Introduction to Periodic Functions

Any activity that repeats on a regular time interval can be described as *periodic*. For example, swinging pendulums and bouncing springs can exhibit *periodic* behavior.

DEFINITION: A ***periodic function*** is a function whose values repeat on regular intervals. Hence, f is periodic if there exists some constant c such that

$$f(t+c) = f(t)$$

for all t in the domain of f such that $f(t+c)$ is defined. (Recall that this means that if the graph of $y = f(t)$ is shifted horizontally c units then it will appear unaffected.)

EXAMPLE 1: Figure 1 shows a graph of a *periodic function*. We know that it is periodic since an interval of the graph repeats over-and-over-and-over; let's highlight that interval on the graph:

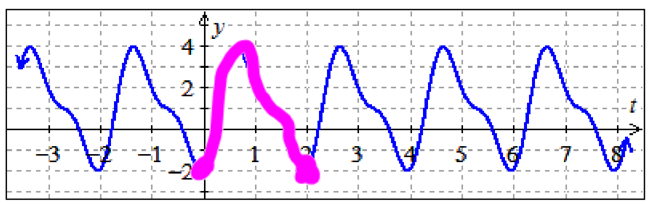


Figure 1

DEFINITION: The ***period*** of a periodic function f is the smallest value $|c|$ such that $f(t+c) = f(t)$ for all t in the domain of f such that $f(t+c)$ is defined.

EXAMPLE 2: Find the period of the function graphed in Figure 1.

The period is 2 units.

DEFINITIONS:

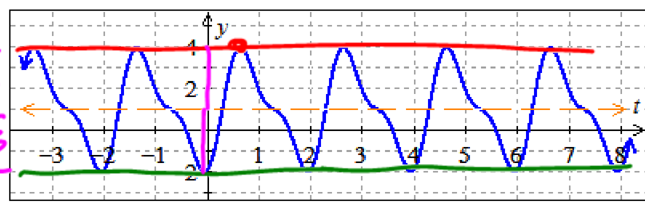
- The ***midline*** of a periodic function is the horizontal line midway between the function's minimum and maximum values.

  If $y = f(t)$ is periodic and $f_{\max}$ and $f_{\min}$ are the maximum and minimum values of f, respectively, then the equation of the midline is $y = \dfrac{f_{\max} + f_{\min}}{2}$

- The ***amplitude*** of a periodic function is the distance between the function's maximum value and the midline (or the function's minimum value and the midline).

EXAMPLE 3: Find the midline and the amplitude of the function graphed in Figure 2.

3 units
3 units



y = 1

Figure 2

Midline

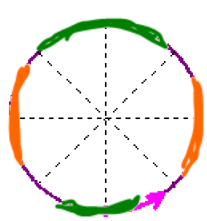
$$y = \frac{4 + (-2)}{2}$$

$$= 1$$

Amplitude

3 units.

EXAMPLE 4: The Amusement Park has a Ferris wheel 200 feet in diameter. The Wheel rotates at a constant rate and completes a rotation once every 40 minutes. Let $h(t)$ represent the height in feet of a Ferris wheel passenger t minutes after boarding the Wheel at ground level. (So $h(0) = 0$.) Sketch a graph of $y = h(t)$ for $0 \le t \le 100$. (Construct a table of values, plot points on the coordinate plane, and consider how best to connect the points.)



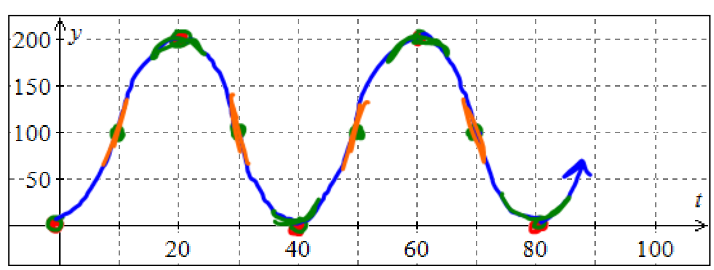


Figure 3: Sketch a graph of $y = h(t)$.