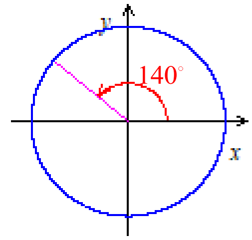
SOLUTIONS: Practice Worksheet: Angles and Arc-length

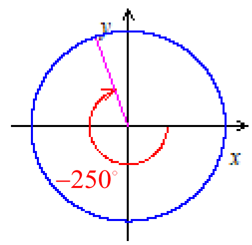
1. a. Draw 140° in standard position on the provided coordinate plane.

Since $90^\circ < 140^\circ < 180^\circ$, 140° falls in Quad 2.



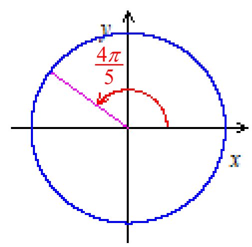
b. Draw -250° in standard position on the provided coordinate plane.

Since $-180^\circ < -250^\circ < -270^\circ$, -250° falls in Quad 2. (Since $-250^\circ < 0^\circ$, the rotation is clockwise.)



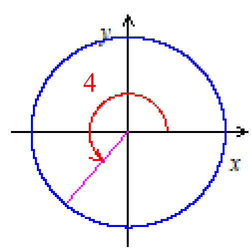
c. Draw $\frac{4\pi}{5}$ radians in standard position on the provided coordinate plane.

Since $\frac{4\pi}{5} > \frac{2.5\pi}{5} = \frac{\pi}{2}$, we know that $\frac{4\pi}{5}$ rotates beyond the border between Quads 1 and 2; and since $\frac{4\pi}{5} < \frac{5\pi}{5} = \pi$ so we know that $\frac{4\pi}{5}$ radians doesn't rotate beyond Quad 2, so it's in Quad 2.



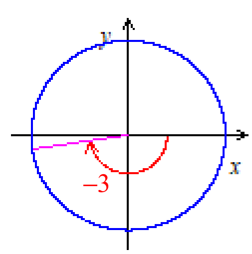
d. Draw 4 radians in standard position on the provided coordinate plane.

Certainly $4 > \pi$ so we know that 4 radians is at least in Quad 3: the only question is whether it's large enough to get into Quad 4. The border between Quads 3 and 4 is $\frac{3\pi}{2} \approx \frac{9}{2}$ and $4 < \frac{9}{2}$ so we know that 4 radians is in Quad 3.



d. Draw -3 radians in standard position on the provided coordinate plane.

-3 is *almost* $-\pi$ but a little bit greater than $-\pi$ so -3 radians rotates *almost* to $-\pi$ but falls a little short, so it lands in Quad 3. (Since $-3 < 0$, the rotation is clockwise.)



2. Complete the table below.

θ (degrees)	0°	30°	45°	60°	90°	180°	270°	360°	3600°
θ (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	20π

3. a. Convert $\frac{\pi}{10}$ radians into degrees.

Let's try this using two different methods:

(1) One option is to use the fact that $\frac{180^\circ}{\pi \text{ rad}} = 1$ (since $180^\circ = \pi$ rad):

$$\begin{aligned}\frac{\pi}{10}\text{rad}\cdot 1 &= \frac{\pi}{10}\cancel{\text{rad}}\cdot\frac{180^\circ}{\pi\ \cancel{\text{rad}}}\\ &= \frac{\cancel{\pi}\cdot 180^\circ}{10\cdot\cancel{\pi}}\\ &= \frac{180^\circ}{10}\\ &= 18^\circ\end{aligned}$$

Thus, $\frac{\pi}{10} = 18^\circ$.

(2) Another option is to recognize that, since π rad $= 180^\circ$,

$$\frac{\pi}{10} = \frac{180^\circ}{10} = 18^\circ.$$

Therefore, $\frac{\pi}{10} = 18^\circ$.

b. Convert 4 radians into degrees.

$$\begin{aligned}4\,\text{rad}\cdot 1 &= 4\,\cancel{\text{rad}}\cdot\frac{180^\circ}{\pi\ \cancel{\text{rad}}}\\ &= \frac{720^\circ}{\pi}\\ &\approx 229.18^\circ\end{aligned}$$

Therefore, 4 radians is exactly equal to $\frac{720^\circ}{\pi}$ and approximately equal to 229.18°.

c. Convert 10° into radians.

$$\begin{aligned} 10^\circ \cdot 1 &= 10^{\not\circ} \cdot \frac{\pi \text{ rad}}{180^{\not\circ}} \\ &= \frac{10\pi}{180} \text{ rad} \\ &= \frac{\pi}{18} \text{ rad} \end{aligned}$$

d. Convert 140° into radians.

$$\begin{aligned} 140^\circ \cdot 1 &= 140^{\not\circ} \cdot \frac{\pi \text{ rad}}{180^{\not\circ}} \\ &= \frac{140\pi}{180} \text{ rad} \\ &= \frac{14\pi}{18} \text{ rad} \\ &= \frac{7\pi}{9} \text{ rad} \end{aligned}$$

e. Convert 1200° into radians.

$$\begin{aligned} 1200^\circ \cdot 1 &= 1200^{\not\circ} \cdot \frac{\pi \text{ rad}}{180^{\not\circ}} \\ &= \frac{1200\pi}{180} \text{ rad} \\ &= \frac{120\pi}{18} \text{ rad} \\ &= \frac{60\pi}{9} \text{ rad} \\ &= \frac{20\pi}{3} \text{ rad} \end{aligned}$$

4. **a.** Find both a positive and a negative angle that are coterminal with 140°. (Answer in degrees.)

To find a positive angle coterminal with 140° we can add 360°:

$$140^\circ + 360^\circ = 500^\circ$$

So 500° is coterminal with 140°.

To find a negative angle coterminal with 140° we can subtract 360°:

$$140^\circ - 360^\circ = -220^\circ$$

So -220° is coterminal with 140°.

b. Find both a positive and a negative angle that are coterminal with $-\frac{5\pi}{8}$. (Answer in radians.)

To find a positive angle coterminal with $-\frac{5\pi}{8}$ we can add 2π:

$$\begin{aligned} -\frac{5\pi}{8} + 2\pi &= -\frac{5\pi}{8} + \frac{16\pi}{8} \\ &= \frac{11\pi}{8} \end{aligned}$$

So $\frac{11\pi}{8}$ is coterminal with $-\frac{5\pi}{8}$.

To find a negative angle coterminal with $-\frac{5\pi}{8}$ we can subtract 2π:

$$\begin{aligned} -\frac{5\pi}{8} - 2\pi &= -\frac{5\pi}{8} - \frac{16\pi}{8} \\ &= -\frac{21\pi}{8} \end{aligned}$$

So $-\frac{21\pi}{8}$ is coterminal with $-\frac{5\pi}{8}$.

c. Try to find a way to represent all of the (infinitely many) different angles that are coterminal with 70°.

To find coterminal angles, we know that we need to add (or subtract) any number of full revolutions. For example,

$$70^\circ + 360^\circ \cdot 1 = 430^\circ$$
$$70^\circ + 360^\circ \cdot 2 = 790^\circ$$
$$70^\circ + 360^\circ \cdot 3 = 1150^\circ$$
etc.

are all coterminal with 70°. To represent all of the infinitely many different possibilities, we can let the symbol k represent any integer (i.e., any whole number) using the notation "$k \in \mathbb{Z}$" (the symbol $\mathbb{Z}$ represents the set of integers and the symbol $\in$ means "is an element of" so this means, "k is an element of the set of integers") and multiply 360° by k to represent all of the different possible quantities of full revolutions. Therefore the expression

$$70^\circ + 360^\circ \cdot k \text{ for all } k \in \mathbb{Z}$$

represents infinitely many different angles that are coterminal with 70°. (Note that $\mathbb{Z}$ includes the negative whole numbers so this also represents coterminal angles resulting from subtracting full revolutions from 70°.)

d. Try to find a way to represent all of the (infinitely many) different angles that are coterminal with $\frac{3\pi}{5}$.

To find coterminal angles, we know that we need to add (or subtract) any number of full revolutions. For example,

$$\frac{3\pi}{5} + 2\pi \cdot 1 = \frac{3\pi}{5} + \frac{10\pi}{5} = \frac{13\pi}{5}$$
$$\frac{3\pi}{5} + 2\pi \cdot 2 = \frac{3\pi}{5} + \frac{20\pi}{5} = \frac{23\pi}{5}$$
$$\frac{3\pi}{5} + 2\pi \cdot 3 = \frac{3\pi}{5} + \frac{30\pi}{5} = \frac{33\pi}{5}$$
etc.

are all coterminal with $\frac{3\pi}{5}$. To represent all of the infinitely many different possibilities, we can define k to be an integer and multiply 2π by k to represent all of the different possible quantities of full revolutions. Therefore the expression

$$\frac{3\pi}{5} + 2\pi \cdot k \text{ for all } k \in \mathbb{Z}$$

represents infinitely many different angles that are coterminal with $\frac{3\pi}{5}$. (Note that we often represent "$2\pi \cdot k$" as "$2k\pi$".)

5. The Greek letters θ and ϕ are often used as variables in mathematics. They are ***different*** symbols so it's important to distinguish between them. Which of these symbols is pronounced "phi"? And which is pronounced "theta"?

θ is "theta."

ϕ is "phi."

6. What is the length of the arc spanned by the angle 4 radians on a circle of radius 20 yards?

We can use the formula $s = r|\theta|$ since the angle 4 is a radian measure:

$$\begin{aligned} s &= r|\theta| \\ &= (20 \text{ yards}) \cdot 4 \\ &= 80 \text{ yards} \end{aligned}$$

Therefore, the length of the arc spanned by the angle 4 on a circle of radius 20 feet is 80 yards.

7. What is the length of the arc spanned by the angle 25° on a circle of radius 30 inches?

Before we can use the formula $s = r|\theta|$, we need to convert the angle 25° into radians:

$$\begin{aligned} 25\cancel{^\circ} \cdot \frac{2\pi \text{ rad}}{360\cancel{^\circ}} &= \frac{50\pi}{360} \text{ rad} \\ &= \frac{5\pi}{36} \text{ rad} \end{aligned}$$

Now we can find the desired arc-length:

$$\begin{aligned} s &= r|\theta| \\ &= (30 \text{ inches}) \cdot \frac{5\pi}{36} \\ &= \frac{25\pi}{6} \text{ inches} \end{aligned}$$

Thus, the length of the arc spanned by the angle 25° on a circle of radius 30 feet is $\frac{25\pi}{6}$ inches.

8. What is the length of the arc spanned by the angle $\frac{4\pi}{5}$ radians on a circle of radius 15 feet?

We can use the formula $s = r|\theta|$ since the angle $\frac{4\pi}{5}$ is a radian measure:

$$\begin{aligned} s &= r|\theta| \\ &= (15 \text{ feet}) \cdot \frac{4\pi}{5} \\ &= 12\pi \text{ feet} \end{aligned}$$

Therefore, the length of the arc spanned by the angle $\frac{4\pi}{5}$ on a circle of radius 15 feet is 12π feet.

9. What is the length of the arc spanned by the angle 135° on a circle of radius 8 meters?

Before we can use the formula $s = r|\theta|$, we need to convert the angle 135° into radians:

$$\begin{aligned} 135^{\circ} \cdot \frac{\pi \text{ rad}}{180^{\circ}} &= \frac{135\pi}{180} \text{ rad} \\ &= \frac{45 \cdot 3\pi}{45 \cdot 4} \text{ rad} \\ &= \frac{3\pi}{4} \text{ rad} \end{aligned}$$

Now we can find the desired arc-length:

$$\begin{aligned} s &= r|\theta| \\ &= (8 \text{ meters}) \cdot \frac{3\pi}{4} \\ &= 6\pi \text{ meters} \end{aligned}$$

Therefore, the length of the arc spanned by the angle 25° on a circle of radius 8 meters is 6π meters.