Class Discussion: Angles and Arc-length

Measuring Angles

Degrees are a "concept":

The "why" and "what" of **radians**:

When angles of various sizes are placed in standard position inside of a circle, there is a direct correspondence between size of the angle and the length of the arc spanned by each angle: larger angles span longer arcs and smaller angles span shorter arcs. See desmos.com/calculator/6lfq0nrc3z.

When an angle is placed in standard position inside circles of various sizes, the pie-shape regions that are created are all **similar figures**; see desmos.com/calculator/qzbnqzettm.

From geometry, when figures are similar *the ratio of corresponding components of the figures is constant*. We use this fact along with what we discussed above about angles and arc-lengths to establish this definition of an angle measured in radians:

if an angle θ spans an arc of length s in a circle of radius r then the radian measure of θ is:
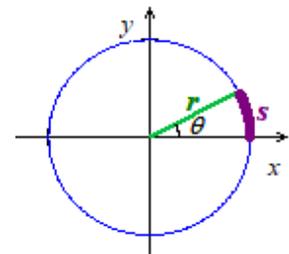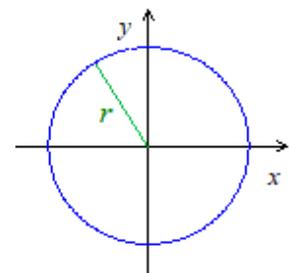
In practice, we aren't going to need to literally measure arc-lengths to find radians measures for angles. For our purposes, we just need to discover one angle's radian measurement and then we can use that to find other radian measures.

We can determine the radian measure of a full revolution (i.e., $360°$) by using the definition of the number π:
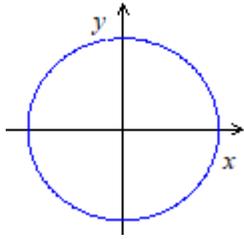
If c is the circumference of a circle and r is the radius of the circle, $\pi = \frac{c}{2r}$.

A full revolution (i.e., $360°$) spans arc equivalent to the entire circumference of the circle: let's use the definition of π to find a formula for the circumference, and then use the definition of the radian measure of an angle to find the radian equivalent of $360°$:
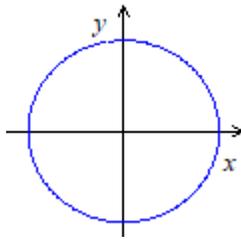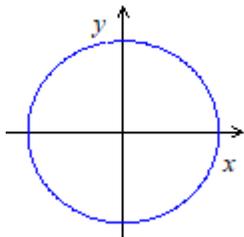
EXAMPLE 1:

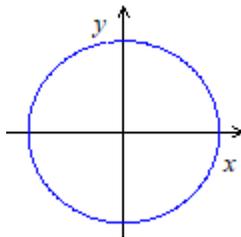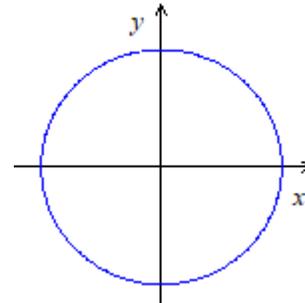|	
2π is about _____ & is a _____ .	π is about _____ & π is _____ 2π .
$\frac{\pi}{2}$ is about _____ & $\frac{\pi}{2}$ is _____ π .	$\frac{3\pi}{2}$ is about _____ & $\frac{3\pi}{2}$ is _____ $\frac{\pi}{2}$.

EXAMPLE 2:

Sketch the given angles in standard position on the circle:

a. 2 radians

b. 3.5 radians

EXAMPLE 3: Convert $\frac{4\pi}{9}$ radians into degrees.