A Summary via Examples

EXAMPLE 1a: Find B and c in the right triangle given in Figure 1. (The triangle might not be drawn to scale.)

Use pythagoras to find c:

$$c^2 = 8^2 + 4^2$$

$$\Rightarrow c = \sqrt{8^2+4^2} = \sqrt{80} = \sqrt{2\cdot 2\cdot 2\cdot 2\cdot 5} = 4\sqrt{5}$$

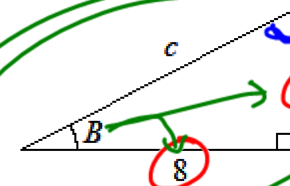


Figure 1

$$\tan(B) = \frac{4}{8} \quad \left(= \frac{\text{opp}}{\text{adj}}\right)$$

$$B = \tan^{-1}\left(\frac{1}{2}\right) \approx 26.6^\circ$$

EXAMPLE 1b: Find a and b in the right triangle given in Figure 2. (The triangle might not be drawn to scale.)

$$\sin(60^\circ) = \frac{b}{10} = \frac{\text{opp}}{\text{hyp}}$$

$$\Rightarrow b = 10\sin(60^\circ) = 10\cdot\frac{\sqrt{3}}{2} = 5\sqrt{3}$$

$$\cos(60^\circ) = \frac{a}{10} = \frac{\text{adj}}{\text{hyp}}$$

$$\Rightarrow a = \boxed{10\cos(60^\circ)} = 10\cdot\frac{1}{2} = 5$$

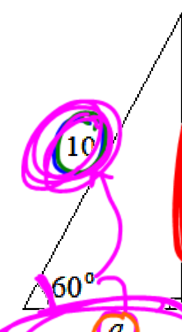


Figure 2

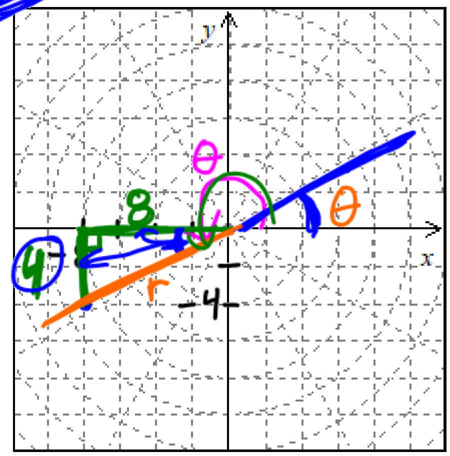
EXAMPLE 2a: Translate the rectangular ordered pair $(-8, -4)$ into polar coordinates (r, θ).

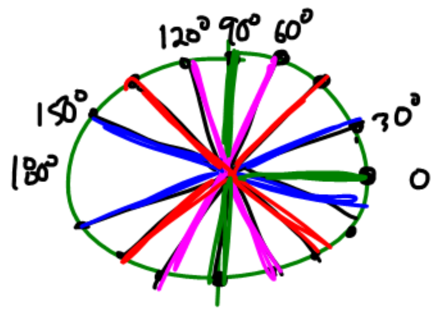
$x \quad y$

$$r = \sqrt{(-8)^2 + (-4)^2}$$
$$= \sqrt{64 + 16}$$
$$= \sqrt{80}$$
$$= 4\sqrt{5}$$

$$\tan(\theta) = \frac{-4}{-8}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{1}{2}\right) + 180^\circ}$$
$$\approx 26.6^\circ + 180^\circ$$
$$= 206.6^\circ$$

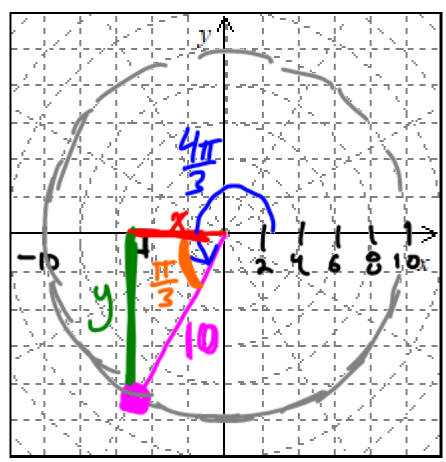




$r \quad \theta$

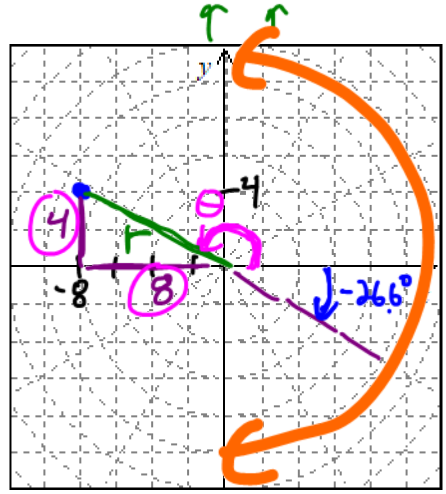
EXAMPLE 2b: Translate the polar ordered pair $\left(10, -\frac{4\pi}{3}\right)$ into rectangular coordinates (x, y).

$$(x, y) = \left(10 \cdot \cos\left(\frac{4\pi}{3}\right), 10 \sin\left(\frac{4\pi}{3}\right)\right)$$
$$= \left(10 \cdot \left(-\frac{1}{2}\right), \quad 10\left(-\frac{\sqrt{3}}{2}\right)\right)$$
$$= \left(-5, \; -5\sqrt{3}\right)$$



EXAMPLE 3a: Find the polar form, $z = r \cdot e^{i \cdot \theta}$, of the complex number $z = -8 + 4i$.

$z = -8 + 4i$ can be identified by point $\boxed{(-8, 4)}$



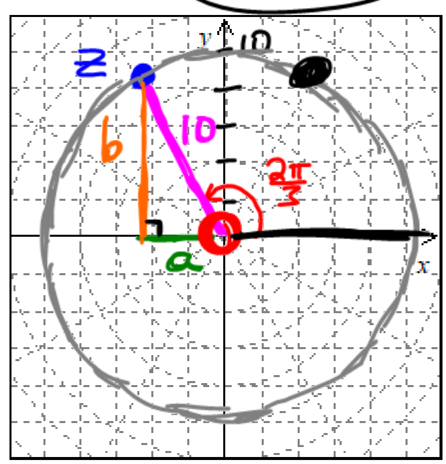
$$r = \sqrt{(-8)^2 + (4)^2}$$
$$= \sqrt{80}$$
$$= 4\sqrt{5}$$

$$\tan(\theta) = \frac{4}{-8}$$
$$\theta = \tan^{-1}\left(-\frac{1}{2}\right) + 180^\circ$$
$$\approx -26.6^\circ + 180^\circ$$
$$\approx 153.4^\circ$$

$$z \approx 4\sqrt{5} \cdot e^{i \cdot (153.4^\circ)}$$

EXAMPLE 3b: Find the rectangular form, $z = a + bi$, of the complex number $z = 10 \cdot e^{i \cdot \frac{2\pi}{3}}$.

We can plot $z = 10e^{i\left(\frac{2\pi}{3}\right)}$ by noticing $r = 10$ & $\theta = \frac{2\pi}{3}$



$$a = 10\cos\left(\frac{2\pi}{3}\right)$$
$$= 10\left(\frac{-1}{2}\right)$$
$$= -5$$

$$b = 10 \cdot \sin\left(\frac{2\pi}{3}\right)$$
$$= 10 \cdot \frac{\sqrt{3}}{2}$$
$$= 5\sqrt{3}$$

$$\therefore z = -5 + 5\sqrt{3} \cdot i$$

EXAMPLE 4a: If $\vec{v} = 8\vec{i} - 4\vec{j}$, find

so $\vec{v} = \langle 8, -4 \rangle$

(1) the direction of $\vec{v}$ (measured in degrees with respect to the positive x-axis)

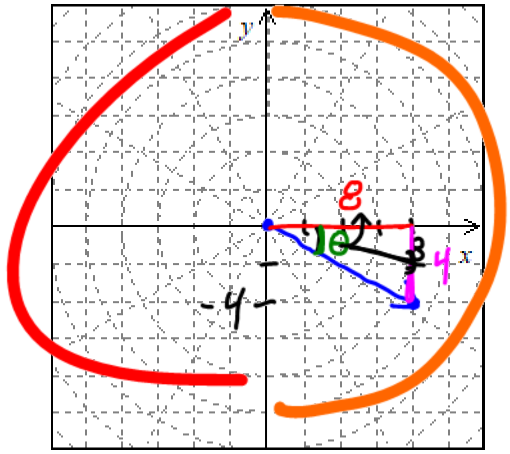
(2) $\|\vec{v}\|$

(1) $\tan(\theta) = \frac{-4}{8} \Rightarrow \theta = \tan^{-1}\left(-\frac{1}{2}\right)$

$\approx -26.6^\circ$

(2) $\|\vec{v}\| = \sqrt{(8)^2 + (-4)^2}$

$= 4\sqrt{5}$



EXAMPLE 4b: Find the components of the vector $\vec{w}$ if $\|\vec{w}\| = 10$ and $\vec{w}$ has a direction of 300° with respect to the positive x-axis.

$\vec{w} = \langle x, y \rangle$

$= \langle 10 \cdot \cos(300^\circ), 10 \cdot \sin(300^\circ) \rangle$

$= \langle 10 \cdot \frac{1}{2}, 10\left(-\frac{\sqrt{3}}{2}\right) \rangle$

$= \langle 5, -5\sqrt{3} \rangle$