# Intro to the Trigonometric Functions: Part 2

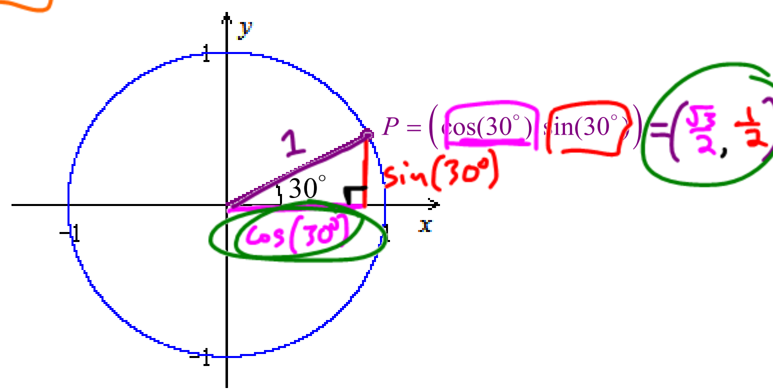
(these notes correspond to Parts 3 & 4 from my MTH 112 Online Lecture Notes)

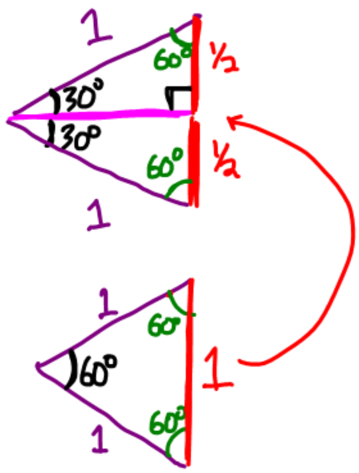
$\boxed{30^\circ} + \boxed{90^\circ} + \boxed{60^\circ} = 180^\circ$

Now let's determine the sine and cosine of some important angles, namely, $30^\circ$, $45^\circ$, and $60^\circ$ (i.e., $\boxed{\frac{\pi}{6}, \frac{\pi}{4}, \text{ and } \frac{\pi}{3}}$). Let's start with $\boxed{30^\circ = \frac{\pi}{6}}$.


$\boxed{\sin(30^\circ) = \frac{1}{2}}$

$\boxed{\cos(30^\circ) = \frac{\sqrt{3}}{2}}$





$\cos(30^\circ) = \frac{\sqrt{3}}{2}$

$$(\cos(30^\circ))^2 + \left(\frac{1}{2}\right)^2 = 1$$

$$\cos^2(30^\circ) + \frac{1}{4} = 1$$

$$\sqrt{\cos^2(30^\circ)} = \sqrt{\frac{3}{4}}$$

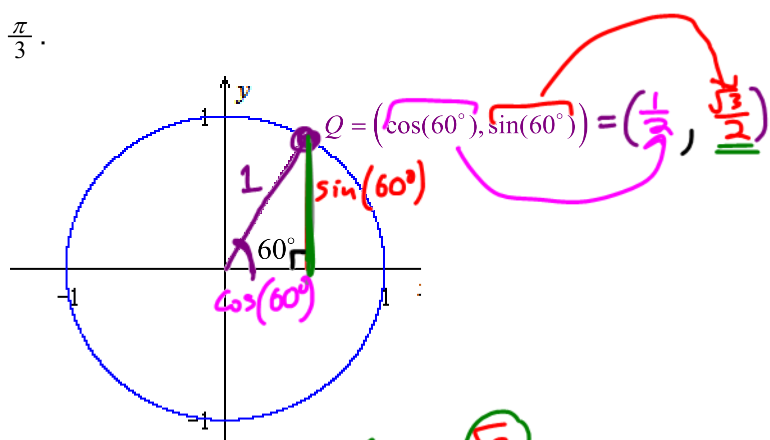
$$\boxed{\cos(30^\circ) = \frac{\sqrt{3}}{2}}$$

$60^\circ + 90^\circ + \boxed{30^\circ} = 180^\circ$

Now let's find the sine and cosine values for $60^\circ = \frac{\pi}{3}$.

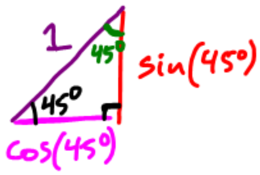

$\sin(60^\circ) = \frac{\sqrt{3}}{2}$

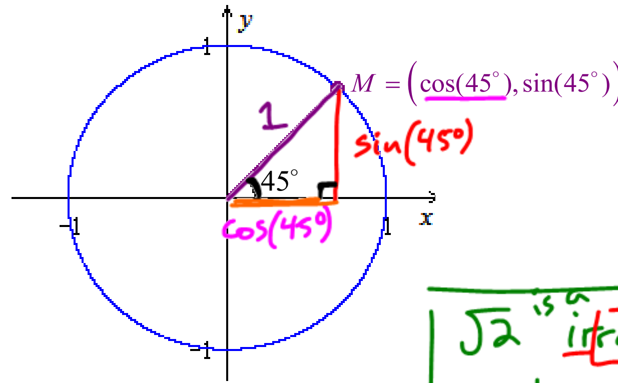
$\cos(60^\circ) = \frac{1}{2}$




$\frac{1}{2} < \frac{\sqrt{3}}{2}$

Now let's find the sine and cosine values for $45^\circ = \frac{\pi}{4}$:





Use pythagoras

$$1 \cdot x^2 + 1x^2 = 1^2$$

$$2x^2 = 1$$

$$\sqrt{x^2} = \sqrt{\tfrac{1}{2}}$$

$$x = \sqrt{\tfrac{1}{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$\sqrt{2}$ is a irrational number. $\sqrt{2} \neq \frac{a}{b}$ where a & b are integers

$$\cos(45^\circ) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sin(45^\circ) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Since we now know the sine and cosine values of $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$ and since sine and cosine represent, respectively, the vertical and horizontal coordinates of points on the circumference of a unit circle, we now know the coordinates of the points on the circumference of the unit circle specified by the angles $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$; let's label the coordinates on the circle in Fig. 1.

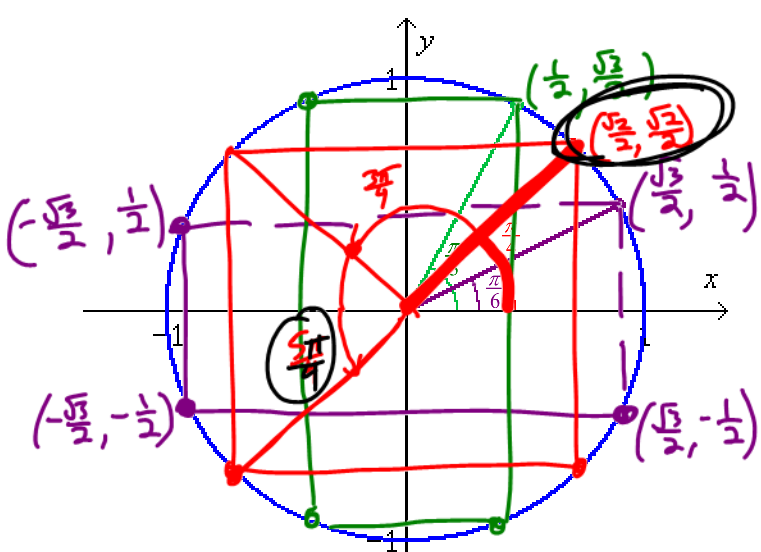



**Figure 1**

Angles: $\frac{\pi}{6}$ (30°) & $\frac{\pi}{3}$ (60°) have sine & cosine values $\frac{1}{2}$ & $\frac{\sqrt{3}}{2}$

Angle $\frac{\pi}{4}$ (45°) has sine & cosine value $\frac{\sqrt{2}}{2}$

Now let's watch the video in Sect. I, Chapter 3, part 4 of the online lecture notes. (18 minutes)

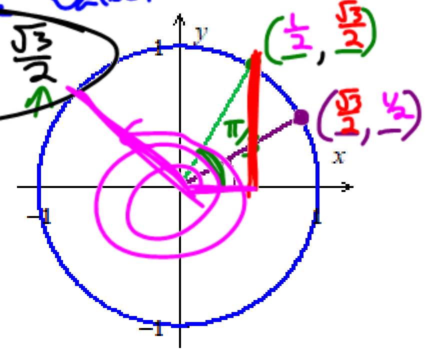
## Finding the sine and cosine of "friendly" angles:

Let's summarize what we've observed about the sine and cosine of multiples of $\frac{\pi}{6}$, $\frac{\pi}{4}$ or $\frac{\pi}{3}$:

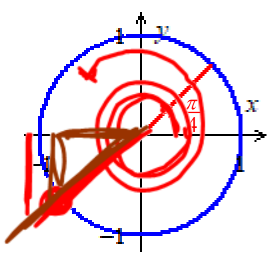
Multiples of $\frac{\pi}{4}$:

All multiples of $\frac{\pi}{4}$ have sine & cosine values involving $\frac{\sqrt{2}}{2}$.

Multiples of $\frac{\pi}{6}$ and $\frac{\pi}{3}$: all have sine & cosine values involving $\frac{1}{2} < \frac{\sqrt{3}}{2}$




**EXAMPLE:** Find $\sin\left(\frac{21\pi}{4}\right)$, $\cos\left(\frac{21\pi}{4}\right)$, and $\tan\left(\frac{21\pi}{4}\right)$.

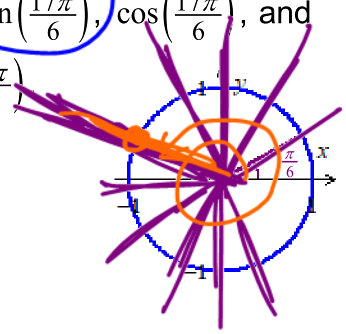



$\sin\left(\frac{21\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\cos\left(\frac{21\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\tan\left(\frac{21\pi}{4}\right) = 1$

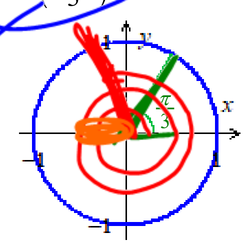
**EXAMPLE:** Find $\sin\left(\frac{17\pi}{6}\right)$, $\cos\left(\frac{17\pi}{6}\right)$, and $\sec\left(\frac{17\pi}{6}\right)$.




$\sin\left(\frac{17\pi}{6}\right) = \frac{1}{2}$

$\frac{17\pi}{6} = \frac{12\pi}{6} + \frac{5\pi}{6}$

**EXAMPLE:** Find $\sin\left(\frac{14\pi}{3}\right)$, $\cos\left(\frac{14\pi}{3}\right)$, and $\csc\left(\frac{14\pi}{3}\right)$.



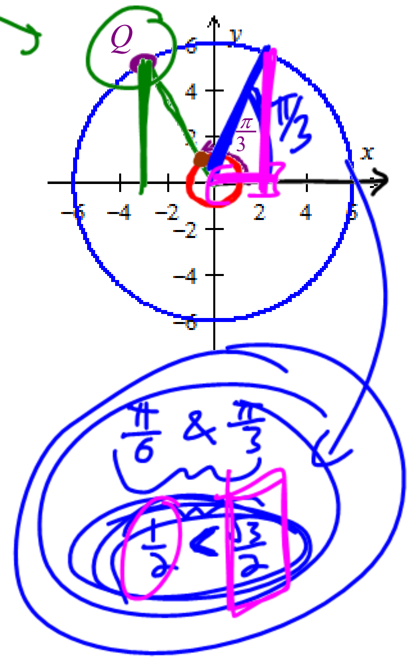

$\cos\left(\frac{14\pi}{3}\right) = -\frac{1}{2}$

$\frac{1}{2} < \frac{\sqrt{3}}{2}$

$\frac{14\pi}{3} = \frac{6\pi}{3} + \frac{6\pi}{3} + \frac{2\pi}{3}$

**EXAMPLE:** A circle with a radius of $6$ units is given below. The point $Q$ is specified by the angle $\frac{2\pi}{3}$. Use the sine and cosine function to find the exact coordinates of point $Q$.

$$Q = \left(6\cdot \cos\left(\tfrac{2\pi}{3}\right), 6\cdot \sin\left(\tfrac{2\pi}{3}\right)\right)$$
$$= \left(6\cdot\left(-\tfrac{1}{2}\right), 6\cdot\left(\tfrac{\sqrt{3}}{2}\right)\right)$$
$$= \left(-3, 3\sqrt{3}\right)$$




## Last slide from video: "Working with Multiples of $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$"