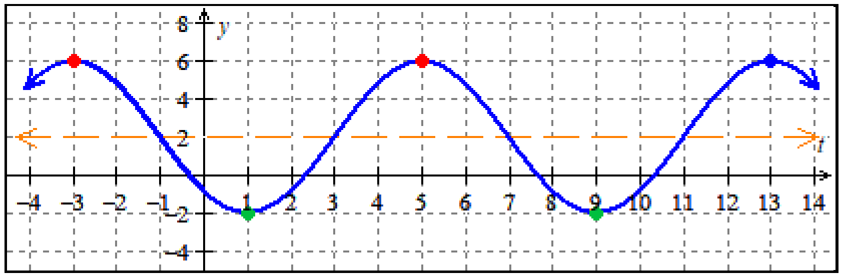
SOLUTIONS: Practice Worksheet: Periodic Functions

1. Determine the period, midline and amplitude of the function $y = f(t)$ graphed below. Note that the following points are on the graph: $(-3, 6)$, $(1, -2)$, $(5, 6)$, $(9, -2)$, and $(13, 6)$.



A graph of $y = f(t)$.

The period of the function graphed above is 8 units. There are a few ways to compute this. For example, we can compare the x-values for the ordered pairs $(\mathbf{-3}, 6)$ and $(\mathbf{5}, 6)$ (colored red in the graph above) or the x-values for the ordered pairs $(\mathbf{1}, -2)$ and $(\mathbf{9}, -2)$ (colored green in the graph above):

$$5 - (-3) = 8 \quad \text{and} \quad 9 - 1 = 8.$$

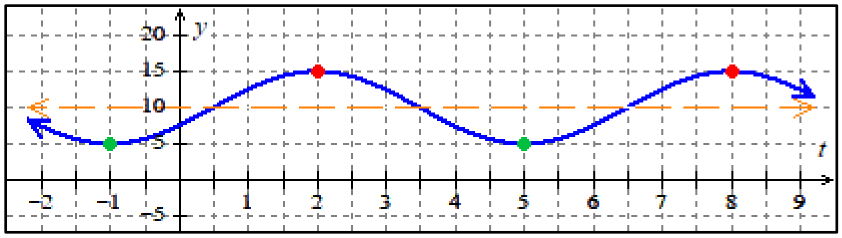
The midline of the graphed function is $y = 2$. There are a few ways to compute this. For example, we can find the average of the y-values for the ordered pairs $(-3, \mathbf{6})$ and $(1, \mathbf{-2})$ (colored red and green in the graph above) since the function reaches, respectively, maximum and minimum values at these points:

$$y = \frac{6 + (-2)}{2} = 2.$$

The amplitude of the graphed function is 4 units. There are a few ways to compute this. For example, we can determine the distance between the midline, $y = \mathbf{2}$, and the y-value of the ordered pair $(-3, \mathbf{6})$ where the function reaches its maximum:

$$6 - 2 = 4.$$

2. Determine the period, midline and amplitude of the function $y = m(t)$ graphed below. Note that the following points are on the graph: $(-1, 5)$, $(2, 15)$, $(5, 5)$, and $(8, 15)$.



A graph of $y = m(t)$.

The period of the function graphed above is 6 units. There are a few ways to compute this. For example, we can compare the x-values for the ordered pairs $(2, 15)$ and $(8, 15)$ (colored red in the graph above) or the x-values for the ordered pairs $(-1, 5)$ and $(5, 5)$ (colored green in the graph above):

$$8 - 2 = 6 \quad \text{and} \quad 5 - (-1) = 6.$$

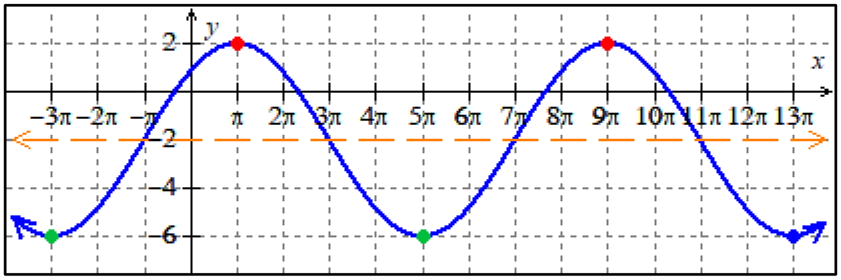
The midline of the graphed function is $y = 10$. There are a few ways to compute this. For example, we can find the average of the y-values for the ordered pairs $(2, 15)$ and $(-1, 5)$ (colored red and green in the graph above) since the function reaches, respectively, maximum and minimum values at these points:

$$y = \frac{15 + (5)}{2} = 10.$$

The amplitude of the graphed function is 5 units. There are a few ways to compute this. For example, we can determine the distance between the midline, $y = 10$, and the y-value of the ordered pair $(2, 15)$ where the function reaches its maximum:

$$15 - (10) = 5.$$

3. Determine the period, midline and amplitude of the function $y = p(x)$ graphed below. Note that the following points are on the graph: $(-3\pi, -6)$, $(\pi, 2)$, $(5\pi, -6)$, and $(9\pi, 2)$.



A graph of $y = p(x)$.

The period of the function graphed above is 8π units. There are a few ways to compute this. For example, we can compare the x-values for the ordered pairs $(\pi, 2)$ and $(9\pi, 2)$ (colored red in the graph above) or the x-values for the ordered pairs $(-3\pi, -6)$ and $(5\pi, -6)$ (colored green in the graph above):

$$9\pi - \pi = 8\pi \quad \text{and} \quad 5\pi - (-3\pi) = 8\pi.$$

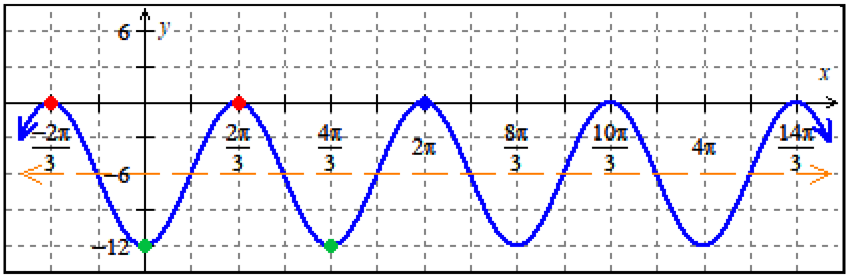
The midline of the graphed function is $y = -2$. There are a few ways to compute this. For example, we can find the average of the y-values for the ordered pairs $(\pi, 2)$ and $(5\pi, -6)$ (colored red and green in the graph above) since the function reaches, respectively, maximum and minimum values at these points:

$$y = \frac{2 + (-6)}{2} = -2.$$

The amplitude of the graphed function is 4 units. There are a few ways to compute this. For example, we can determine the distance between the midline, $y = -2$, and the y-value of the ordered pair $(\pi, 2)$ where the function reaches its maximum:

$$2 - (-2) = 4.$$

4. Determine the period, midline and amplitude of the function $y = g(t)$ graphed below. Note that the following points are on the graph: $\left(-\frac{2\pi}{3}, 0\right)$, $(0, -12)$, $\left(\frac{2\pi}{3}, 0\right)$, $\left(\frac{4\pi}{3}, -12\right)$, and $(2\pi, 0)$.



A graph of $y = g(t)$.

The period of the function graphed above is $\frac{4\pi}{3}$ units. There are a few ways to compute this. For example, we can compare the x-values for the ordered pairs $\left(-\frac{2\pi}{3}, 0\right)$ and $\left(\frac{2\pi}{3}, 0\right)$ (colored red in the graph above) or the x-values for the ordered pairs $(0, -12)$ and $\left(\frac{4\pi}{3}, -12\right)$ (colored green in the graph above):

$$\frac{2\pi}{3} - \left(-\frac{2\pi}{3}\right) = \frac{4\pi}{3} \quad \text{and} \quad \frac{4\pi}{3} - 0 = \frac{4\pi}{3}.$$

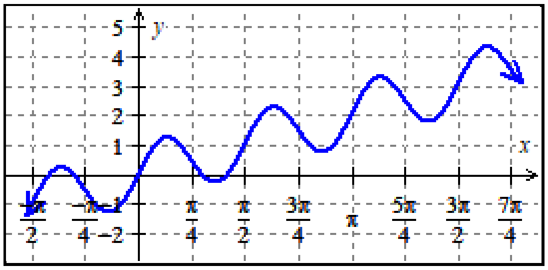
The midline of the graphed function is $y = -6$. There are a few ways to compute this. For example, we can find the average of the y-values for the ordered pairs $\left(-\frac{2\pi}{3}, 0\right)$ and $(0, -12)$ (colored red and green in the graph above) since the function reaches, respectively, maximum and minimum values at these points:

$$y = \frac{0 + (-12)}{2} = -6.$$

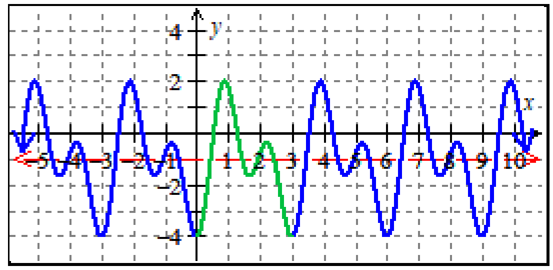
The amplitude of the graphed function is 6 units. There are a few ways to compute this. For example, we can determine the distance between the midline, $y = -6$, and the y-value of the ordered pair $\left(-\frac{2\pi}{3}, 0\right)$ where the function reaches its maximum:

$$0 - (-6) = 6.$$

5. Determine which of the functions graphed below are periodic functions and find the period, midline, and amplitude of the periodic functions.



$y = A(x)$ is **not** periodic.

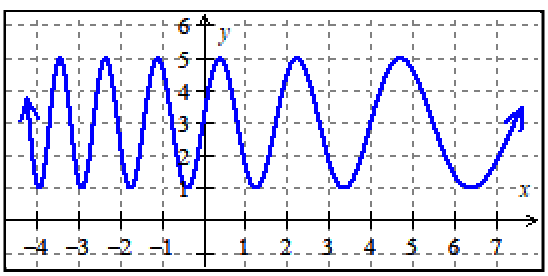


$y = B(x)$ is periodic.

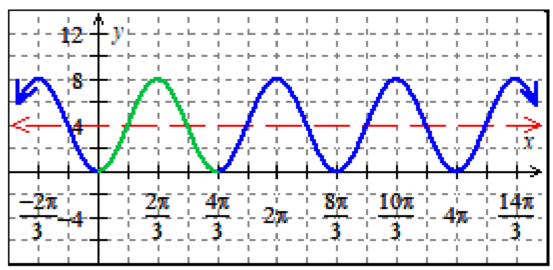
The **period** of $y = B(x)$ is 3 units. In the graph above, we've colored one period green.

The **midline** of $y = B(x)$ is $y = -1$. In the graph above, we've colored the midline red.

The **amplitude** of $y = B(x)$ is 3 units. This is half of the distance between the maximum y-value ($y = 2$) and the minimum y-value ($y = -4$).



$y = C(x)$ is **not** periodic.

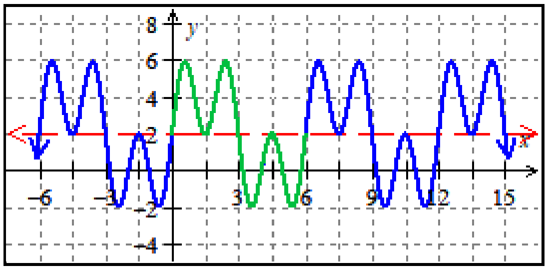


$y = D(x)$ is periodic.

The **period** of $y = D(x)$ is $\frac{4\pi}{3}$ units. In the graph above, we've colored one period green.

The **midline** of $y = D(x)$ is $y = 4$. In the graph above, we've colored the midline red.

The **amplitude** of $y = D(x)$ is 4 units. This is half of the distance between the maximum y-value ($y = 8$) and the minimum y-value ($y = 0$).

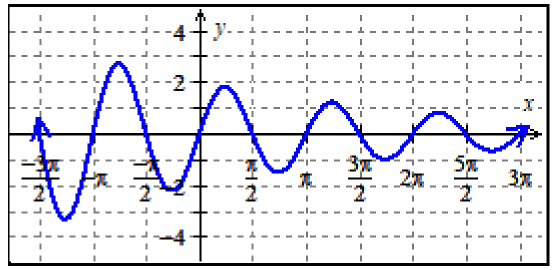


$y = E(x)$ is periodic.

The **period** of $y = E(x)$ is 6 units. In the graph above, we've colored one period green.

The **midline** of $y = E(x)$ is $y = 2$. In the graph above, we've colored the midline red.

The **amplitude** of $y = E(x)$ is 4 units. This is half of the distance between the maximum y-value ($y = 6$) and the minimum y-value ($y = -2$).



$y = F(x)$ is **not** periodic.

6. There's a Ferris wheel with a diameter of 80 feet. The wheel rotates at a constant rate, and completes a full rotation every 20 minutes. The wheel is lifted 10 feet above the ground level, and passengers load into carriages at the lowest point in the wheel's travel (so passengers start their trip 10 feet above the ground). Determine the period, midline and amplitude of the function that associates amount of time a passenger spends in a carriage travelling around the wheel with the height (in feet) above the ground of such a passenger.

The period of the function described above is 20 minutes since that's how long it takes the Ferris wheel to complete a full rotation.

The midline of the function is $y = 50$. We can compute this by finding the vertical-center of the Ferris wheel and adding 10 feet because it's lifted up 10 feet:

$$y = \frac{80}{2} + 10 = 50.$$

The amplitude of the function is 40 feet since that is half of the wheel's diameter.

7. Describe an activity in your life or that you're familiar with that's periodic (or approximately periodic). What's the period? Is there a midline and amplitude? (Not all periodic behaviors have midlines or amplitudes.) If you can't think of anything else, describe the high temperature in Portland each day.

A function that outputs the high temperature in Portland each day will (approximately) repeat itself every 365 days since Earth's weather cycle is determined by the Earth's rotation about the sun, which takes 365 days to complete. So the period is 365 days.

The highest temperature we get in Portland is about 105 degrees Fahrenheit, and the lowest temperature is about 10 degrees Fahrenheit. The average of 105 and 10 is 57.5. So the midline is about $y = 57.5$ degrees Fahrenheit.

The amplitude is the distance between the midline and the maximum output (or the midline and the minimum output value): $105 - 57.5 = 47.5$, so the amplitude is about 47.5 degrees Fahrenheit.