Graphing Sinusoidal Functions

Below is a summary of what is studied in MTH 111 about graph transformations; see Section I, Units 6–8 from my online notes for MTH 111 to review graph transformations.

SUMMARY OF GRAPH TRANSFORMATIONS

Suppose that f and g are functions such that $g(t) = A \cdot f\left(\omega(t-h)\right) + k$ and $A, \omega, h, k \in \mathbb{R}$. In order to transform the graph of the function f into the graph of g:

1st: horizontally stretch/compress the graph of f by a factor of $\frac{1}{|\omega|}$ and, if $\omega < 0$, reflect it about the y-axis. (Stretch if $|\omega| < 1$; compress if $|\omega| > 1$.)

2nd: shift the graph horizontally h units (shift right if $h > 0$; shift left if $h < 0$).

3rd: vertically stretch/compress the graph by a factor of $|A|$ and, if $A < 0$, reflect it about the t-axis. (Stretch if $|A| > 1$; compress if $|A| < 1$.)

4th: shift the graph vertically k units (shift up if k is positive and down if k is negative).

(The *order* in which these transformations are performed *matters*.)

When we apply these graph transformations to the graphs of $y = \sin(t)$ and $y = \cos(t)$ we obtain *sinusoidal functions*:

DEFINITION: A **sinusoidal function** is function f of the form

$$f(t) = A\sin\left(\omega(t-h)\right) + k \quad \text{or} \quad f(t) = A\cos\left(\omega(t-h)\right) + k$$

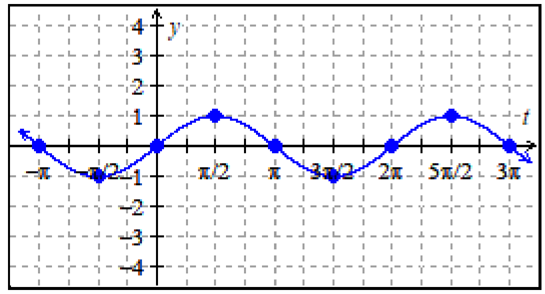
where $A, \omega, h, k \in \mathbb{R}$, $A \neq 0$, and $\omega \neq 0$.

A sinusoidal function of this form has the following properties:

•	•
•	
•	

(We'll use the examples below to determine the properties for the box above.)

EXAMPLE: The graph of $f(t) = \sin(t)$ is given below. Sketch a graph of $y = 3\sin(t)$.

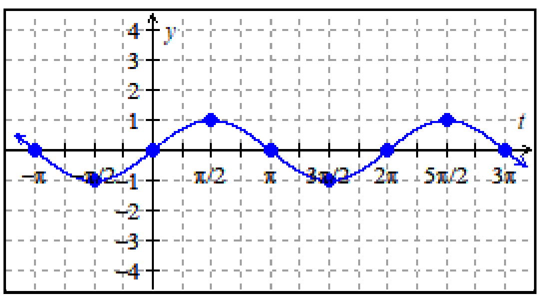


Sketch a graph of $y = 3\sin(t)$.

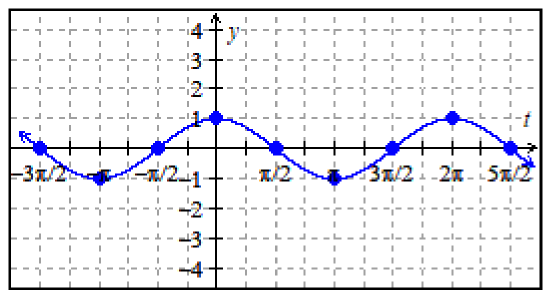
Use Desmos to graph $f(t) = \sin(t)$ & $y = A\sin(t)$ and $g(t) = \cos(t)$ & $y = A\cos(t)$ for various values of $A > 0$; then complete the following sentence:

- The graphs of $y = A\sin(t)$ and $y = A\cos(t)$ have ______________________________.

EXAMPLE: The graph of $f(t) = \sin(t)$ is given below; sketch a graph of $y = -2\sin(t)$; also the graph of $g(t) = \cos(t)$ is given below; sketch a graph of $y = -4\cos(t)$.



Sketch a graph of $y = -2\sin(t)$.

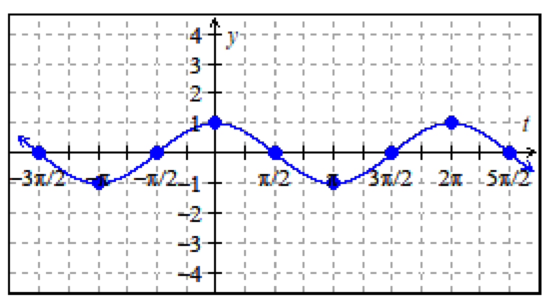


Sketch a graph of $y = -4\cos(t)$.

Observation: The reflected sine wave "starts" ___________________________________

while the reflected cosine wave "starts" ____.

EXAMPLE: The graph of $g(t) = \cos(t)$ is given below. Sketch a graph of $y = \cos(t) + 2$.

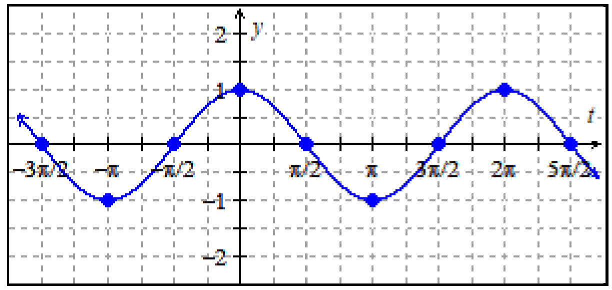


Sketch a graph of $y = \cos(t) + 2$.

Use Desmos to graph $g(t) = \cos(t)$ & $y = \cos(t) + k$ and $f(t) = \sin(t)$ & $y = \sin(t) + k$ for various values of k; then complete the following sentence:

- The graphs of $y = \cos(t) + k$ and $y = \sin(t) + k$ have ________________________.

EXAMPLE: The graph of $g(t) = \cos(t)$ is given below. Sketch a graph of $y = \cos(2t)$.

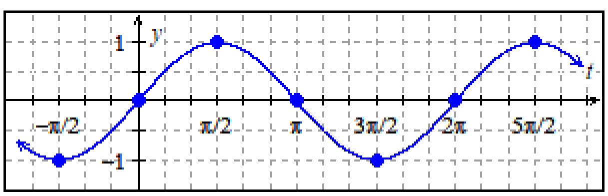


Sketch a graph of $y = \cos(2t)$.

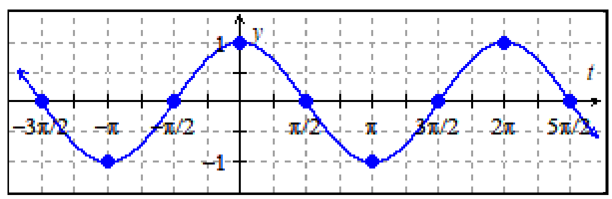
Use [Desmos](https://www.desmos.com) to graph $g(t) = \cos(t)$ & $y = \cos(\omega \cdot t)$ and $f(t) = \sin(t)$ & $y = \sin(\omega \cdot t)$ for various values of $\omega > 0$; then complete the following sentence:

- The graphs of $y = \cos(\omega \cdot t)$ and $y = \sin(\omega \cdot t)$ …

EXAMPLE: The graphs of $f(t) = \sin(t)$ and $g(t) = \cos(t)$ are given below; sketch graphs of $y = \sin\left(t - \frac{\pi}{3}\right)$ and $y = \cos\left(t + \frac{\pi}{4}\right)$.



Sketch a graph of $y = \sin\left(t - \frac{\pi}{3}\right)$.

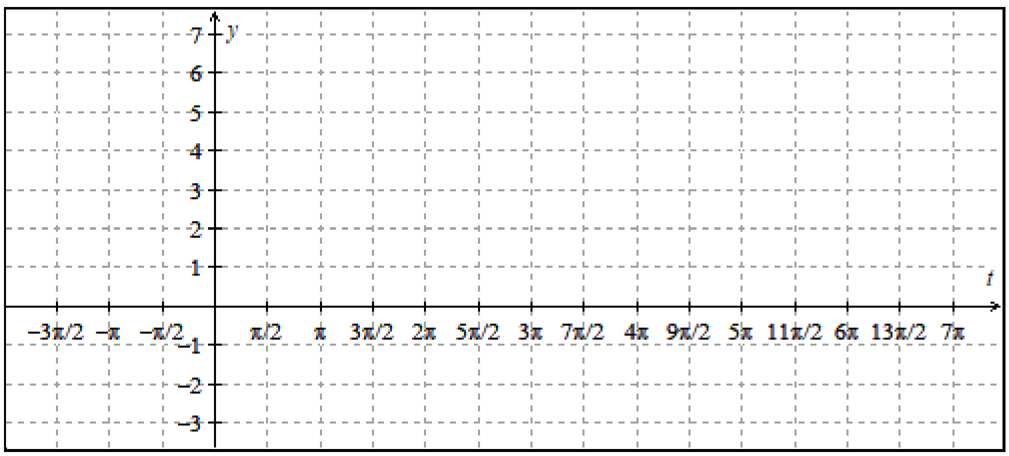


Sketch a graph of $y = \cos\left(t + \frac{\pi}{4}\right)$

EXAMPLE: Use [Desmos](https://www.desmos.com) to compare $p(t) = \cos\left(2t - \frac{\pi}{3}\right)$ and $q(t) = \cos\left(2\left(t - \frac{\pi}{3}\right)\right)$. Determine the appropriate horizontal shift for each function.

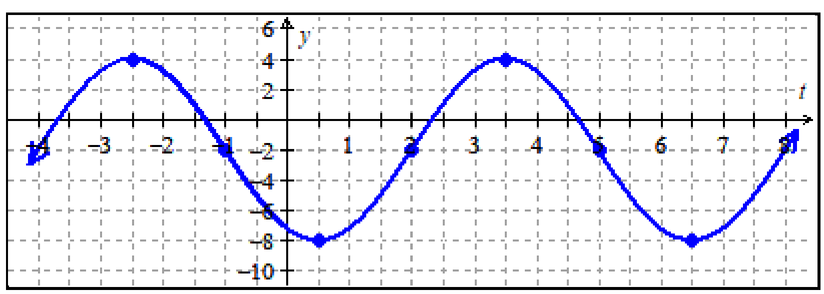
<u>Observation</u>: *To determine the horizontal shift, first factor the input of the trig function.*

EXAMPLE: Sketch a graph of $m(t) = 2\sin\left(\frac{1}{2}t + \frac{\pi}{4}\right) + 3$. State the period, midline, and amplitude of m.



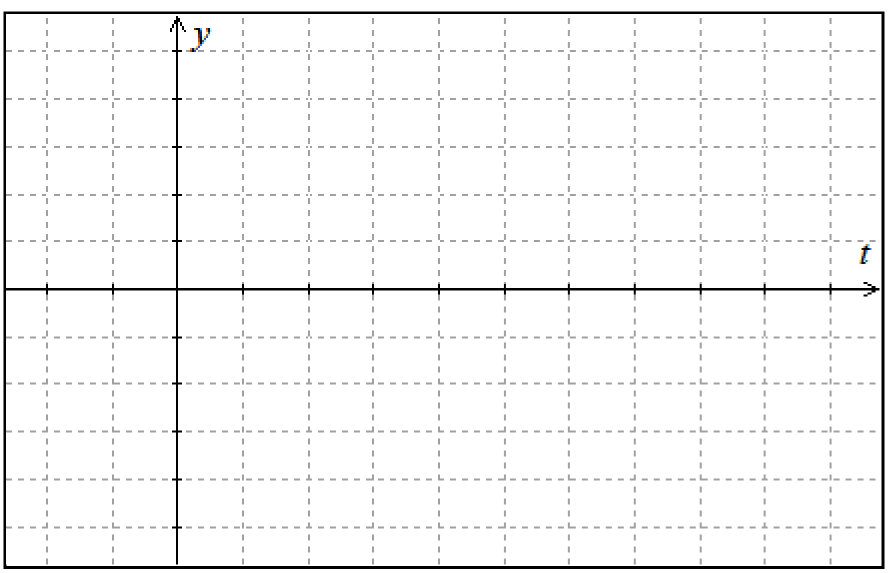
Sketch a graph of $m(t) = 2\sin\left(\frac{1}{2}t + \frac{\pi}{4}\right) + 3$.

EXAMPLE: Find (at least) two algebraic rules (i.e., “formulas”), one involving sine and one involving cosine, for the sinusoidal function n whose graph is given below.



The graph of $y = n(t)$.

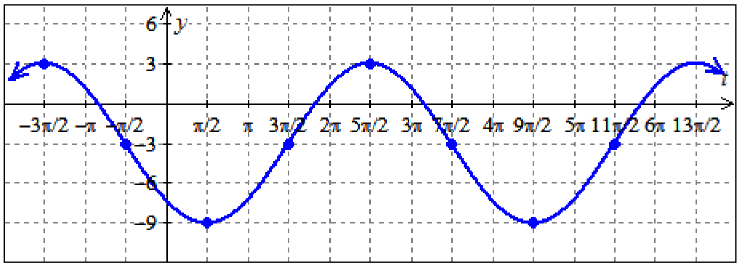
EXAMPLE: Sketch a graph of $f(t) = 2\sin\left(\pi t - \frac{\pi}{4}\right) - 3$ on the coordinate plane below.



Sketch a graph of $f(t) = 2\sin\left(\pi t - \frac{\pi}{4}\right) - 3$.

More Practice: Graphing Sinusoidal Functions

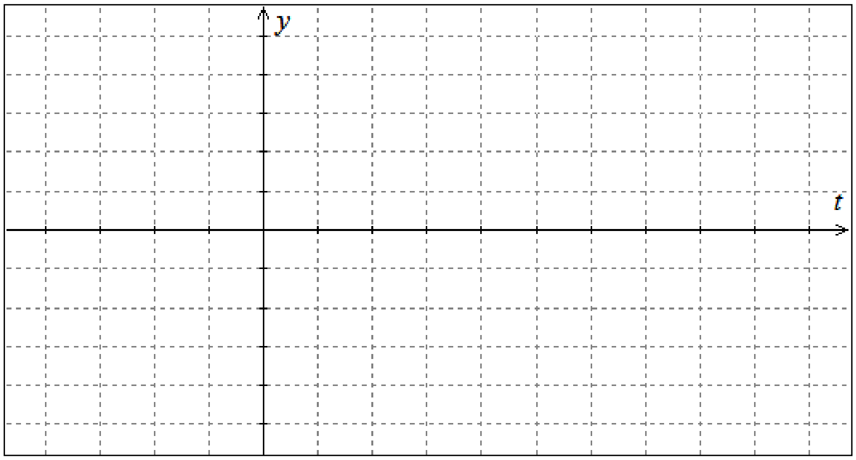
a. Find an algebraic rule (i.e., a "formula") for the function f whose graph is given below.



The graph of $y = f(t)$.

b. Sketch a graph of $g(t) = 3\cos\left(\frac{\pi}{2}t - \frac{\pi}{4}\right) - 1$ on the coordinate plane below.

List the *period*, *amplitude*, *midline*, and *horizontal shift*.



Sketch a graph of $g(t) = 3\cos\left(\frac{\pi}{2}t - \frac{\pi}{4}\right) - 1$.