180^\circ \\
\Rightarrow\quad & B = 180^\circ - 90^\circ - A \\
\Rightarrow\quad & B \approx 180^\circ - 90^\circ - 19.47^\circ \\
\Rightarrow\quad & B \approx 70.53^\circ
\end{aligned}$$

2. a. Find the exact value of all six trig functions for the angles θ and ϕ in the triangle in Fig. 2. (The triangle may not be drawn to scale.)

First let's use the Pythagorean Theorem to find s:

$$4^2 + 6^2 = s^2$$
$$\Rightarrow 16 + 36 = s^2$$
$$\Rightarrow 52 = s^2$$
$$\Rightarrow s = \sqrt{52} = 2\sqrt{13}$$

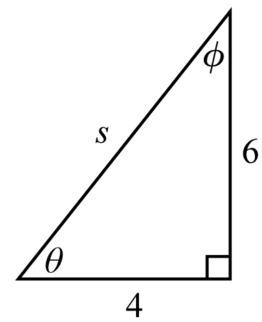

Figure 2

Now, let's find the value of all six trig functions for the angle θ:

$\sin(\theta) = \frac{\text{OPP}}{\text{HYP}} = \frac{6}{2\sqrt{13}} = \frac{3}{\sqrt{13}}$	$\cos(\theta) = \frac{\text{ADJ}}{\text{HYP}} = \frac{4}{2\sqrt{13}} = \frac{2}{\sqrt{13}}$	$\tan(\theta) = \frac{\text{OPP}}{\text{ADJ}} = \frac{6}{4} = \frac{3}{2}$
$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{\frac{3}{\sqrt{13}}} = \frac{\sqrt{13}}{3}$	$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{\frac{2}{\sqrt{13}}} = \frac{\sqrt{13}}{2}$	$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

Finally, let's find the value of all six trig functions for the angle ϕ:

$\sin(\phi) = \frac{\text{OPP}}{\text{HYP}} = \frac{4}{2\sqrt{13}} = \frac{2}{\sqrt{13}}$	$\cos(\phi) = \frac{\text{ADJ}}{\text{HYP}} = \frac{6}{2\sqrt{13}} = \frac{3}{\sqrt{13}}$	$\tan(\phi) = \frac{\text{OPP}}{\text{ADJ}} = \frac{4}{6} = \frac{2}{3}$
$\csc(\phi) = \frac{1}{\sin(\phi)} = \frac{1}{\frac{2}{\sqrt{13}}} = \frac{\sqrt{13}}{2}$	$\sec(\phi) = \frac{1}{\cos(\phi)} = \frac{1}{\frac{3}{\sqrt{13}}} = \frac{\sqrt{13}}{3}$	$\cot(\phi) = \frac{1}{\tan(\phi)} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

b. Solve the triangle in Figure 2 by finding approximate measurements (in degrees) of angles θ and ϕ and the exact length of the side s.

In part (**a**) we discovered that $s = 2\sqrt{13}$.

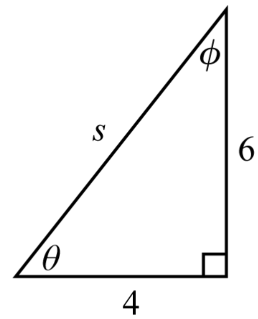

Figure 2 (again)

To find angle θ we can use the given information and the tangent function (I'm choosing to use tangent instead of sine or cosine since then I can use the given information, rather than information that I've derived; this way, if I've made a mistake on a previous portion of the problem, it won't impact this part of the problem):

$$\tan(\theta) = \frac{\text{OPP}}{\text{ADJ}}$$

$$\Rightarrow \quad \tan(\theta) = \frac{6}{4} = \frac{3}{2}$$

$$\Rightarrow \quad \tan^{-1}\left(\tan(\theta)\right) = \tan^{-1}\left(\frac{3}{2}\right)$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{3}{2}\right) \approx 56.3^\circ$$

Now we can find angle ϕ we can use the fact that the total angle measure in a triangle is always 180°:

$$\theta + \phi + 90^\circ = 180^\circ$$

$$\Rightarrow \quad \phi = 180^\circ - 90^\circ - \theta$$

$$\Rightarrow \quad \phi \approx 180^\circ - 90^\circ - 56.3^\circ$$

$$\Rightarrow \quad \phi \approx 33.7^\circ$$

3. Find the values of c, A, and B in the triangle in Figure 3. You should approximate the values (in degrees for the angles) and denote your approximations correctly.
(The triangle may not be drawn to scale.)

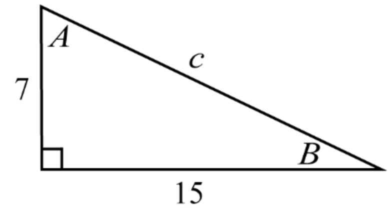

Figure 3

First let's use the Pythagorean Theorem to find the length of the side c:

$$\begin{aligned} & c^2 = 7^2 + 15^2 \\ \Rightarrow\ & c^2 = 49 + 225 \\ \Rightarrow\ & c^2 = 274 \\ \Rightarrow\ & c = \sqrt{274} \approx 16.55 \end{aligned}$$

Now we can use the tangent function to get an equation involving angle B and then solve the equation for B:

$$\begin{aligned} & \tan(B) = \tfrac{7}{15} \\ \Rightarrow\ & B = \tan^{-1}\left(\tfrac{7}{15}\right) \approx 25.02^\circ \end{aligned}$$

Note that we've chosen to use tangent (instead of sine or cosine) to find B since that allows us to use the given side-lengths. If we use sine or cosine, we would need to use the value for c that we found above but it's possible that we made a mistake so, whenever possible, it's more sensible rely on the given information rather than on information that we've found ourselves.

Finally, we can use the rule that the sum of the angles in a triangle is always 180° to find angle A:

$$\begin{aligned} & A + B + 90^\circ = 180^\circ \\ \Rightarrow\ & A + 25.02^\circ + 90^\circ \approx 180^\circ \\ \Rightarrow\ & A \approx 180^\circ - 90^\circ - 25.02^\circ \\ \Rightarrow\ & A \approx 64.98^\circ \end{aligned}$$

4. Find the values of p, q, and θ in the triangle in Figure 4. You should approximate the values (in degrees for the angles) and denote your approximations correctly. (The triangle may not be drawn to scale.)

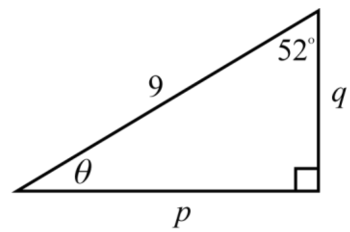

Figure 4

First we'll use the rule that the sum of the angles in a triangle is always 180° to find θ:

$$\theta + 52^\circ + 90^\circ = 180^\circ$$

$$\Rightarrow \quad \theta = 180^\circ - 90^\circ - 52^\circ$$

$$\Rightarrow \quad \theta = 38^\circ$$

Now we'll use the sine function to create an equation involving side p and then solve the equation for p:

$$\sin(52^\circ) = \frac{p}{9}$$

$$\Rightarrow \quad p = 9 \cdot \sin(52^\circ)$$

$$\Rightarrow \quad p \approx 7.09$$

Now we can use the cosine function to get an equation involving angle q and then solve the equation for q:

$$\cos(52^\circ) = \frac{q}{9}$$

$$\Rightarrow \quad q = 9 \cdot \cos(52^\circ)$$

$$\Rightarrow \quad q \approx 5.54$$

(We can check that $(7.09)^2 + (5.54)^2 \approx 9^2$ so at least this isn't an obviously-impossible result!)

5. Find the values of X and Y in the triangle in Figure 5. You should approximate the values (in degrees for the angles) and denote your approximations correctly.
(The triangle may not be drawn to scale.)

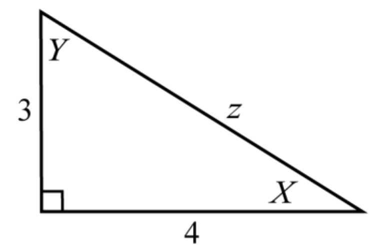

Figure 5

First let's use the Pythagorean Theorem to find the length of the side z

$$
\begin{aligned}
& z^2 = 3^2 + 4^2 \\
\Rightarrow\ & z^2 = 9 + 16 \\
\Rightarrow\ & z^2 = 25 \\
\Rightarrow\ & z = 5
\end{aligned}
$$

There are a variety of ways find the angles X and Y; here's one option in which we only use the sine function:

First we'll use the sine function to get an equation involving angle X and then solve the equation for X:

$$
\begin{aligned}
& \sin(X) = \frac{\text{OPP OF } X}{\text{HYP}} \\
\Rightarrow\ & \sin(X) = \tfrac{3}{5} \\
\Rightarrow\ & X = \sin^{-1}\left(\tfrac{3}{5}\right) \\
\Rightarrow\ & X \approx 36.87^\circ
\end{aligned}
$$

Now we can use the sine function to get an equation involving angle Y and then solve the equation for Y:

$$
\begin{aligned}
& \sin(Y) = \frac{\text{OPP OF } Y}{\text{HYP}} \\
\Rightarrow\ & \sin(Y) = \tfrac{4}{5} \\
\Rightarrow\ & Y = \sin^{-1}\left(\tfrac{4}{5}\right) \\
\Rightarrow\ & Y \approx 53.13^\circ
\end{aligned}
$$

(We can check that $90^\circ + 36.87^\circ + 53.13^\circ = 180^\circ$ so at least this isn't an obviously-impossible result!)

6. Find the values of c, A, and B in the triangle in Figure 6. You should approximate the values (in degrees for the angles) and denote your approximations correctly.
(The triangle may not be drawn to scale.)

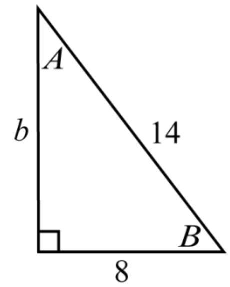

Figure 6

First let's use the Pythagorean Theorem to find the length of the side b:

$$b^2 + 8^2 = 14^2$$
$$\Rightarrow \quad b^2 = 196 - 64$$
$$\Rightarrow \quad b^2 = 132$$
$$\Rightarrow \quad b = \sqrt{132} \approx 11.49$$

Now we can use the cosine function to get an equation involving angle B and then solve the equation for B:

$$\cos(B) = \frac{8}{14}$$
$$\Rightarrow \quad B = \cos^{-1}\left(\frac{8}{14}\right) \approx 55.15^\circ$$

Note that we've chosen to use cosine (instead of sine or tangent) to find B because that allows us to use the given side-lengths. If we use sine or tangent, we would need to use the value for b that we found above but it's possible that we made a mistake so, whenever possible, it's more sensible rely on the given information rather than on information that we've found ourselves.

Finally, we can use the rule that the sum of the angles in a triangle is always 180° to find angle A:

$$A + B + 90^\circ = 180^\circ$$
$$\Rightarrow \quad A + 55.15^\circ + 90^\circ \approx 180^\circ$$
$$\Rightarrow \quad A \approx 180^\circ - 90^\circ - 55.15^\circ$$
$$\Rightarrow \quad A \approx 34.85^\circ$$