$\sqrt[3]{x^3} = \sqrt[3]{10}$ vs $x^3 = 8$

$x = \sqrt[3]{10}$ $\quad$ $x = 2$

"unfriendly" $\quad$ "friendly"

Solving Trig Equations

EXAMPLE: Find all of the solutions to the equation $8\cos(t) + 3 = -1$.

$8\cos(t) + 3 = -1$ $\quad$ isolate trig function

$8\cos(t) = -4$

$\cos(t) = \frac{-4}{8} = -\frac{1}{2}$

$\cos^{-1}(\cos(t)) = \cos^{-1}\left(-\frac{1}{2}\right)$

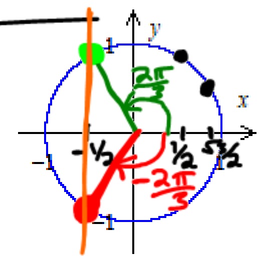
$\boxed{t = \frac{2\pi}{3} + 2k\pi \quad \text{OR} \quad t = -\frac{2\pi}{3} + 2k\pi \quad k \in \mathbb{Z}}$

set of integers (whole numbers)

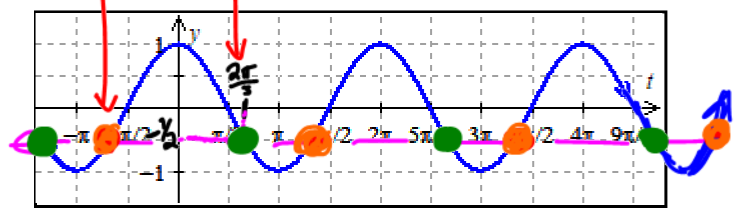
"is an element of"

Identity:

$\cos(\theta) = \cos(-\theta)$



$y = -\frac{1}{2}$



A graph of $y = \cos(t)$.

EXAMPLE: Find all of the solutions to the equation $4\sin(t) + 2\sqrt{3} = 0$.

$4\sin(t) + 2\sqrt{3} = 0$

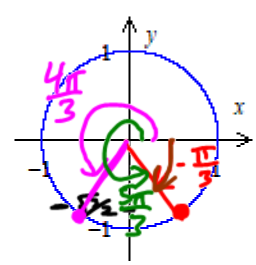
$4\sin(t) = -2\sqrt{3}$

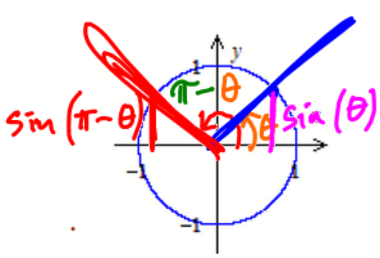
$\sin(t) = -\frac{2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$

$\sin^{-1}(\sin(t)) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

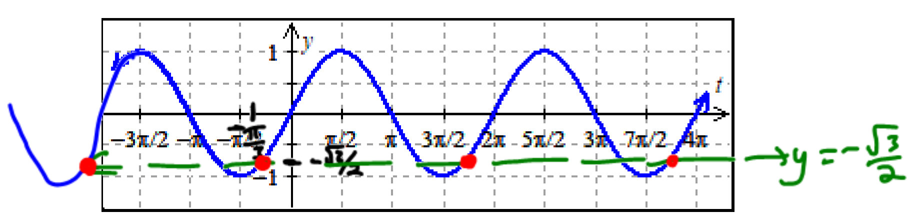
$2\pi \cdot k = 2k\pi$

$\boxed{t = -\frac{\pi}{3} + 2k\pi \quad \text{OR} \quad t = \pi - \left(-\frac{\pi}{3}\right) + 2k\pi = \frac{4\pi}{3} + 2k\pi \quad \text{for } k \in \mathbb{Z}}$





$\sin(\theta) = \sin(\pi - \theta)$



A graph of $y = \sin(t)$.

$y=\cos(3t)$ has period: $P = 2\pi \cdot \frac{1}{3} = \frac{2\pi}{3}$

EXAMPLE: First find *all* of the solutions to the equation $2\sqrt{2}\cos(2t) = -2$. Then find the particular solutions on the interval $[0, 2\pi)$.

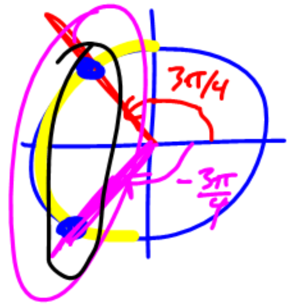
$[0,\pi)$

$$2\sqrt{2}\cos(3t) = -2$$

$$\cos(3t) = -\frac{2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\cos(3t) = -\frac{\sqrt{2}}{2}$$

$$\cos^{-1}(\cos(3t)) = \cos^{-1}\left(-\sqrt{2}/2\right)$$



$$\frac{3t}{3} = \frac{3\pi}{4\cdot 3} + \frac{2k\pi}{3} \quad \text{OR} \quad \frac{3t}{3} = -\frac{3\pi}{4\cdot 3} + \frac{2k\pi}{3}, \; k \in \mathbb{Z}$$

$$\boxed{t = \frac{\pi}{4} + \frac{2\pi}{3}\cdot k \quad \text{OR} \quad t = -\frac{\pi}{4} + \frac{2k\pi}{3}, \; k \in \mathbb{Z}}$$

Find solutions on $[0, 2\pi) = [0, \frac{24\pi}{12})$

$k=0$: $t = \frac{\pi}{4} + \frac{2\cdot 0\cdot\pi}{3} = \frac{\pi}{4}$ ✓ OR $t = -\frac{\pi}{4} + \frac{2\cdot 0\,\pi}{3} = -\frac{\pi}{4}$ ✗

$k=1$: $t = \frac{\pi}{4} + \frac{2\cdot 1\pi}{3} = \frac{3\pi}{12} + \frac{8\pi}{12} = \frac{11\pi}{12}$ ✓ $\qquad t = -\frac{\pi}{4} + \frac{2\cdot 1\,\pi}{3} = -\frac{3\pi}{12} + \frac{8\pi}{12} = \frac{5\pi}{12}$ ✓

$k=2$: $t = \frac{\pi}{4} + \boxed{\frac{2\cdot 2\pi}{3}} = \frac{3\pi}{12} + \frac{16\pi}{12} = \frac{19\pi}{12}$ ✓ $\qquad t = -\frac{\pi}{4} + \boxed{\frac{2\cdot 2\pi}{3}} = -\frac{3\pi}{12} + \frac{16\pi}{12} = \frac{13\pi}{12}$ ✓

$k=3$: $t = \frac{\pi}{4} + \frac{2\cdot 3\pi}{3} = \frac{3\pi}{12} + \frac{24\pi}{12} = \frac{27\pi}{12} > \frac{24\pi}{12} = 2\pi$ ✗ $\qquad t = -\frac{\pi}{4} + \frac{2\cdot\cancel{3}\,\pi}{\cancel{3}} = \frac{7\pi}{4}$ ✓

$k=4$: NOT WORTH TRYING $\qquad t = -\frac{\pi}{4} + \frac{2\cdot 4\pi}{3} = \frac{3\pi}{12} + \frac{32\pi}{12} = \frac{35\pi}{12} > 2\pi$ ✗

Solution set is:

$$\left\{\frac{\pi}{4}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{7\pi}{4}\right\}$$

$k = -1$: $t = \frac{\pi}{4} + \frac{2(-1)\pi}{3}$

$= \frac{3\pi}{12} + \frac{-8\pi}{12}$

$= \frac{-5\pi}{12} < 0$ ✗

$t < 0$ ✗