Proving Trigonometric Identities

This quarter we've studied many important trigonometric identities. Because these identities are so useful, it is worthwhile to learn (or memorize) most of them (e.g., the Pythagorean Identity). But there are many other identities that aren't particularly important (so they aren't worth memorizing) but they exist and provide us an opportunity to learn another skill: proving mathematical statements. Today we will *prove* that some equations are in fact identities.

An **identity** is an equation that's true for all values in the domains of the involved expressions. To prove an identity, we need to show that both sides of the equation are *always* equal. To accomplish this, we need to start with the expression on one side of the equation and use the rules of algebra and the identities that we've already studied to manipulate the expression until it is identical to the expression on the other side of the equation.

EXAMPLE 1: Prove the identity $\sin(x) = \dfrac{\tan(x)}{\sec(x)}$.

EXAMPLE 2: Prove the identity $\cot(x) + \tan(x) = \csc(x)\sec(x)$.

EXAMPLE 3: Prove the identity $\dfrac{1}{1-\cos(t)} + \dfrac{1}{1+\cos(t)} = 2\csc^2(t)$

EXAMPLE 4: Prove the identity $\dfrac{\cos(\theta)}{1-\sin(\theta)} = \dfrac{1+\sin(\theta)}{\cos(\theta)}$.