Right Triangle Trigonometry

As we studied in "Intro to the Trigonometric Functions: Part 1," if we put the same angle in the center of two circles of different radii, we can construct two *similar triangles*; see Figure 1.

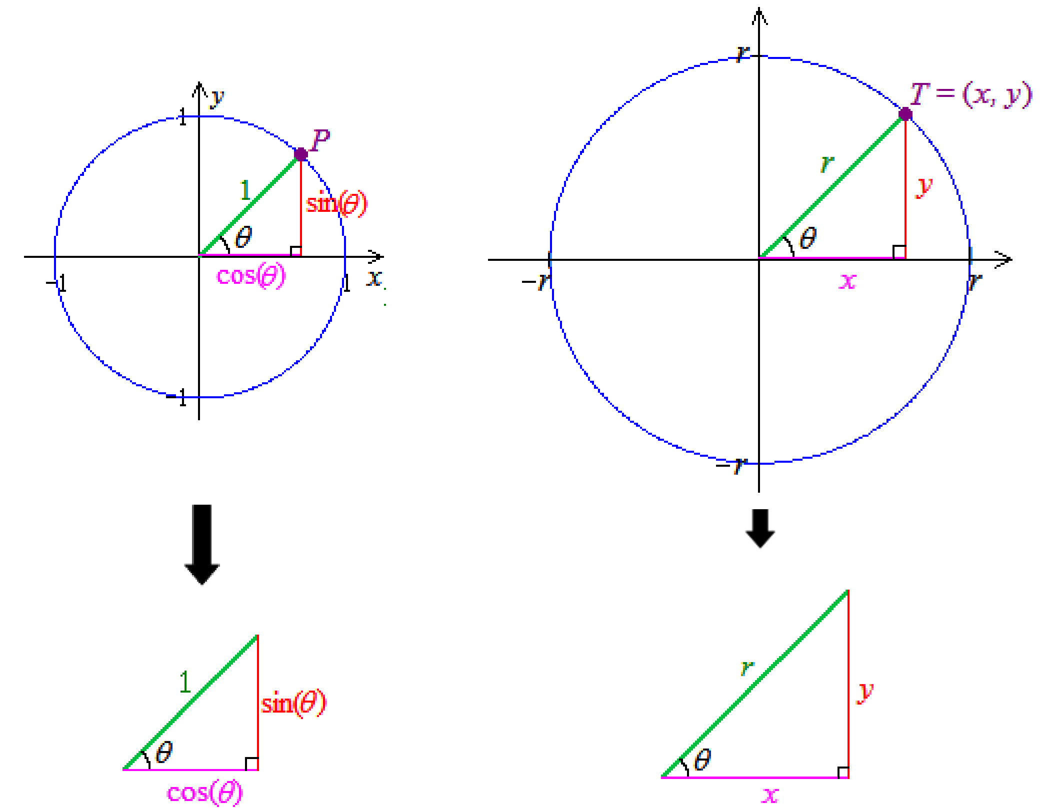

Figure 1: The angle θ in both a unit circle and in a circle of radius r, inducing similar right triangles.

We can use these similar triangles to obtain the following ratios (which we can use to derive expressions for $\sin(\theta)$ and $\cos(\theta)$):

To help remember these ratios, it's best to imagine yourself standing at angle θ looking into the triangle. Then the side labeled y is on the **opposite** side of the triangle while the side labeled x is **adjacent** to you. We use these descriptions (as well as the fact that the side labeled r is the **hypotenuse** of the triangle) to refer to the sides of the triangles in Figure 1.

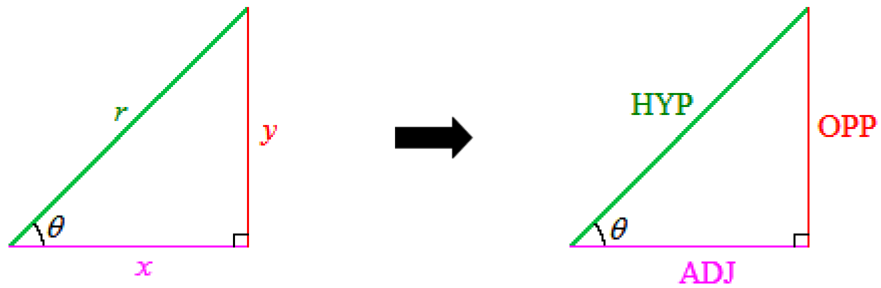

Figure 2: We use the terms **opposite** (or **OPP**), **adjacent** (or **ADJ**), and **hypotenuse** (or **HYP**) to refer to the sides of a right triangle.

DEFINITION: If θ is the angle given in the right triangles in Figure 2, then

$\sin(\theta) =$ $\cos(\theta) =$ $\tan(\theta) =$

Consequently, the other trigonometric functions can be defined as follows:

$\cot(\theta) =$ $\sec(\theta) =$ $\csc(\theta) =$

EXAMPLE 1: Find value for all six trigonometric functions of the angle α given in the right triangle in Figure 3. (The triangle may not be drawn to scale.)

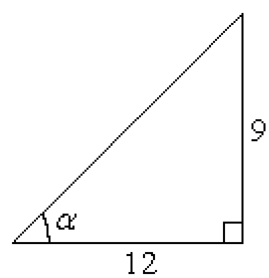

Figure 3

We can use the trigonometric functions, along with the Pythagorean Theorem to "**solve a right triangle**," i.e., find the missing side-lengths and missing angle-measures for a triangle.

EXAMPLE 2: Solve the right triangle given in Figure 4 by finding A, b, and c. (The triangle might not be drawn to scale.)

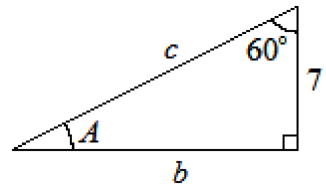

Figure 4

EXAMPLE 3: Solve the right triangle given in Figure 5 by finding c, α, and β. (The triangle might not be drawn to scale.)

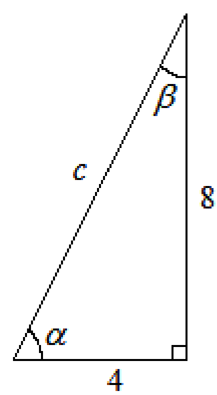

Figure 5