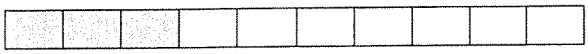
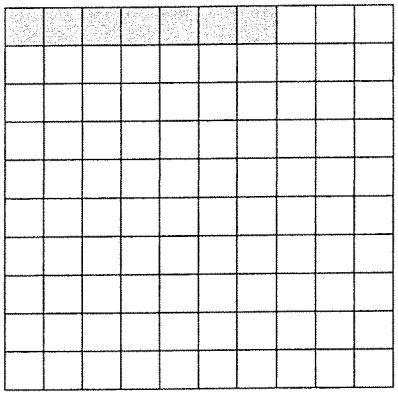
Section 4.1 An Introduction to Decimals

Let's compare some fractions to their decimal equivalents.



$$\frac{3}{10} = 0.3$$



$$\frac{7}{100} = 0.07$$

Place Value Chart

Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths
2	1	0	,	9	8	7	,	6	5	4	,	3	2	1	.	2	3	4	5	6

This Chart shows the place value of the number 210,987,654,321.23456

This is how you say it.

Two hundred ten billion, nine hundred eighty seven million, six hundred fifty four thousand, three hundred twenty one, and twenty three thousand four hundred fifty six hundred thousandths.

Math-Aids.Com

Example: Consider the decimal number 3,576.12489 0000

a. What is the place value of the digit 4?

thousandth

b. Which digit tells the number of millionths.

No digit is shown in the millionths place, but we can write a zero there.

The place values of the columns for the fractional part of a decimal each have a value that is $\frac{1}{10}$ of the value of the place directly to its left. For example, the value of the tenths column is $\frac{1}{10}$ the value of the ones column: $1 \cdot \frac{1}{10} = \frac{1}{10}$. The value of the hundredths column is $\frac{1}{10}$ of the value of the tenths column: $\frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$. The value of the thousandths column is $\frac{1}{10}$ of the value of the hundredths column: $\frac{1}{100} \cdot \frac{1}{10} = \frac{1}{1000}$.

The meaning of the decimal 3.456 becomes clear when we write it in **expanded form** (also called **expanded notation**).

3.456 = 3 ones + 4 tenths + 5 hundredths + 6 thousandths

which can be written as:

$$3.456 = 3 + \frac{4}{10} + \frac{5}{100} + \frac{6}{1000}$$

The word *decimal* comes from the Latin word *decima*, meaning a tenth part.

Example: Write the decimal 1,274.3659 in expanded form.

1,274.3659 = 1 thousand + 2 hundred + 7 tens + 4 ones
+ 3 tenths + 6 hundredths + 5 thousandths
+ 9 ten-thousandths

To understand how to read a decimal, let's examine the expanded form of 3.456 in more detail. Recall that $3.456 = 3 + \frac{4}{10} + \frac{5}{100} + \frac{6}{1000}$. To add the fractions, we need to build a common denominator. The LCD is 1000.

$3.456 = 3 + \frac{4}{10} \cdot \frac{100}{100} + \frac{5}{100} \cdot \frac{10}{10} + \frac{6}{1000}$

$= 3 + \frac{400}{1000} + \frac{50}{1000} + \frac{6}{1000}$

$= 3 + \frac{456}{1000}$

$= 3\frac{456}{1000}$

We are not going to reduce the fraction since we are thinking about how to read the decimal.

Notice that the last digit in 3.456 is in the thousandths place. This observation suggests the following method for reading decimals.

Reading Decimals
To read a decimal:
1. Look to the left of the decimal point and say the name of the whole number.
2. The decimal is read as "and."
3. Say the fractional part of the decimal as a whole number followed by the name of the last place-value column of the digit that is the farthest to the right.

Example: Write each decimal in words and then as a fraction or mixed number. You do not have to simplify the fraction.

a. One gallon of milk weights 8.6 pounds.

Eight and six-tenths

$8\frac{6}{10}$

b. The smallest freshwater fish is the dwarf pygmy goby. Adult males weigh 0.00014 ounce.

Fourteen-hundred thousandths.

$\frac{14}{100,000}$

Example: Write each number in standard form.

a. Sixty-seven and thirty-eight hundredths.

67.38

b. Nineteen and twenty-five thousandths.

19.025

Comparing Decimals

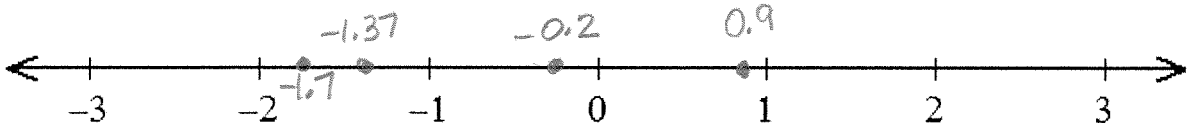
To compare two decimals:

1. Make sure both numbers have the same number of decimal places to the right of the decimal point. Write any additional zeros necessary to achieve this.
2. Compare the digits of each decimal, column by column, working from left to right.
3. *If the decimals are positive:* When two digits differ, the decimal with the greatest digit is the greater number. *If the decimals are negative:* When two digits differ, the decimal with the smaller digit is the greater number.

Example: Place a < or > symbol between the numbers to make a true statement.

a. 2.7446 > 2.7439 **b.** 39.200 < 39.203 **c.** -10.450 < -10.419

Example: Graph -1.37, 0.9, -1.7, and -0.2 on a number line.



Rounding a Decimal

1. To round a decimal to a certain decimal place value, locate the **rounding digit** in that place.
2. Look at the **test digit** directly to the right of the rounding digit.
3. If the test digit is 5 or greater, round up by adding 1 to the rounding digit and dropping the digits to its right. If the test digit is less than 5, round down by keeping the rounding digit and dropping all the digits to its right.

Example: Round 47.0676 to the nearest thousandth.

test digit

47.068

Example: Round -420.139 to the nearest tenth.

-420.1

Example: Round 56.095 to the nearest hundredth.

56.10

Section 4.2 Adding and Subtracting Decimals

To add or subtract objects, they must be similar. When we add decimals in a vertical format, we make sure the decimal points are lined up. Then we can add the numbers as we would whole numbers, writing the decimal point directly below the decimal points in the addends.

Example: Add: 36.824 + 8.5 + 79 + 12.65

$$\begin{array}{r} 36.824 \\ 8.500 \\ 79.000 \\ +\ 12.650 \\ \hline 136.974 \end{array}$$

$36.824 + 8.5 + 79 + 12.65 = 136.974$

Example: Subtract: 356.4 – 123.6

$$\begin{array}{r} 356.4 \\ -\ 123.6 \\ \hline 232.8 \end{array}$$

$356.4 - 123.6 = 232.8$

Example: Subtract 42.318 from 45.6.

$$\begin{array}{r} 45.600 \\ -\ 42.318 \\ \hline 3.282 \end{array}$$

$45.6 - 42.318 = 3.282$

Example: Add: $-3.4+(-8.3)$

$-3.4+(-8.3)=-11.7$

Scratch work

$$\begin{array}{r} 3.4 \\ +\,8.3 \\ \hline 11.7 \end{array}$$

Example: Add: $15.21+(-29.8)$

$15.21+(-29.8)=-14.59$

$$\begin{array}{r} {}^{7}\cancel{29.8}\,{}^{1}0 \\ -\,15.21 \\ \hline 14.59 \end{array}$$

Example: Subtract: $-46.8-7.6$

$-46.8-7.6=-46.8+(-7.6)$

$=-54.4$

$$\begin{array}{r} {}^{1}\,{}^{1} \\ 46.8 \\ +\,7.6 \\ \hline 54.4 \end{array}$$

Example: Subtract: $-2.98-(-27.3)$

$-2.98-(-27.3)=-2.98+27.3$

$=24.32$

$$\begin{array}{r} {}^{6}\,{}^{12} \\ 27.\cancel{3}0 \\ -\,2.98 \\ \hline 24.32 \end{array}$$

Example: Evaluate: $-33.5-(-16.3+7.9)$

$-33.5-(-16.3+7.9)=-33.5-(-8.4)$

$=-33.5+8.4$

$=-25.1$

$$\begin{array}{r} {}^{5} \\ 16.3 \\ -\,7.9 \\ \hline 8.4 \end{array}$$

$$\begin{array}{r} 33.5 \\ -\,8.4 \\ \hline 25.1 \end{array}$$

Section 4.3 Multiplying Decimals

Multiply $0.7 \cdot 0.13$ by first writing the decimals as fractions and then using the rules for multiplying factions. Write the final answer as a decimal.

$$0.7 \cdot 0.13 = \frac{7}{10} \cdot \frac{13}{100}$$

$$= \frac{91}{1000}$$

$$= 0.091$$

$$\begin{array}{r} {}^{2}13 \\ \times\ 7 \\ \hline 91 \end{array}$$

Compare the number of decimal places in the factors with the number of decimal places in the product.

0.091
3 decimal places

0.7 (1 decimal place) 0.13 (2 decimal places)

1 + 2 = 3

Multiplying Decimals

1. Multiply the decimals as if they were whole numbers.
2. Find the total number of decimal places in both factors.
3. Insert a decimal point in the result from step 1 so that the answer has the same number of decimal places as the total found in step 2.

Example: Multiply: $6.4 \cdot 3.7$

$6.4 \cdot 3.7 = 23.68$

$$\begin{array}{r} {}^{2}64 \\ \times\ 37 \\ \hline 448 \\ {}^{1}1920 \\ \hline 2368 \end{array}$$

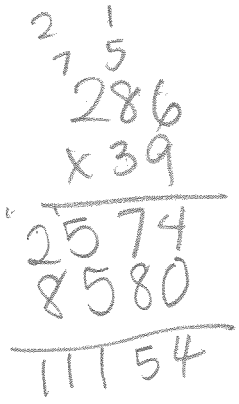
Example: Multiply: $0.004(2.3)$

$0.004(2.3) = 0.0092$

$$\begin{array}{r} {}^{1}23 \\ \times\ 4 \\ \hline 92 \end{array}$$

Example: Multiply: $286(3.9)$

$286(3.9) = 1115.4$



Consider the following.

2.345	2.345	2.345
$\times$ 10	$\times$ 100	$\times$1000
23.450	234.500	2345.000

Multiplying a Decimal by 10, 100, 1000, and so on
To find the product of a decimal and 10, 100, 1000, and so on, move the decimal point to the right the same number of places as there are zeros in the power of 10.

Example: Multiply: $3.725 \cdot 100$

$3.725 \cdot 100 = 372.5$

Example: Multiply: $46.3(100{,}000)$

$46.3(100{,}000) = 4{,}630{,}000$

Consider the following.

3.14	3.14	3.14
×0.1	×0.01	×0.001
0.314	0.0314	0.00314

Multiplying a Decimal by 0.1, 0.01, 0.001, and so on
To find the product of a decimal and 0.1, 0.01, 0.001, and so on, move the decimal point to the left the same number of decimal places as there are in the power of 10.

Example: Multiply: $542.6 \cdot 0.0001$ → 4 decimal places

$542.6 \cdot 0.0001 = 0.05426$

Example: Multiply: $0.83(0.01)$ → 2 decimal places

$0.83(0.01) = 0.0083$

Example: Evaluate: $(-0.07)^2$

$(-0.07)^2 = (-0.07)(-0.07)$

$= 0.0049$

Example: Evaluate: $(-6.3)(3)-(1.2)^2$

$(-6.3)(3)-(1.2)^2 = (-6.3)(3) - 1.44$

$= -18.9 - 1.44$

$= -18.9 + (-1.44)$

$= -20.34$

$1.2 \times 1.2 = 144$

$63 \times 3 = 189$

$18.90 + 1.44 = 20.34$

Example: Evaluate: $(-0.9)^2 + 4|-7.1+6.8|$

$(-0.9)^2 + 4|-7.1+6.8| = (-0.9)(-0.9) + 4|-0.3|$

$= 0.81 + 4(0.3)$

$= 0.81 + 1.2$

$= 2.01$

$7.1 - 6.8 = 0.3$

$1.20 + 0.81 = 2.01$