SOLUTIONS: Practice Worksheet: Solving Trig Equations

1. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to ***directly communicate*** what the given expressions equal.

a. $\sin(t) = -\frac{\sqrt{2}}{2}$

$$\sin(t) = -\frac{\sqrt{2}}{2}$$

$$\Rightarrow \quad t = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) + 2k\pi \quad \text{or} \quad t = \pi - \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad t = -\frac{\pi}{4} + 2k\pi \quad \text{or} \quad t = \pi - \left(-\frac{\pi}{4}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad t = -\frac{\pi}{4} + 2k\pi \quad \text{or} \quad t = \frac{5\pi}{4} + 2k\pi, \quad k \in \mathbb{Z}$$

(In the step indicated by a red arrow ($\Rightarrow$) we've used the identity $\sin(t) = \sin(\pi - t)$ in order to generate the second family of solutions Instead of using the inverse trig function in this step, you might choose to obtain the step indicated by a pink arrow ($\Rightarrow$) by utilizing your awareness of these two facts: $\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ and $\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ although using "awareness" might inspire you to use "$\frac{7\pi}{4}$" instead of "$-\frac{\pi}{4}$" for the first family of solutions, which will to lead you to $t = \frac{7\pi}{4} + 2k\pi$" instead of $t = -\frac{\pi}{4} + 2k\pi$ for the second family of solutions)

b. $2\cos(x) - \sqrt{3} = 0$

$$2\cos(x) - \sqrt{3} = 0$$

$$\Rightarrow \quad 2\cos(x) = \sqrt{3}$$

$$\Rightarrow \quad \cos(x) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + 2k\pi \quad \text{or} \quad x = -\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = -\frac{\pi}{6} + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = -\frac{\pi}{6} + 2k\pi, \quad k \in \mathbb{Z}$$

(In the step indicated by a red arrow ($\Rightarrow$) we've used the identity $\cos(x) = -\cos(x)$ in order to generate the second family of solutions. Instead of using the inverse trig function in this step, you might choose to obtain the step indicated by a pink arrow ($\Rightarrow$) by utilizing your awareness of these two facts: $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$, although using "awareness" might inspire you to use "$\frac{11\pi}{6}$" instead of "$-\frac{\pi}{6}$" for the second family of solutions, which will to lead you to $x = \frac{11\pi}{6} + 2k\pi$ instead of $x = -\frac{\pi}{6} + 2k\pi$.)

2. Find the solutions on the interval $[0, 2\pi)$ for the equations below; provide *exact* solutions.

a. $\cos(t) = -\frac{1}{2}$

$$\cos(t) = -\frac{1}{2}$$

$$\Rightarrow \quad t = \cos^{-1}\left(-\frac{1}{2}\right) + 2k\pi \quad \text{or} \quad x = -\cos^{-1}\left(-\frac{1}{2}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad t = \frac{2\pi}{3} + 2k\pi \quad \text{or} \quad x = -\frac{2\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$$

Now we need to substitute particular values of k in order to determine which solutions fall on the interval $[0, 2\pi)$:

$k = -1$: $x = \frac{2\pi}{3} + 2(-1)\pi = \frac{2\pi}{3} - \frac{6\pi}{3} = -\frac{4\pi}{3}$ or $x = -\frac{2\pi}{3} + 2(-1)\pi = -\frac{2\pi}{3} - \frac{6\pi}{3} = -\frac{8\pi}{3}$

Since both of these values are negative, they aren't in the interval $[0, 2\pi)$. There's no reason to try smaller values of k since they'll produce yet smaller solutions which will certainly be outside the interval.

$k = 0$: $x = \frac{2\pi}{3} + 2(0)\pi = \frac{2\pi}{3}$ or $x = -\frac{2\pi}{3} + 2(0)\pi = -\frac{2\pi}{3}$

Only $\frac{2\pi}{3}$ is in the given interval.

$k = 1$: $x = \frac{2\pi}{3} + 2(1)\pi = \frac{2\pi}{3} + 2\pi > 2\pi$ or $x = -\frac{2\pi}{3} + 2(1)\pi = -\frac{2\pi}{3} + \frac{6\pi}{3} > 2\pi = \frac{4\pi}{3}$

Only $\frac{2\pi}{3}$ is in the given interval. There's no reason to try larger values of k since they'll produce yet greater solutions which will certainly be outside the given interval.

Therefore, the solution set for $\cos(t) = -\frac{1}{2}$ on the interval $[0, 2\pi)$ is $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$.

b. $\dfrac{\sin(x)}{2} - \dfrac{\sqrt{3}}{4} = 0$

$$\frac{\sin(x)}{2} - \frac{\sqrt{3}}{4} = 0$$

$$2 \cdot \frac{\sin(x)}{2} = \frac{\sqrt{3}}{4} \cdot 2$$

$$\sin(x) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad t = \sin^{-1}\left(\tfrac{\sqrt{3}}{2}\right) + 2k\pi \text{ or } t = \pi - \sin^{-1}\left(\tfrac{\sqrt{3}}{2}\right) + 2k\pi, \ k \in \mathbb{Z}$$

$$\Rightarrow \quad t = \tfrac{\pi}{3} + 2k\pi \quad \text{or} \quad t = \pi - \left(\tfrac{\pi}{3}\right) + 2k\pi, \ k \in \mathbb{Z}$$

$$\Rightarrow \quad t = \tfrac{\pi}{3} + 2k\pi \quad \text{or} \quad t = \tfrac{2\pi}{3} + 2k\pi, \ k \in \mathbb{Z}$$

Now we need to substitute particular values of k in order to determine which solutions fall on the interval $[0, 2\pi)$:

$k = -1$: $x = \tfrac{\pi}{3} + 2(-1)\pi = \tfrac{\pi}{3} - \tfrac{6\pi}{3} = -\tfrac{5\pi}{3}$ or $x = \tfrac{2\pi}{3} + 2(-1)\pi = \tfrac{2\pi}{3} - \tfrac{6\pi}{3} = -\tfrac{4\pi}{3}$

Since both of these values are negative, they aren't in the interval $[0, 2\pi)$. There's no reason to try smaller values of k since they'll produce yet smaller solutions which will certainly be outside the interval.

$k = 0$: $x = \tfrac{\pi}{3} + 2(0)\pi = \tfrac{\pi}{3}$ or $x = \tfrac{2\pi}{3} + 2(0)\pi = \tfrac{2\pi}{3}$

Both $\tfrac{\pi}{3}$ and $\tfrac{2\pi}{3}$ are in the given interval.

$k = 1$: $x = \tfrac{\pi}{3} + 2(1)\pi = \tfrac{\pi}{3} + 2\pi > 2\pi$ or $x = \tfrac{2\pi}{3} + 2(1)\pi = \tfrac{2\pi}{3} + 2\pi > 2\pi$

Both of these solutions are outside the given interval. There's no reason to try larger values of k since they'll produce yet greater solutions which will certainly be outside the given interval.

Therefore, the solution set for $\dfrac{\sin(x)}{2} - \dfrac{\sqrt{3}}{4} = 0$ on the interval $[0, 2\pi)$ is $\left\{\tfrac{\pi}{3}, \tfrac{2\pi}{3}\right\}$.

3. Find *all* of the solutions to the equations below; provide *exact* solutions.

a. $\sin(6t) = -\frac{\sqrt{3}}{2}$

$$\sin(6t) = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad 6t = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + 2k\pi \quad \text{or} \quad 6t = \pi - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad 6t = -\frac{\pi}{3} + 2k\pi \quad \text{or} \quad 6t = \pi - \left(-\frac{\pi}{3}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad \frac{6t}{6} = \frac{-\frac{\pi}{3}}{6} + \frac{2k\pi}{6} \quad \text{or} \quad \frac{6t}{6} = \frac{\frac{4\pi}{3}}{6} + \frac{2k\pi}{6}, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad t = -\frac{\pi}{18} + \frac{k\pi}{3} \quad \text{or} \quad t = \frac{2\pi}{9} + \frac{k\pi}{3}, \quad k \in \mathbb{Z}$$

b. $5 + 4\cos(2\theta) = 1$

$$5 + 4\cos(2\theta) = 1$$

$$\Rightarrow \quad 4\cos(2\theta) = -4$$

$$\Rightarrow \quad \cos(2\theta) = -1$$

$$\Rightarrow \quad 2\theta = \cos^{-1}(-1) + 2k\pi, \quad k \in \mathbb{Z}$$

(there's only one "family" of solutions since the cosine function achieves the output -1 only once in each period)

$$\Rightarrow \quad 2\theta = \pi + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad \frac{2\theta}{2} = \frac{\pi}{2} + \frac{2k\pi}{2}, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad \theta = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}$$

c. $16\cos(4x) + 11 = 3$

$$16\cos(4x) + 11 = 3$$

$$\Rightarrow \quad 16\cos(4x) = -8$$

$$\Rightarrow \quad \cos(4x) = \tfrac{-8}{16} = -\tfrac{1}{2}$$

$$\Rightarrow \quad 4x = \cos^{-1}\left(-\tfrac{1}{2}\right) + 2k\pi \quad \text{or} \quad 4x = -\cos^{-1}\left(-\tfrac{1}{2}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad 4x = \tfrac{2\pi}{3} + 2k\pi \quad \text{or} \quad 4x = -\tfrac{2\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad \frac{4x}{4} = \frac{\frac{2\pi}{3}}{4} + \frac{2k\pi}{4} \quad \text{or} \quad \frac{4x}{4} = \frac{-\frac{2\pi}{3}}{4} + \frac{2k\pi}{4}, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad x = \tfrac{2\pi}{3\cdot 4} + \tfrac{2k\pi}{4} \quad \text{or} \quad x = -\tfrac{2\pi}{3\cdot 4} + \tfrac{k\pi}{2}, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad x = \tfrac{\pi}{6} + \tfrac{k\pi}{2} \quad \text{or} \quad x = -\tfrac{\pi}{6} + \tfrac{k\pi}{2}, \quad k \in \mathbb{Z}$$

d. $16 - 24\sin(8t) = 4$

$$16 - 24\sin(8t) = 4$$

$$\Rightarrow \quad -24\sin(8t) = -12$$

$$\Rightarrow \quad \sin(8t) = \tfrac{-12}{-24} = \tfrac{1}{2}$$

$$\Rightarrow \quad 8t = \sin^{-1}\left(\tfrac{1}{2}\right) + 2k\pi \quad \text{or} \quad 8t = \pi - \sin^{-1}\left(\tfrac{1}{2}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad 8t = \tfrac{\pi}{6} + 2k\pi \quad \text{or} \quad 8t = \pi - \tfrac{\pi}{6} + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad \frac{8t}{8} = \frac{\frac{\pi}{6}}{8} + \frac{2k\pi}{8} \quad \text{or} \quad \frac{8t}{8} = \frac{\frac{5\pi}{6}}{8} + \frac{2k\pi}{8}, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad t = \tfrac{\pi}{48} + \tfrac{k\pi}{4} \quad \text{or} \quad t = \tfrac{5\pi}{48} + \tfrac{k\pi}{4}, \quad k \in \mathbb{Z}$$

4. Find the solutions on the interval $[0, 2\pi)$ to following equations.

a. $5 + 4\cos(2\theta) = 1$

$$5 + 4\cos(2\theta) = 1$$

$$\Rightarrow \quad 4\cos(2\theta) = -4$$

$$\Rightarrow \quad \cos(2\theta) = -1$$

$$\Rightarrow \quad 2\theta = \cos^{-1}(-1) + 2k\pi, \ k \in \mathbb{Z}$$

(there's only one "family" of solutions since the cosine function achieves the output -1 only once in each period)

$$\Rightarrow \quad 2\theta = \pi + 2k\pi, \ k \in \mathbb{Z}$$

$$\Rightarrow \quad \frac{2\theta}{2} = \frac{\pi}{2} + \frac{2k\pi}{2}, \ k \in \mathbb{Z}$$

$$\Rightarrow \quad \theta = \tfrac{\pi}{2} + k\pi, \ k \in \mathbb{Z}$$

Now we need to substitute particular values of k in order to determine which solutions fall on the interval $[0, 2\pi)$:

$k = -1$: $\theta = \tfrac{\pi}{2} + (-1)\cdot\pi = -\tfrac{\pi}{2} < 0$ so $-\tfrac{\pi}{2} \notin [0, 2\pi)$.

We could try smaller values of k but it should be clear that since $k = -1$ produced a value of θ that's too small, smaller values of k will produces even smaller values of θ so they won't produce solutions in the given interval so there's no need in trying smaller values of k.

$k = 0$: $\theta = \tfrac{\pi}{2} + 0\cdot\pi = \tfrac{\pi}{2} \in [0, 2\pi)$, so $\tfrac{\pi}{2}$ is a solution in the given interval.

$k = 1$: $\theta = \tfrac{\pi}{2} + 1\cdot\pi = \tfrac{3\pi}{2} \in [0, 2\pi)$, so $\tfrac{3\pi}{2}$ is a solution in the given interval.

$k = 2$: $\theta = \tfrac{\pi}{2} + 2\cdot\pi = \tfrac{5\pi}{2} > 2\pi$ so $\tfrac{5\pi}{2} \notin [0, 2\pi)$.

We could try larger values of k but it should be clear that since $k = 2$ produced a value θ that's too large, larger values of k will produces even larger values of θ, so they won't produce solutions in the given interval so there's no need in trying larger values of k.

Therefore, the solution set to $5 + 4\cos(2\theta) = 1$ on the interval $[0, 2\pi)$ is $\left\{\tfrac{\pi}{2}, \tfrac{3\pi}{2}\right\}$.

b. $4 - 6\sin(2x) = 7$

$$4 - 6\sin(2x) = 7$$

$$\Rightarrow \quad -6\sin(2x) = 3$$

$$\Rightarrow \quad \sin(2x) = \tfrac{3}{-6} = -\tfrac{1}{2}$$

$$\Rightarrow \; 2x = \sin^{-1}\left(-\tfrac{1}{2}\right) + 2k\pi \text{ or } 2x = \pi - \sin^{-1}\left(-\tfrac{1}{2}\right) + 2k\pi, \; k \in \mathbb{Z}$$

$$\Rightarrow \; 2x = -\tfrac{\pi}{6} + 2k\pi \quad \text{or} \quad 2x = \pi - \left(-\tfrac{\pi}{6}\right) + 2k\pi, \; k \in \mathbb{Z}$$

$$\Rightarrow \frac{2x}{2} = \frac{-\frac{\pi}{6}}{2} + \frac{2k\pi}{2} \quad \text{or} \quad \frac{2x}{2} = \frac{\frac{7\pi}{6}}{2} + \frac{2k\pi}{2}, \; k \in \mathbb{Z}$$

$$\Rightarrow \; x = -\tfrac{\pi}{12} + k\pi \quad \text{or} \quad x = \tfrac{7\pi}{12} + k\pi, \; k \in \mathbb{Z}$$

Now we need to substitute particular values of k into these equations to determine which solutions fall on the interval $[0, 2\pi)$:

$k = -1$: $x = -\tfrac{\pi}{12} + (-1)\pi = -\tfrac{\pi}{12} - \tfrac{12\pi}{12} = -\tfrac{13\pi}{12} < 0$ or $x = \tfrac{7\pi}{12} + (-1)\pi = \tfrac{7\pi}{12} - \tfrac{12\pi}{12} = -\tfrac{5\pi}{12} < 0$

Since both of these values are negative, they aren't in the interval $[0, 2\pi)$. We could try smaller values of k but it should be clear that since $k = -1$ resulted in a values of α that are too small, smaller values of k will produces even smaller values of α so they won't produce solutions in the given interval so there's no need in trying smaller values of k.

$k = 0$: $x = -\tfrac{\pi}{12} + (0)\pi = -\tfrac{\pi}{12} < 0$ or $x = \tfrac{7\pi}{12} + (0)\pi = \tfrac{7\pi}{12}$

Only $\tfrac{7\pi}{12}$ is in the given interval.

$k = 1$: $x = -\tfrac{\pi}{12} + (1)\pi = -\tfrac{\pi}{12} + \tfrac{12\pi}{12} = \tfrac{11\pi}{12}$ or $x = \tfrac{7\pi}{12} + (1)\pi = \tfrac{7\pi}{12} + \tfrac{12\pi}{12} = \tfrac{19\pi}{12}$

Both $\tfrac{11\pi}{12}$ and $\tfrac{19\pi}{12}$ are in the given interval.

$k = 2$: $x = -\frac{\pi}{12} + (2)\pi$ or $x = \frac{7\pi}{12} + (2)\pi$

$= -\frac{\pi}{12} + \frac{24\pi}{12}$ $= \frac{7\pi}{12} + \frac{24\pi}{12}$

$= \frac{23\pi}{12}$ $= \frac{31\pi}{12} > 0$

Only $\frac{23\pi}{12}$ is in the given interval.

$k = 3$: $x = -\frac{\pi}{12} + (3)\pi$ or $x = \frac{7\pi}{12} + (3)\pi$

$= -\frac{\pi}{12} + \frac{36\pi}{12}$ $= \frac{7\pi}{12} + \frac{36\pi}{12}$

$= \frac{35\pi}{12} > 2\pi$ $= \frac{43\pi}{12} > 2\pi$

Since both of these values are greater than 2π, they aren't in the interval $[0, 2\pi)$. We could try larger values of k but it should be clear that since $k = 3$ resulted in a values of x that are too larger, larger values of k will produces even larger values of x so they won't produce solutions in the given interval so there's no need in trying larger values of k.

Thus, the solution set to $4 - 6\sin(2x) = 7$ on the interval $[0, 2\pi)$ is $\left\{\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}\right\}$.

c. $6\sqrt{2}\cos(3\alpha) + 10 = 4$

$$6\sqrt{2}\cos(3\alpha) + 10 = 4$$

$$\Rightarrow \quad 6\sqrt{2}\cos(3\alpha) = -6$$

$$\Rightarrow \quad \cos(3\alpha) = \tfrac{-6}{6\sqrt{2}} = -\tfrac{1}{\sqrt{2}}$$

$$\Rightarrow \quad 3\alpha = \cos^{-1}\left(-\tfrac{1}{\sqrt{2}}\right) + 2k\pi \quad \text{or} \quad 3\alpha = -\cos^{-1}\left(-\tfrac{1}{\sqrt{2}}\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad 3\alpha = \tfrac{3\pi}{4} + 2k\pi \quad \text{or} \quad 3\alpha = -\tfrac{3\pi}{4} + 2k\pi, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad \frac{3\alpha}{3} = \frac{\frac{3\pi}{4}}{3} + \frac{2k\pi}{3} \quad \text{or} \quad \frac{3\alpha}{3} = \frac{-\frac{3\pi}{4}}{3} + \frac{2k\pi}{3}, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad \alpha = \tfrac{3\pi}{4\cdot 3} + \tfrac{2k\pi}{3} \quad \text{or} \quad \alpha = -\tfrac{3\pi}{4\cdot 3} + \tfrac{2k\pi}{3}, \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad \alpha = \tfrac{\pi}{4} + \tfrac{2k\pi}{3} \quad \text{or} \quad \alpha = -\tfrac{\pi}{4} + \tfrac{2k\pi}{3}, \quad k \in \mathbb{Z}$$

Now we need to substitute particular values of k into these equations to determine which solutions fall on the interval $[0, 2\pi)$:

$k = -1$: $\alpha = \tfrac{\pi}{4} + \tfrac{2(-1)\pi}{3} = \tfrac{3\pi}{12} - \tfrac{8\pi}{12} = -\tfrac{5\pi}{12}$ or $\alpha = -\tfrac{\pi}{4} + \tfrac{2(-1)\pi}{3} = -\tfrac{3\pi}{12} - \tfrac{8\pi}{12} = -\tfrac{11\pi}{12}$

Since both of these values are negative, they aren't in the interval $[0, 2\pi)$. We could try smaller values of k but it should be clear that since $k = -1$ resulted in a values of α that are too small, smaller values of k will produces even smaller values of α so they won't produce solutions in the given interval so there's no need in trying smaller values of k.

$k = 0$: $\alpha = \tfrac{\pi}{4} + \tfrac{2\cdot 0\cdot \pi}{3} = \tfrac{\pi}{4}$ or $\alpha = -\tfrac{\pi}{4} + \tfrac{2\cdot 0\cdot \pi}{3} = -\tfrac{\pi}{4}$

Only $\tfrac{\pi}{4}$ is in the given interval.

$k = 1$: $\alpha = \tfrac{\pi}{4} + \tfrac{2\cdot 1\cdot \pi}{3} = \tfrac{3\pi}{14} + \tfrac{8\pi}{12} = \tfrac{11\pi}{12}$ or $\alpha = -\tfrac{\pi}{4} + \tfrac{2\cdot 1\cdot \pi}{3} = -\tfrac{3\pi}{12} + \tfrac{8\pi}{12} = \tfrac{5\pi}{12}$

Both $\tfrac{11\pi}{12}$ and $\tfrac{5\pi}{12}$ are in the given interval.

$k = 2$: $\alpha = \frac{\pi}{4} + \frac{2 \cdot 2 \cdot \pi}{3}$ or $\alpha = -\frac{\pi}{4} + \frac{2 \cdot 2 \cdot \pi}{3}$

$= \frac{3\pi}{12} + \frac{16\pi}{12}$ $\quad$ $= -\frac{3\pi}{12} + \frac{16\pi}{12}$

$= \frac{19\pi}{12}$ $\quad$ $= \frac{13\pi}{12}$

Both $\frac{19\pi}{12}$ and $\frac{13\pi}{12}$ are in the given interval.

$k = 3$: $\alpha = \frac{\pi}{4} + \frac{2 \cdot 3 \cdot \pi}{3}$ or $\alpha = -\frac{\pi}{4} + \frac{2 \cdot 3 \cdot \pi}{3}$

$= \frac{\pi}{4} + \frac{8\pi}{4}$ $\quad$ $= -\frac{\pi}{4} + \frac{8\pi}{4}$

$= \frac{9\pi}{4} > 2\pi$ $\quad$ $= \frac{7\pi}{4}$

Only $\frac{7\pi}{4}$ is in the given interval.

$k = 4$: $\alpha = \frac{\pi}{4} + \frac{2 \cdot 4 \cdot \pi}{3}$ or $\alpha = -\frac{\pi}{4} + \frac{2 \cdot 4 \cdot \pi}{3}$

$= \frac{3\pi}{12} + \frac{32\pi}{12}$ $\quad$ $= -\frac{3\pi}{12} + \frac{32\pi}{12}$

$= \frac{35\pi}{12} > 2\pi$ $\quad$ $= \frac{29\pi}{12} > 2\pi$

Since both of these values are greater than 2π, they aren't in the interval $[0, 2\pi)$. We could try larger values of k but it should be clear that since $k = 4$ resulted in a values of α that are too larger, larger values of k will produces even larger values of α so they won't produce solutions in the given interval so there's no need in trying larger values of k.

Thus, the solution set to $6\sqrt{2}\cos(3\alpha) + 10 = 4$ on the interval $[0, 2\pi)$ is $\left\{\frac{\pi}{4}, \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}, \frac{7\pi}{4}\right\}$.