SOLUTIONS: Practice Worksheet: Inverse Trig Functions

1. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to ***directly communicate*** what the given expressions equal.

a. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ (since $\frac{\sqrt{3}}{2} = \sin\left(\frac{\pi}{3}\right)$ and $-\frac{\pi}{2} \le \frac{\pi}{3} \le \frac{\pi}{2}$)

b. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ (since $\frac{\sqrt{2}}{2} = \cos\left(\frac{\pi}{4}\right)$ and $0 \le \frac{\pi}{4} \le \pi$)

c. $\sin^{-1}\left(-\frac{1}{2}\right)$

$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ (since $-\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right)$ and $-\frac{\pi}{2} \le -\frac{\pi}{6} \le \frac{\pi}{2}$)

d. $\cos^{-1}(0)$

$\cos^{-1}(0) = \frac{\pi}{2}$ (since $0 = \cos\left(\frac{\pi}{2}\right)$ and $0 \le \frac{\pi}{2} \le \pi$)

e. $\tan^{-1}\left(\sqrt{3}\right)$

$\tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$ (since $\sqrt{3} = \tan\left(\frac{\pi}{3}\right)$ and $-\frac{\pi}{2} < \frac{\pi}{3} < \frac{\pi}{2}$)

f. $\tan^{-1}(-1)$

$\tan^{-1}(-1) = -\frac{\pi}{4}$ (since $-1 = \tan\left(-\frac{\pi}{4}\right)$ and $-\frac{\pi}{2} < -\frac{\pi}{4} < \frac{\pi}{2}$)

g. $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$ (since $-\frac{1}{\sqrt{3}} = \tan\left(-\frac{\pi}{6}\right)$ and $-\frac{\pi}{2} < -\frac{\pi}{6} < \frac{\pi}{2}$)

2. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to ***directly communicate*** what the given expressions equal.

a. $\sin\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

$$\sin\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) = \sin\left(\frac{\pi}{3}\right) \quad \left(\text{since } \frac{\sqrt{3}}{2} = \sin\left(\frac{\pi}{3}\right) \text{ and } -\frac{\pi}{2} \le \frac{\pi}{3} \le \frac{\pi}{2}\right)$$
$$= \frac{\sqrt{3}}{2} \quad \left(\text{since } \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}\right)$$

b. $\cos\left(\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$

$$\cos\left(\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right) = \cos\left(\frac{3\pi}{4}\right) \quad \left(\text{since } -\frac{\sqrt{2}}{2} = \cos\left(\frac{3\pi}{4}\right) \text{ and } 0 \le \frac{3\pi}{4} \le \pi\right)$$
$$= -\frac{\sqrt{2}}{2} \quad \left(\text{since } \cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}\right)$$

c. $\cos^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right)$

$$\cos^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right) = \cos^{-1}\left(\frac{1}{2}\right) \quad \left(\text{since } \cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}\right)$$
$$= \frac{\pi}{3} \quad \left(\text{since } \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \text{ and } 0 \le \frac{\pi}{3} \le \pi\right)$$

d. $\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right)$

$$\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) \quad \left(\text{since } \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}\right)$$
$$= -\frac{\pi}{3} \quad \left(\text{since } \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \text{ and } -\frac{\pi}{2} \le -\frac{\pi}{3} \le \frac{\pi}{2}\right)$$

e. $\tan^{-1}\left(\tan\left(\frac{5\pi}{4}\right)\right)$

$$\tan^{-1}\left(\tan\left(\frac{5\pi}{4}\right)\right) = \tan^{-1}(1) \quad \left(\text{since } \tan\left(\frac{5\pi}{4}\right) = 1\right)$$
$$= \frac{\pi}{4} \quad \left(\text{since } \tan\left(\frac{\pi}{4}\right) = 1 \text{ and } -\frac{\pi}{2} < \frac{\pi}{4} < \frac{\pi}{2}\right)$$

3. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to ***directly communicate*** what the given expressions equal.

a. $\sin^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right)$

$$\sin^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \quad \left(\text{since } \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}\right)$$
$$= \frac{\pi}{3} \quad \left(\text{since } \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \text{ and } -\frac{\pi}{2} \le \frac{\pi}{3} \le \frac{\pi}{2}\right)$$

b. $\sin\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

$$\sin\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \sin\left(\frac{5\pi}{6}\right) \quad \left(\text{since } -\frac{\sqrt{3}}{2} = \cos\left(\frac{5\pi}{6}\right) \text{ and } 0 \le \frac{5\pi}{6} \le \pi\right)$$
$$= \frac{1}{2} \quad \left(\text{since } \sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}\right)$$

c. $\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right)$

$$\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right) = \tan^{-1}\left(-\sqrt{3}\right) \quad \left(\text{since } \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}\right)$$
$$= -\frac{\pi}{3} \quad \left(\text{since } \tan\left(-\frac{\pi}{3}\right) = -\sqrt{3} \text{ and } -\frac{\pi}{2} < -\frac{\pi}{3} < \frac{\pi}{2}\right)$$

d. $\cos^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$

$$\cos^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right) = \cos^{-1}(-1) \quad \left(\text{since } \tan\left(\frac{3\pi}{4}\right) = -1\right)$$
$$= \pi \quad \left(\text{since } \cos(\pi) = -1 \text{ and } 0 \le \pi \le \pi\right)$$

e. $\tan^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right)$

$$\tan^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right) = \tan^{-1}(1) \quad \left(\text{since } \sin\left(\frac{\pi}{2}\right) = 1\right)$$
$$= \frac{\pi}{4} \quad \left(\text{since } \tan\left(\frac{\pi}{4}\right) = 1 \text{ and } -\frac{\pi}{2} < \frac{\pi}{4} < \frac{\pi}{2}\right)$$

f. $\sin\left(\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$

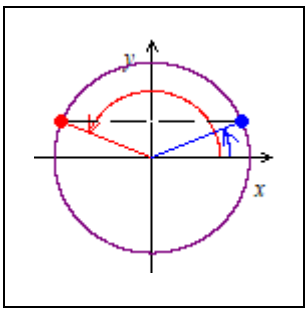
$$\sin\left(\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right) = \sin\left(\frac{\pi}{6}\right) \qquad \left(\text{since } \frac{1}{\sqrt{3}} = \tan\left(\frac{\pi}{6}\right) \text{ and } -\frac{\pi}{2} < \frac{\pi}{6} < \frac{\pi}{2}\right)$$
$$= \frac{1}{2} \qquad \left(\text{since } \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}\right)$$

4. Find the exact value of each of the following expressions; do not use a calculator. Be sure to use proper notation to ***directly communicate*** what the given expressions equal.

a. $\sin^{-1}\left(\sin\left(\frac{7\pi}{8}\right)\right)$

$\frac{7\pi}{8}$ isn't a "friendly angle" so we aren't familiar with its sine so we need to rely on our conceptual understanding of the sine function along with the symmetry of a circle:

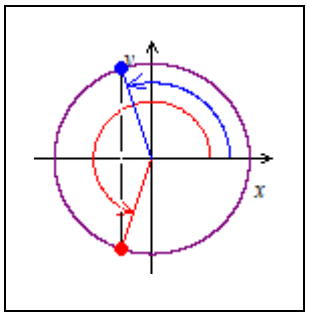
$$\sin^{-1}\left(\sin\left(\frac{7\pi}{8}\right)\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{8}\right)\right) \qquad \text{(using the symmetry of a circle)}$$
$$= \frac{\pi}{8} \qquad \left(\text{since } -\frac{\pi}{2} \le \frac{\pi}{8} \le \frac{\pi}{2}\right)$$



b. $\cos^{-1}\left(\cos\left(\frac{7\pi}{5}\right)\right)$

As in part (**a**) above, $\frac{7\pi}{5}$ isn't a "friendly angle" so we need to rely on our conceptual understanding of the cosine function along with the symmetry of a circle:

$$\cos^{-1}\left(\cos\left(\frac{7\pi}{5}\right)\right) = \cos^{-1}\left(\cos\left(\frac{3\pi}{5}\right)\right) \qquad \text{(using the symmetry of a circle)}$$
$$= \frac{3\pi}{5} \qquad \left(\text{since } 0 \le \frac{3\pi}{5} \le \pi\right)$$



c. $\sin^{-1}\left(\sin\left(\frac{9\pi}{7}\right)\right)$

As in parts (**a**) and (**b**) above, $\frac{9\pi}{7}$ isn't a "friendly angle" so we need to rely on our conceptual understanding of the sine function along with the symmetry of a circle:

$$\sin^{-1}\left(\sin\left(\frac{9\pi}{7}\right)\right) = \sin^{-1}\left(\sin\left(-\frac{2\pi}{7}\right)\right) \qquad \text{(using the symmetry of a circle)}$$
$$= -\frac{2\pi}{7} \qquad \left(\text{since } -\frac{\pi}{2} \le -\frac{2\pi}{7} \le \frac{\pi}{2}\right)$$