Some Additional Practice: Final Exam

This isn't meant to be a “practice Final”, nor is it meant to be “the only thing you need to study to be prepared for the Final.” It's just some additional practice problems covering the material that we studied after the Midterm. If you're comfortable with all of these problems – and with all of the problems included in the “Some Additional Practice for the Midterm” – then that's a good sign that you're prepared for the Final. But you should also study the Lecture Notes, the Extra Practice problems associated with the Lecture Notes, the Worksheets, the Quizzes, and the suggested practice problems from the textbook.

1. Suppose that $\sin(\alpha) = \frac{5}{13}$ and $\cos(\beta) = \frac{3}{5}$, and where $0 < \alpha < \frac{\pi}{2}$ and $\frac{3\pi}{2} < \beta < 2\pi$. Find the exact value of:

a. $\sin(\alpha + \beta)$.

b. $\cos(\alpha - \beta)$.

c. $\sin(2\beta)$.

d. $\cos(2\beta)$.

e. $\sin\left(\frac{\beta}{2}\right)$.

f. $\cos\left(\frac{\beta}{2}\right)$.

2. Prove the following identities.

a. $\tan(x) + \cot(x) = \sec(x)\csc(x)$

b. $\tan^2(x) - \sin^2(x) = \tan^2(x)\sin^2(x)$

c. $\cos(2x) = \cos^4(x) - \sin^4(x)$

3. Convert the following polar ordered pairs into Cartesian (i.e., rectangular) coordinates.

a. $\left(3, \frac{\pi}{2}\right)$

b. $\left(\pi, \frac{5\pi}{3}\right)$

c. $(10, -10^\circ)$

4. Convert the following Cartesian (i.e., rectangular) ordered pairs into polar coordinates.

a. $(10, -10)$

b. $(-3, 0)$

c. $\left(-8, -8\sqrt{3}\right)$

5. Find an equation involving polar coordinates whose graph is equivalent to the Cartesian equation $y = 3x - 1$.

6. Find an equation involving polar coordinates whose graph is equivalent to the Cartesian equation $y = x^2$.

7. Translate the complex number $z = -3 + 3\sqrt{3} \cdot i$ into its polar form $z = re^{i\theta}$.

8. Translate the polar form of the complex number $z = 4e^{i\cdot\frac{5\pi}{6}}$ into its rectangular form $z = a + bi$.

9. Determine the magnitude and direction (with respect to the positive x-axis) of the vector $\vec{v} = \langle -3, -7 \rangle$.

10. Suppose $\vec{v} = \langle -4, 1 \rangle$ and $\vec{u} = \langle 3, -6 \rangle$.

a. Find $\vec{w} = \vec{v} - 2\vec{u}$.

b. Use the *dot product* to find the angle between $\vec{v} = \langle -4, 1 \rangle$ and $\vec{u} = \langle 3, -6 \rangle$?

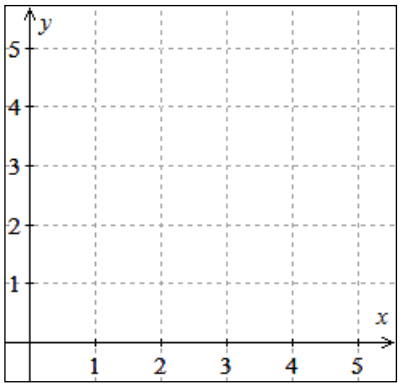
11. **a.** Find the horizontal and vertical components of the vector $\vec{v}$ that starts at the point $P = (5, 6)$ and ends at the point $Q = (2, 2)$.

b. Find the magnitude, $\|\vec{v}\|$, and the direction (with respect to the positive x-axis) of the vector $\vec{v}$ that you found in part **a**?

12. The tables below represent the x- and y-coordinates of the motion of a robot as a function of time, t, in seconds. Sketch the graph of the motion of the robot; use arrows to indicate the direction of travel.

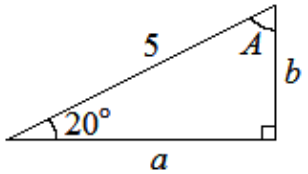
t	$x = f(t)$
0	2
1	1
2	0
3	1
4	1

t	$y = g(t)$
0	1
1	1
2	1
3	2
4	3

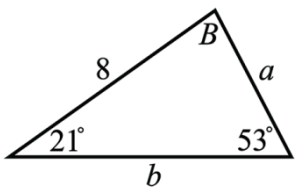


13. Find the missing side(s) and missing angle(s) for the triangles given below. (The triangles may not be drawn to scale.)

a.



b.



c.