Core Connections: Courses 1–3
Extra Practice

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ADDITION AND SUBTRACTION OF FRACTIONS #1

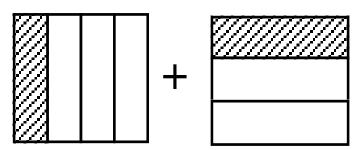
Before fractions can be added or subtracted, the fractions must have the same denominator, that is, a common denominator. There are three methods for adding or subtracting fractions.

AREA MODEL METHOD

Step 1: Copy the problem.

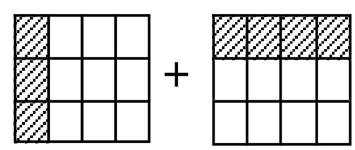
$\frac{1}{4} + \frac{1}{3}$

Step 2: Draw and divide equal-sized rectangles for each fraction. One rectangle is cut horizontally into an equal number of pieces based on the first denominator (bottom). The other is cut vertically, using the second denominator. The number of shaded pieces in each rectangle is based on the numerator (top). Label each rectangle, with the fraction it represents.



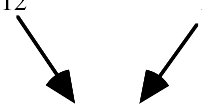
$\frac{1}{4} \quad + \quad \frac{1}{3}$

Step 3: Superimpose the lines from each rectangle onto the other rectangle, as if one rectangle is placed on top of the other one.

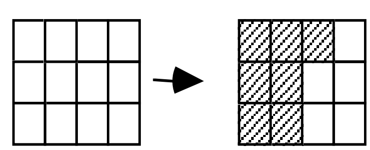


Step 4: Rename the fractions as twelfths, because the new rectangles are divided into twelve equal parts. Change the numerators to match the number of twelfths in each figure.

$\frac{3}{12} \quad + \quad \frac{4}{12}$



Step 5: Draw an empty rectangle with twelfths, then combine all twelfths by shading the same number of twelfths in the new rectangle as the total that were shaded in both rectangles from the previous step.

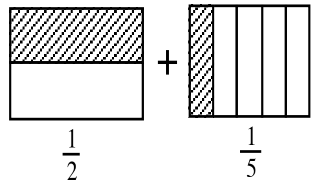


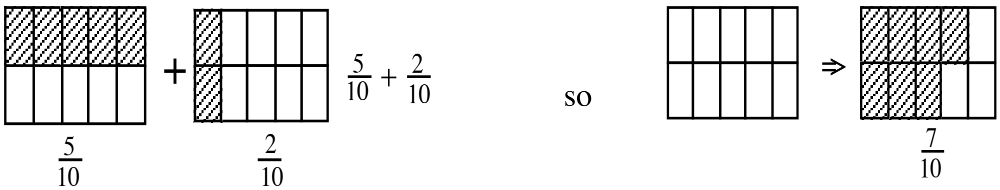
$\frac{7}{12}$

Step 6: Simplify if necessary.

Example 1

$\frac{1}{2}+\frac{1}{5}$ can be modeled as:

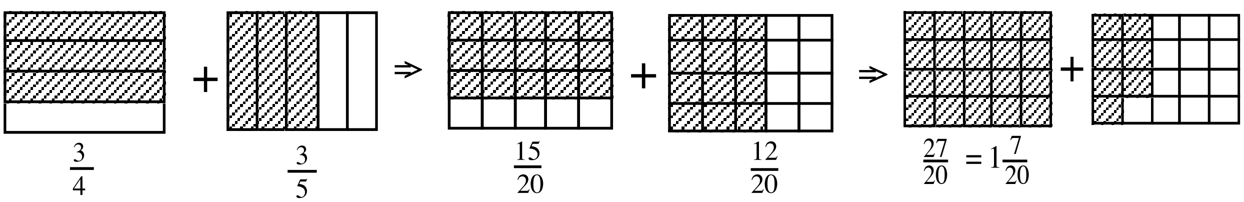




Thus, $\frac{1}{2}+\frac{1}{5}=\frac{7}{10}$.

Example 2

$\frac{3}{4}+\frac{3}{5}$ would be:



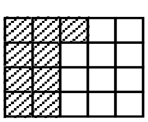
Problems

Use the area model method to add the following fractions.

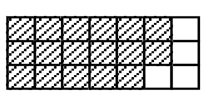
1. $\frac{1}{4}+\frac{1}{5}$
2. $\frac{2}{3}+\frac{1}{7}$
3. $\frac{1}{3}+\frac{1}{4}$

Answers

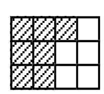
1. $\frac{5}{20}+\frac{4}{20}=\frac{9}{20}$



2. $\frac{14}{21}+\frac{3}{21}=\frac{17}{21}$



3. $\frac{4}{12}+\frac{3}{12}=\frac{7}{12}$



IDENTITY PROPERTY OF MULTIPLICATION (Giant 1) METHOD

The Giant One, known in mathematics as the Identity Property of Multiplication, uses a fraction with the same numerator and denominator ($\frac{3}{3}$, for example) to write an equivalent fraction that helps to create common denominators.

Example

Add $\frac{2}{3}+\frac{1}{4}$ using the Giant One.

Step 1: Multiply both $\frac{2}{3}$ and $\frac{1}{4}$ by Giant Ones to get a common denominator.

$$\frac{2}{3}\cdot\frac{4}{4}+\frac{1}{4}\cdot\frac{3}{3}=\frac{8}{12}+\frac{3}{12}$$

Step 2: Add the numerators of both fractions to get the answer.

$$\frac{8}{12}+\frac{3}{12}=\frac{11}{12}$$

RATIO TABLE METHOD

The **least common multiple**, that is, the smallest positive integer divisible by both (or all) of the denominators, is found by using ratio tables. The least common multiple is used as the common denominator of the fractions. The Giant One or another ratio table can be used to find the new numerators.

Example

Solve $\frac{3}{4}-\frac{1}{6}$ using a ratio table to find the least common denominator of the fractions.

Use a ratio table to find the least common denominator of the fractions. (This is the same as finding the least common multiple of the denominators, 4 and 6.)

4	8	12	16
6	12	18	24

You then use the Giant One to find the new numerator.

$$\frac{3}{4}-\frac{1}{6}\Rightarrow\frac{3}{4}\cdot\frac{3}{3}-\frac{1}{6}\cdot\frac{2}{2}\Rightarrow\frac{9}{12}-\frac{2}{12}\Rightarrow\frac{7}{12}$$

Problems

Find each sum or difference. Use the method of your choice.

1. $\frac{1}{3}+\frac{3}{5}$	2. $\frac{5}{6}+\frac{1}{3}$	3. $\frac{5}{9}-\frac{1}{3}$	4. $\frac{1}{4}+\frac{5}{7}$
5. $\frac{3}{9}+\frac{3}{4}$	6. $\frac{5}{12}+\frac{2}{3}$	7. $\frac{4}{5}-\frac{2}{3}$	8. $\frac{3}{4}-\frac{2}{5}$
9. $\frac{5}{8}+\frac{3}{8}$	10. $\frac{1}{4}+\frac{2}{3}$	11. $\frac{1}{6}+\frac{2}{3}$	12. $\frac{7}{8}+\frac{3}{4}$
13. $\frac{5}{7}-\frac{1}{3}$	14. $\frac{3}{4}-\frac{2}{3}$	15. $\frac{4}{5}+\frac{1}{4}$	16. $\frac{6}{7}-\frac{3}{4}$
17. $\frac{2}{3}-\frac{3}{4}$	18. $\frac{3}{5}-\frac{9}{15}$	19. $\frac{4}{5}-\frac{2}{3}$	20. $\frac{4}{6}-\frac{11}{23}$

Answers

1. $\frac{14}{15}$	2. $\frac{7}{6}=1\frac{1}{6}$	3. $\frac{2}{9}$	4. $\frac{27}{28}$	5. $\frac{39}{36}=1\frac{3}{36}=1\frac{1}{12}$
6. $\frac{13}{12}=1\frac{1}{12}$	7. $\frac{2}{15}$	8. $\frac{7}{20}$	9. $\frac{49}{40}=1\frac{9}{40}$	10. $\frac{11}{12}$
11. $\frac{5}{6}$	12. $\frac{13}{8}=1\frac{5}{8}$	13. $\frac{8}{21}$	14. $\frac{1}{12}$	15. $\frac{21}{20}=1\frac{1}{20}$
16. $\frac{3}{28}$	17. $-\frac{1}{12}$	18. 0	19. $\frac{2}{15}$	20. $-\frac{3}{12}=-\frac{1}{4}$

To summarize addition and subtraction of fractions:

1. Rename each fraction with equivalents that have a common denominator.
2. Add or subtract only the numerators, keeping the common denominator.
3. Simplify if possible.

SUBTRACTING MIXED NUMBERS

To subtract mixed numbers, change the mixed numbers into fractions greater than one, find a common denominator, then subtract.

Example

Find the difference: $2\frac{1}{5} - 1\frac{4}{3}$.

$$\begin{array}{rl} 2\frac{1}{5} & = \frac{11}{5} \cdot \frac{3}{3} = \frac{33}{15} \\ -1\frac{2}{3} & \quad \frac{5}{3} \cdot \frac{5}{5} = \frac{25}{15} \\ \hline & \qquad\qquad \frac{8}{15} \end{array}$$

Problems

Find each difference.

1. $2\frac{1}{8} - 1\frac{3}{4}$
2. $4\frac{1}{3} - 2\frac{3}{6}$
3. $1\frac{1}{6} - \frac{3}{5}$
4. $4\frac{3}{4} - 2\frac{4}{5}$
5. $6 - 1\frac{2}{5}$
6. $4\frac{1}{8} - 1\frac{2}{3}$

Answers

1. $\frac{17}{8} - \frac{7}{4} \Rightarrow \frac{17}{8} - \frac{14}{8} \Rightarrow \frac{3}{8}$
2. $\frac{13}{3} - \frac{15}{6} \Rightarrow -\frac{15}{6} \Rightarrow \frac{11}{6}$ or $1\frac{5}{6}$
3. $\frac{7}{6} - \frac{3}{5} \Rightarrow \frac{35}{30} - \frac{18}{30} \Rightarrow \frac{17}{30}$
4. $\frac{19}{4} - \frac{14}{5} \Rightarrow \frac{95}{20} - \frac{56}{20} \Rightarrow \frac{39}{20}$ or $\frac{19}{20}$
5. $\frac{6}{1} - \frac{7}{5} \Rightarrow \frac{30}{5} - \frac{7}{5} \Rightarrow \frac{23}{5}$ or $4\frac{3}{5}$
6. $\frac{33}{8} - \frac{5}{3} \Rightarrow \frac{99}{24} - \frac{40}{24} \Rightarrow \frac{59}{24}$ or $2\frac{11}{24}$

AREA OF A CIRCLE #2

The **AREA** of a circle is the measure of the region inside it. To find the area of a circle when given the **radius**, use this formula:

$$\boldsymbol{A = r \cdot r \cdot \pi = r^2 \cdot \pi}$$

The radius of a circle needs to be identified in order to find the area of the circle. The radius is half the diameter. Next square the radius and multiply the result by π.

Example 1

Find the area of a circle with $r = 17$ feet.

$$\begin{aligned} A &= r^2 \cdot \pi \\ &\approx (17 \cdot 17)(3.14) \\ &\approx 907.46 \text{ ft}^2 \end{aligned}$$

Example 2

Find the area of a circle with $d = 84$ cm.

$$\begin{aligned} \text{radius} &= \text{diameter} \div 2 \\ &= 84 \div 2 \\ &= 42 \text{ cm} \end{aligned}$$

$$\begin{aligned} A &= r^2 \cdot \pi \\ &\approx (42 \cdot 42) \cdot 3.14 \\ &\approx 5538.96 \text{ cm}^2 \end{aligned}$$

Example 3

Find the radius of a circle with area 78.5 square meters.

$$78.5 = r^2 \cdot \pi$$
$$78.5 = 3.14r^2$$
$$r^2 = \tfrac{78.5}{3.14} \approx 24.89$$
$$r \approx \sqrt{24.89}$$
$$r \approx 5 \text{ meters}$$

Example 4

Find the radius of a circle with area 50.24 square centimeters.

$$50.24 = r^2 \cdot \pi$$
$$50.24 = 3.14r^2$$
$$r^2 = \tfrac{50.24}{3.14} = 16$$
$$r = \sqrt{16}$$
$$r = 4 \text{ centimeters}$$

Problems

Find the area of the circles with the following radius or diameter lengths. Use $\pi = 3.14$. Round your answers to the nearest hundredth.

1. $r = 9$ cm
2. $r = 5.2$ in.
3. $d = 20$ ft
4. $d = \frac{3}{5}$ cm
5. $r = \frac{1}{4}$ m
6. $r = 7.2$ in.
7. $r = 4.6$ cm
8. $r = 6\frac{1}{4}$ in.
9. $d = 26.4$ ft
10. $r = 13.7$ m

Find the radius of each circle given the following areas. Use $\pi = 3.14$. Round your answers to the nearest tenth.

11. $A = 314$ m^2
12. $A = 55.39$ cm^2
13. $A \approx 140.95$ ft^2
14. $A \approx 262.31$ in.^{2}
15. $A = 660.19$ km^2

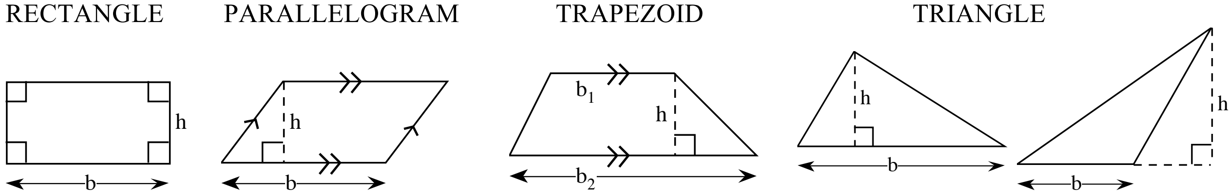
Answers

1. 254.34 cm^2
2. 84.91 in.^{2}
3. 314 ft^2
4. 0.28 cm^2
5. 0.20 m^2
6. 162.78 in.^{2}
7. 66.44 cm^2
8. 122.66 in.^{2}
9. 547.11 ft^2
10. 589.35 m^2
11. $r = 10$ m
12. $r = 4.2$ cm
13. $r \approx 6.7$ ft
14. $r = 9.1$ in.
15. $r = 14.5$ km

AREA OF POLYGONS #3

AREA is the number of square units in a flat region. The formulas to calculate the area of several kinds of polygons are:

RECTANGLE PARALLELOGRAM TRAPEZOID TRIANGLE



$A = bh$ $A = bh$ $A = \frac{1}{2}(b_1 + b_2)h$ $A = \frac{1}{2}bh$

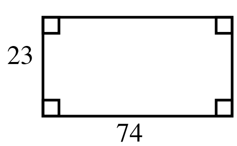
Note that the legs of any right triangle form a base and a height for the triangle (see Example 1, part (c)).

The area of a more complicated figure may be found by breaking it into smaller regions of the types shown above, calculating each area, and finding the sum or difference of the areas.

Example 1

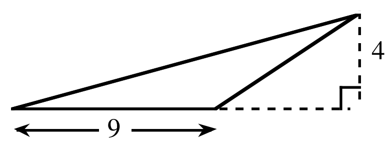
Find the area of each figure. All lengths are centimeters.

a.



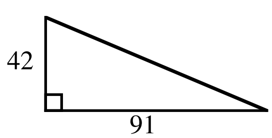
$A = bh = (74)(23) = 1702 \text{ cm}^2$

b.



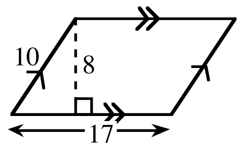
$A = \frac{1}{2}bh = \frac{1}{2}(9)(4) = 18 \text{ cm}^2$

c.



$A = \frac{1}{2}bh = \frac{1}{2}(91)(42) = 1911 \text{ cm}^2$

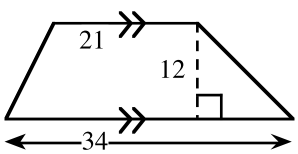
d.



$A = bh = (17)8 = 136 \text{ cm}^2$

Note that 10 is a <u>side</u> of the parallelogram, <u>not</u> the height.

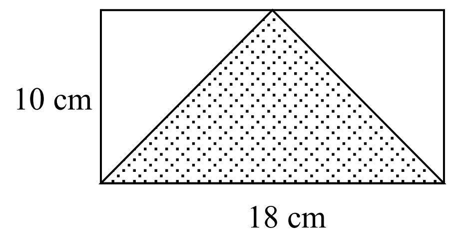
e.



$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(21 + 34)12 = \frac{1}{2}(55)(12) = 330 \text{ cm}^2$

Example 2

Find the area of the shaded region.



The area of the shaded region is the area of the triangle minus the area of the rectangle.

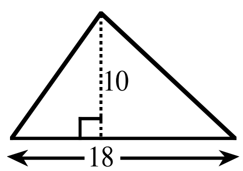
large triangle: $A = \frac{1}{2}(18)(10) = 90 \text{ cm}^2$

rectangle: $A = 9(4) = 36 \text{ cm}^2$

shaded region: $A = 90 - 36 = 54 \text{ cm}^2$

Problems

Find the area of the following triangles, parallelograms and trapezoids. Pictures are not drawn to scale. Round answers to the nearest tenth.

1.


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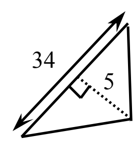

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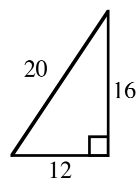

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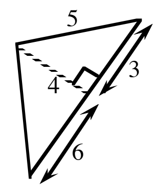

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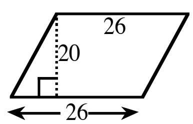

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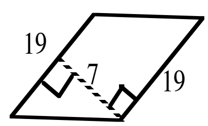

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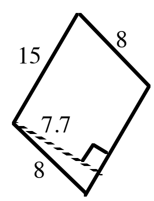

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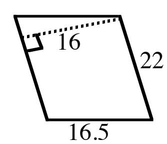

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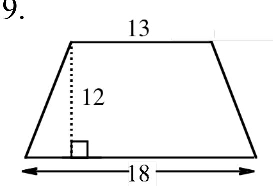

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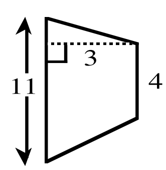

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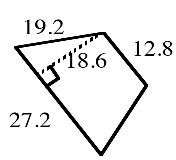

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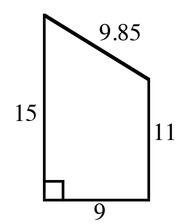

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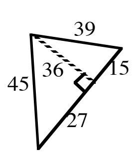

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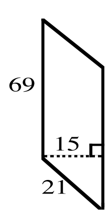

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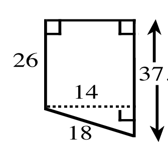

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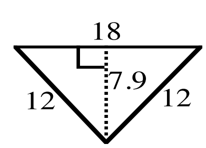
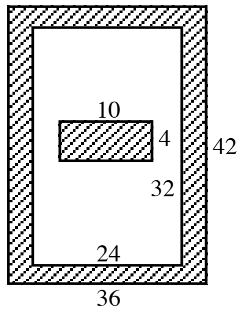


Find the area of each shaded region.

17.



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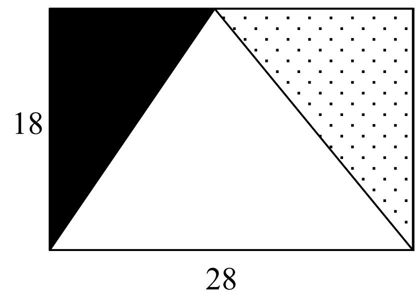

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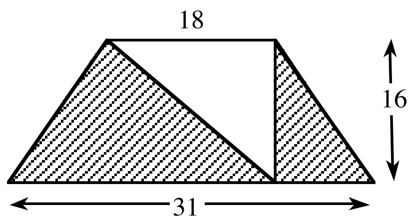

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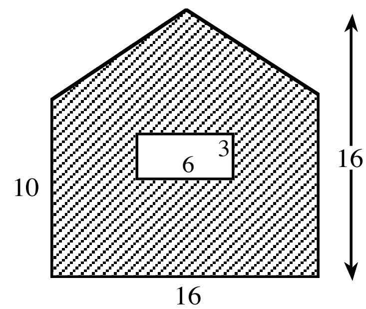
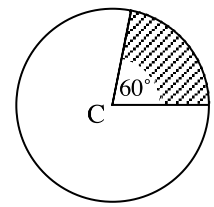


Answers (in square units)

1. 90	2. 85	3. 96	4. 18	5. 520	6. 133
7. 115.5	8. 352	9. 186	10. 22.5	11. 372	12. 117
13. 756	14. 1035	15. 443.1	16. 71.1	17. 784	18. 252
19. 248	20. 190				

AREA OF SECTORS (SUBPROBLEMS WITH CIRCLES) #4

A **SECTOR** of a circle is formed by the two radii of a central angle and the arc between their endpoints on the circle. When we find the area of a sector, we are finding a fractional part of the circle.



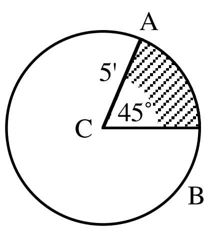
A_{sector} = (fractional part)(area of circle)

For example, a 60˚ sector looks like a slice of pizza. 60º is the part of the circle. The whole circle is 360º.

$\frac{\text{part}}{\text{whole}} = \frac{60}{360} = \frac{1}{6}$ The sector is $\frac{1}{6}$ of the circle, so $A_{\text{sector}} = \frac{1}{6}(\text{Area Circle})$.

Example 1

Find the area of the 45º sector.



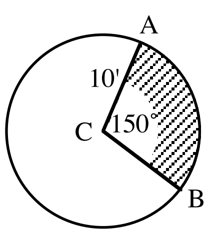
Fractional part of circle: $\frac{\text{part}}{\text{whole}} = \frac{45^\circ}{360^\circ} = \frac{1}{8}$

Area of the circle: $A = 5^2 \cdot \pi = 25\pi \approx 78.5 \text{ ft}^2$

Area of the sector: $\frac{1}{8} \cdot 78.5 \approx 9.81 \text{ ft}^2$

Example 2

Find the area of the 150º sector.



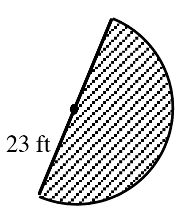
Fractional part of circle: $\frac{\text{part}}{\text{whole}} = \frac{150^\circ}{360^\circ} = \frac{5}{12}$

Area of circle is $A = r^2 \cdot \pi = 10^2 \cdot \pi = 100\pi \approx 314 \text{ ft}^2$

Area of the sector: $\frac{5}{12} \cdot 314 \approx 130.83 \text{ ft}^2$

Example 3

Find the area of the semicircle.



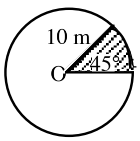
Fractional part of circle: $\frac{\text{part}}{\text{whole}} = \frac{180^\circ}{360^\circ} = \frac{1}{2}$

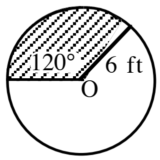
Area of circle: $A = r^2 \cdot \pi = 23^2 \cdot \pi = 529\pi \approx 1661.06 \text{ ft}^2$

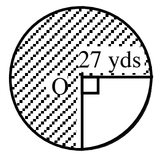
Area of the sector: $\frac{1}{2} \cdot 1661.06 \approx 830.53 \text{ ft}^2$

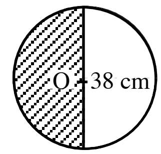
Problems

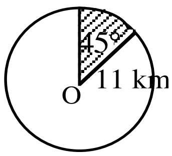
Calculate the area of the following shaded sectors. Point O is the center of each circle. Use $\pi = 3.14$ in all problems.

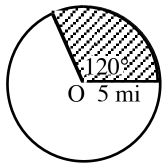
1. 

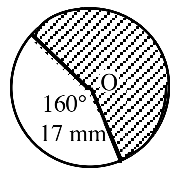
2. 

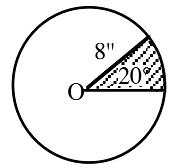
3. 

4. 

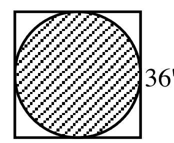
5. 

6. 

7. 

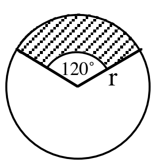
8. 

9. Find the area of a circular garden if the diameter of the garden is 60 feet.

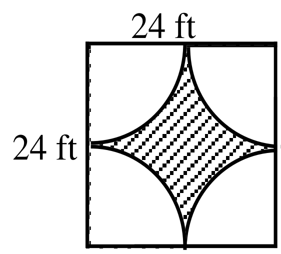


10. Find the area of a circle inscribed in a square whose edge is 36 feet long.

11. Find the radius. The shaded area is 157.84 cm^2.



12. Find the area of the shaded region.



Answers

1. 39.25 m^2	2. 37.68 ft^2	3. 1716.80 yd^2	4. 566.77 cm^2
5. 47.49 km^2	6. 26.17 mi^2	7. 806.63 mm^2	8. 11.16 $in.^2$
9. 2826 ft^2	10. 1017.36 ft^2	11. 12.28 cm	12. 123.84 ft^2

ARITHMETIC OPERATIONS WITH DECIMALS #5

ADDING AND SUBTRACTING DECIMALS: Write the problem in column form with the decimal points in a vertical column. Write in zeros so that all decimal parts of the number have the same number of digits. Add or subtract as with whole numbers. Place the decimal point in the answer aligned with those above.

MULTIPLYING DECIMALS: Multiply as with whole numbers. In the product, the number of decimal places is equal to the total number of decimal places in the factors (numbers you multiplied). Sometimes zeros need to be added to place the decimal point.

DIVIDING DECIMALS: When dividing a decimal by a whole number, place the decimal point in the answer space directly above the decimal point in the number being divided. Divide as with whole numbers. Sometimes it is necessary to add zeros to the number being divided to complete the division.

When dividing decimals or whole numbers by a decimal, the divisor must be multiplied by a power of ten to make it a whole number. The dividend must be multiplied by the same power of ten. Then divide following the same rules for division by a whole number.

Example 1

Add 47.37, 28.9, 14.56, and 7.8.

$$\begin{array}{r} 47.37 \\ 28.9\mathbf{0} \\ 14.56 \\ +\ 7.8\mathbf{0} \\ \hline 98.63 \end{array}$$

Example 2

Subtract 198.76 from 473.2.

$$\begin{array}{r} 473.2\mathbf{0} \\ -\ 198.76 \\ \hline 274.44 \end{array}$$

Example 3

Multiply 27.32 by 14.53.

$$\begin{array}{rl} 27.32 & \text{(2 decimal places)} \\ \times\ 14.53 & \text{(2 decimal places)} \\ \hline 8196 & \\ 13660 & \\ 10928 & \\ 2732 & \\ \hline 396.9596 & \text{(4 decimal places)} \end{array}$$

Example 4

Multiply 0.37 by 0.00004.

$$\begin{array}{rl} 0.37 & \text{(2 decimal places)} \\ \times\ 0.0004 & \text{(4 decimal places)} \\ \hline \mathbf{0.000}148 & \text{(6 decimal places)} \end{array}$$

Example 5

Divide 32.4 by 8.

$$\begin{array}{r} 4.05 \\ 8\overline{)\,32.4\mathbf{0}} \\ 32 \\ \hline 0\ 40 \\ 40 \\ \hline 0 \end{array}$$

Example 6

Divide 27.42 by 1.2. First multiply each number by 10^1 or 10.

$1.2\overline{)27.42} \Rightarrow 12\overline{)174.2}$

$$\begin{array}{r} 22.85 \\ 12\overline{)274.20} \\ 24 \\ \hline 34 \\ 24 \\ \hline 10\,2 \\ 9\,6 \\ \hline 60 \\ 60 \\ \hline 0 \end{array}$$

Problems

1. $4.7 + 7.9$
2. $3.93 + 2.82$
3. $38.72 + 6.7$
4. $58.3 + 72.84$
5. $4.73 + 692$
6. $428 + 7.392$
7. $42.1083 + 14.73$
8. $9.87 + 87.47936$
9. $9.999 + 0.001$
10. $0.0001 + 99.9999$
11. $0.0137 + 1.78$
12. $2.037 + 0.09387$
13. $15.3 + 72.894$
14. $47.9 + 68.073$
15. $289.307 + 15.938$
16. $476.384 + 27.847$
17. $15.38 + 27.4 + 9.076$
18. $48.32 + 284.3 + 4.638$
19. $278.63 + 47.0432 + 21.6$
20. $347.68 + 28.00476 + 84.3$
21. $8.73 – 4.6$
22. $9.38 – 7.5$
23. $8.312 – 6.98$
24. $7.045 – 3.76$
25. $6.304 – 3.68$
26. $8.021 – 4.37$
27. $14 – 7.431$
28. $23 – 15.37$
29. $10 – 4.652$
30. $18 – 9.043$
31. $0.832 – 0.47$
32. $0.647 – 0.39$
33. $1.34 – 0.0538$
34. $2.07 – 0.523$
35. $4.2 – 1.764$
36. $3.8 – 2.406$
37. $38.42 – 32.605$
38. $47.13 – 42.703$
39. $15.368 + 14.4 – 18.5376$
40. $87.43 – 15.687 – 28.0363$
41. $7.34 \cdot 6.4$
42. $3.71 \cdot 4.03$
43. $0.08 \cdot 4.7$
44. $0.04 \cdot 3.75$
45. $41.6 \cdot 0.302$
46. $9.4 \cdot 0.0053$
47. $3.07 \cdot 5.4$
48. $4.023 \cdot 3.02$
49. $0.004 \cdot 0.005$
50. $0.007 \cdot 0.0004$
51. $0.235 \cdot 0.43$
52. $4.32 \cdot 0.0072$
53. $0.0006 \cdot 0.00013$
54. $0.0005 \cdot 0.00026$
55. $8.38 \cdot 0.0001$
56. $47.63 \cdot 0.000001$
57. $0.078 \cdot 3.1$
58. $0.043 \cdot 4.2$
59. $350 \cdot 0.004$
60. $421 \cdot 0.00005$

Divide. Round answers to the hundredth, if necessary.

61. 14.3 ÷ 8	62. 18.32 ÷ 5	63. 147.3 ÷ 6
64. 46.36 ÷ 12	65. 100.32 ÷ 24	66. 132.7 ÷ 28
67. 47.3 ÷ 0.002	68. 53.6 ÷ 0.004	69. 500 ÷ 0.004
70. 420 ÷ 0.05	71. 1.32 ÷ 0.032	72. 3.486 ÷ 0.012
73. 46.3 ÷ 0.011	74. 53.7 ÷ 0.023	75. 25.46 ÷ 5.05
76. 26.35 ÷ 2.2	77. 6.042 ÷ 0.006	78. 7.035 ÷ 0.005
79. 207.3 ÷ 4.4	80. 306.4 ÷ 3.2	

Answers

1. 12.6	2. 6.75	3. 45.42	4. 131.14	5. 696.73
6. 435.392	7. 56.8383	8. 97.34936	9. 10.000	10. 100.0000
11. 1.7937	12. 2.13087	13. 88.194	14. 115.973	15. 305.245
16. 504.231	17. 51.856	18. 337.258	19. 347.2732	20. 459.98476
21. 4.13	22. 1.88	23. 1.332	24. 3.285	25. 2.624
26. 3.651	27. 6.569	28. 7.63	29. 5.348	30. 8.957
31. 0.362	32. 0.257	33. 1.2862	34. 1.547	35. 2.436
36. 1.394	37. 5.815	38. 4.427	39. 11.2304	40. 43.7067
41. 46.976	42. 14.9513	43. 0.376	44. 0.15	45. 12.5632
46. 0.04982	47. 16.578	48. 12.14946	49. 0.000020	50. 0.0000028
51. 0.10105	52. 0.031104	53. 0.000000078	54. 0.000000130	55. 0.000838
56. 0.0004763	57. 0.2418	58. 0.1806	59. 1.4	60. 0.02105
61. 1.7875 or 1.79	62. 3.664 or 3.66	63. 24.55	64. $3.86\overline{3}$ or 3.86	65. 4.18
66. 4.74	67. 23,650	68. 13,400	69. 125,000	70. 8400
71. 41.25	72. 29.05	73. 4209.09	74. 2334.78	75. 5.04
76. 11.98	77. 1007	78. 1407	79. 47.11	80. 95.75

ARITHMETIC OPERATIONS WITH INTEGERS #6

ADDITION OF INTEGERS

Add numbers two at a time. If the signs are the same, add the numbers and keep the same sign. If the signs are different, ignore the signs (that is, use the absolute value of each number) and find the difference of the two numbers. The sign of the answer is determined by the number farthest from zero, that is, the number with the greater absolute value.

Follow the same rules for fractions and decimals.

Remember to apply the correct order of operations when you are working with more than one operation.

Example 1: Same signs

a. $2 + 3 = 5$ or $3 + 2 = 5$

b. $-2 + (-3) = -5$ or $-3 + (-2) = -5$

Example 2: Different signs

a. $-2 + 3 = 1$ or $3 + (-2) = 1$

b. $-3 + 2 = -1$ or $2 + (-3) = -1$

Problems: Addition

Simplify the following expression using the rules above without using a calculator.

1. $5 + (-2)$	2. $4 + (-1)$	3. $9 + (-7)$
4. $-10 + 5$	5. $-9 + 2$	6. $-12 + 8$
7. $-3 + (-7)$	8. $-12 + (-4)$	9. $-13 + (-16)$
10. $-7 + (-14)$	11. $-7 + 13$	12. $-24 + 11$
13. $-4 + 3 + 6$	14. $8 + (-10) + (-5)$	15. $5 + (-4) + (-2) + (-9)$
16. $-9 + (-3) + (-2) + 11$	17. $10 + (-7) + (-6) + 5 + (-8)$	18. $14 + (-13) + 18 + (-22)$
19. $55 + (-65) + 30$	20. $19 + (-16) + (-5) + 15$	

Answers

1. 3	2. 3	3. 2	4. –5	5. –7
6. –4	7. –10	8. –16	9. –29	10. –21
11. 6	12. –13	13. 5	14. –7	15. –10
16. –3	17. –6	18. –3	19. 20	20. 13

SUBTRACTION OF INTEGERS

To find the difference of two values, change the subtraction sign to addition, change the sign of the number being subtracted, then follow the rules for addition.

Follow the same rules for fractions and decimals.

Remember to apply the correct order of operations when you are working with more than one operation.

Example 1

a. $2 - 3 \Rightarrow 2 + (-3) = -1$

b. $-2 - (-3) \Rightarrow -2 + (+3) = 1$

Example 2

a. $-2 - 3 \Rightarrow -2 + (-3) = -5$

b. $2 - (-3) \Rightarrow 2 + (+3) = 5$

Problems: Subtraction

Use the rule stated above to find each difference.

1. $8 - (-3)$	2. $8 - 3$	3. $-8 - 3$
4. $-8 - (-3)$	5. $-38 - 62$	6. $-38 - (-62)$
7. $38 - 62$	8. $38 - (-62)$	9. $-5 - (-3) - 4 - 7$
10. $5 - (-8) - 3 - (-7)$	11. $5 - (-8)$	12. $18 - 25$
13. $-7 - 5$	14. $-26 - 7$	15. $-3 - (-7)$
16. $10 - (-5)$	17. $-58 - 24$	18. $-62 - 73$
19. $-74 - (-47)$	20. $-37 - (-55)$	

Answers

1. 11	2. 5	3. –11	4. –5	5. –100
6. 24	7. –24	8. 100	9. –13	10. 17
11. 13	12. –7	13. –12	14. –33	15. 4
16. 15	17. –82	18. –135	19. –27	20. 18

MULTIPLICATION AND DIVISION OF INTEGERS

Multiply and divide integers two at a time. If the signs are the same, their product will be positive. If the signs are different, their product will be negative.

Follow the same rules for fractions and decimals.

Remember to apply the correct order of operations when you are working with more than one operation.

Examples

a. $2 \cdot 3 = 6$ or $3 \cdot 2 = 6$

b. $-2 \cdot (-3) = 6$ or $(+2) \cdot (+3) = 6$

c. $2 \div 3 = \frac{2}{3}$ or $3 \div 2 = \frac{3}{2}$

d. $(-2) \div (-3) = \frac{2}{3}$ or $(-3) \div (-2) = \frac{3}{2}$

e. $(-2) \cdot 3 = -6$ or $3 \cdot (-2) = -6$

f. $(-2) \div 3 = -\frac{2}{3}$ or $3 \div (-2) = -\frac{3}{2}$

g. $9 \cdot (-7) = -63$ or $-7 \cdot 9 = -63$

h. $-63 \div 9 = -7$ or $9 \div (-63) = -\frac{1}{7}$

Problems: Multiplication and Division

Use the rules above to find each product or quotient.

1. $(-4)(2)$
2. $(-3)(4)$
3. $(-12)(5)$
4. $(-21)(8)$
5. $(4)(-9)$
6. $(13)(-8)$
7. $(45)(-3)$
8. $(105)(-7)$
9. $(-7)(-6)$
10. $(-7)(-9)$
11. $(-22)(-8)$
12. $(-127)(-4)$
13. $(-8)(-4)(2)$
14. $(-3)(-3)(-3)$
15. $(-5)(-2)(8)(4)$
16. $(-5)(-4)(-6)(-3)$
17. $(-2)(-5)(4)(8)$
18. $(-2)(-5)(-4)(-8)$
19. $(-2)(-5)(4)(-8)$
20. $2(-5)(4)(-8)$
21. $10 \div (-5)$
22. $18 \div (-3)$
23. $96 \div (-3)$
24. $282 \div (-6)$
25. $-18 \div 6$
26. $-48 \div 4$
27. $-121 \div 11$
28. $-85 \div 85$
29. $-76 \div (-4)$
30. $-175 \div (-25)$
31. $-108 \div (-12)$
32. $-161 \div 23$
33. $-223 \div (-223)$
34. $354 \div (-6)$
35. $-1992 \div (-24)$
36. $-1819 \div (-17)$
37. $-1624 \div 29$
38. $1007 \div (-53)$
39. $994 \div (-14)$
40. $-2241 \div 27$

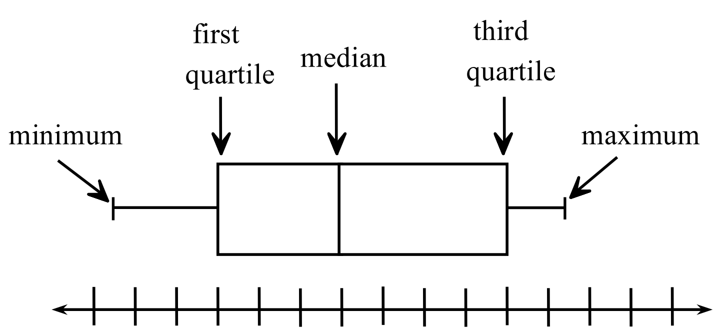
Answers

1.	–8	2.	–12	3.	–60	4.	–168	5.	–36
6.	–104	7.	–135	8.	–735	9.	42	10.	63
11.	176	12.	508	13.	64	14.	–27	15.	320
16.	360	17.	320	18.	320	19.	–320	20.	320
21.	–2	22.	–6	23.	–32	24.	–47	25.	–3
26.	–12	27.	–11	28.	–1	29.	19	30.	7
31.	9	32.	7	33.	1	34.	–59	35.	83
36.	107	37.	–56	38.	–19	39.	–71	40.	–83

BOX PLOTS #7

A way to display data that shows how the data is grouped or clustered is a **box plot**. The box plot displays the data using quartiles.

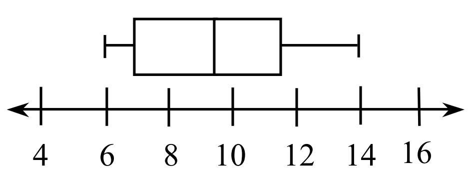
- Put the data set in order, lowest to highest.
- Create a number line that is slightly greater than the range of the data.
- Find the median of the data set. Place a vertical line segment (about two centimeters long) above the median.
- Find the median of the lower half of the data, that is, the numbers to the left of the median. Place a vertical line segment above this number. This number marks the **first quartile** of the data.
- Find the median of the upper half of the data, that is, the numbers to the right of the median. Place a vertical line segment above this number. This number marks the **third quartile** of the data.
- Draw a box between the first quartile and the third quartile, using the line segments you drew above each quartile as the vertical sides. The line segment for the median will be inside the box.
- Place two short vertical segments above the number line, one that labels the minimum (the smallest number) and another that labels the maximum (the largest number) values of the data set.
- Draw a horizontal line segment from the first quartile to the short segment representing the minimum value and a horizontal line segment from the third quartile to the short segment representing the maximum value.



Example 1

Display this data in a box plot:
6, 8, 10, 9, 7, 7, 11, 12, 6, 12, 14, and 10.

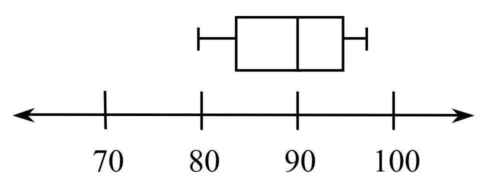
- Place the data in order from least to greatest: 6, 6, 7, 7, 8, 9, 10, 10, 11, 12, 12, and 14. The range is 14 – 6 = 8. Thus you start with a number line with equal intervals from 4 to 16.
- The median of the set of data is 9.5. Draw a vertical line segment at this value above the number line.
- The median of the lower half of the data (the first quartile) is 7. Draw a vertical line segment at this value above the number line.
- The median of the upper half of the data (the third quartile) is 11.5. Draw a vertical line segment at this value above the number line.
- Draw a box between the first and third quartiles.
- Place a short vertical segment at the minimum value (6) and another at the maximum value (14).



Example 2

Display this data in a box plot:
80, 90, 85, 83, 83, 92, 97, 91, and 95.

- Place the data in order from least to greatest: 80, 83, 83, 85, 90, 91, 92, 96, and 97. The range is 97 – 80 = 17. Thus you want a number line with equal intervals from 70 to 100.
- Find the median of the set of data: 90 Draw the vertical line segment.
- Find the first quartile: 83. Draw the vertical line segment.
- Find the third quartile: 92 + 96 = 188; 188 ÷ 2 = 94. Draw the vertical line segment.
- Draw the box connecting the first and lthird quartiles. Place a short vertical segment at the minimum value (80) and another at the maximum value (97). Draw the segments to the highest and lowest values.



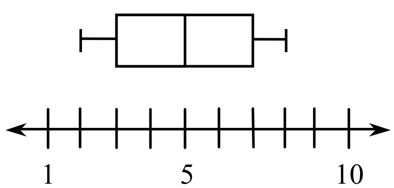
Problems

Make a box plot for each set of data.

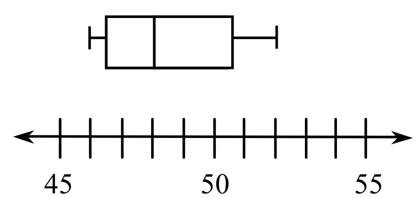
1. 5, 8, 3, 2, 7, 3, 7, 4, and 6.
2. 47, 52, 50, 47, 51, 46, 49, 46, and 48.
3. 20, 35, 16, 19, 25, 32, 17, 38, 16, and 36.
4. 70, 63, 62, 74, 67, 62, 70, 72, 60, and 61.
5. 72, 63, 70, 42, 50, 53, 65, 38, and 39.
6. 76, 90, 75, 72, 93, 82, 70, 85, and 80.

Answers

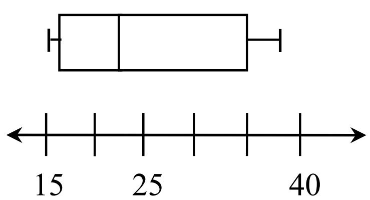
1.



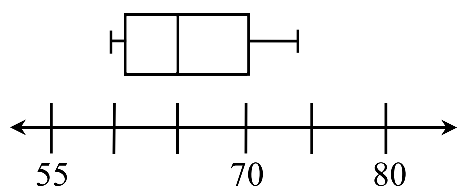
2.



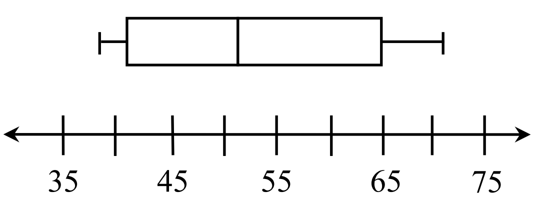
3.



4.



5.



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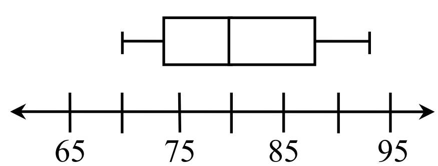
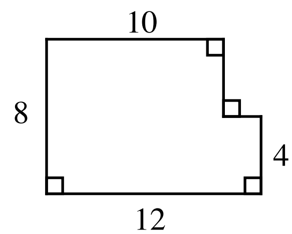


CALCULATING COMPOUND AREAS USING SUBPROBLEMS #8

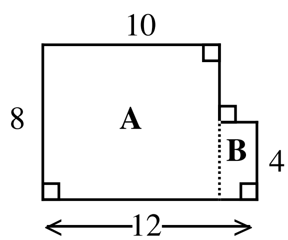
Every polygon can be dissected (or broken up) into rectangles and triangles that have no interior points in common. This is an example of the problem solving strategy of **SUBPROBLEMS**. Finding simpler problems that you know how to solve will help you solve the larger problem.

Example 1

Find the area of the figure at right.



Method #1



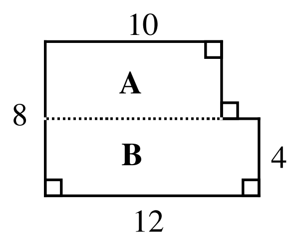
Subproblems:

1. Find the area of rectangle A: $8 \cdot 10 = 80$ square units.

2. Find the area of rectangle B: $4 \cdot (12 - 10) = 4 \cdot 2 =$ 8 square units.

3. Add the area of rectangle A to the area of rectangle B: $80 + 8 = 88$ square units.

Method #2



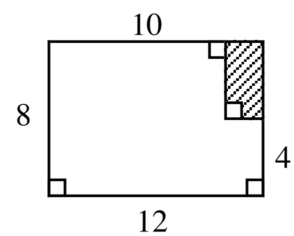
Subproblems:

1. Find the area of rectangle A: $10 \cdot (8 - 4) = 10 \cdot 4$ $= 40$ square units.

2. Find the area of rectangle B: $12 \cdot 4 = 48$ square units.

3. Add the area of rectangle A to the area of rectangle B: $40 + 48 = 88$ square units.

Method #3



Subproblems:

1. Make a large rectangle by enclosing the upper right corner.

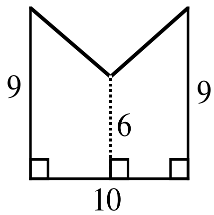
2. Find the area of the new, larger rectangle: $8 \cdot 12 = 96$ square units.

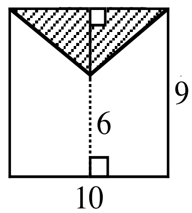
3. Find the area of the shaded rectangle: $(8 - 4) \cdot (12 - 10)$ $= 4 \cdot 2 = 8$ square units.

4. Subtract the shaded rectangle from the larger rectangle: $96 - 8 = 88$ square units.

Example 2

Find the area of the figure below.





Subproblems:

1. Make a rectangle out of the figure by enclosing the top.

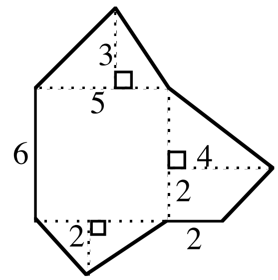
2. Find the area of the entire rectangle: $10 \cdot 9 = 90$ square units

3. Find the area of the shaded triangle. Use the formula $A = \frac{1}{2}bh$.
   $b = 10$ and $h = 9 - 6 = 3$,
   so $A = \frac{1}{2}(10 \cdot 3) = \frac{30}{2}$
   $= 15$ square units

4. Subtract the area of the triangle from the area of the rectangle:
   $90 - 15 = 75$ square units

Example 3

Find the area of the figure below.



This figure consists of five familiar figures: a central rectangle, 5 units by 6 units; three triangles, one on top with b = 5 and h = 3, one on the right with b = 4 and h = 4, and one on the bottom with b = 5 and h = 2; and a trapezoid with an upper base of 4, a lower base of 2 and a height of 2.

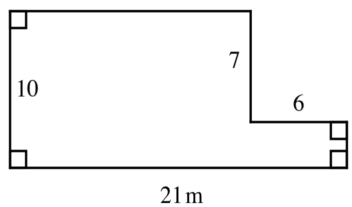
The area is:

rectangle	$5 \cdot 6$	$= 30$
top triangle	$\frac{1}{2} \cdot 5 \cdot 3$	$= 7.5$
bottom triangle	$\frac{1}{2} \cdot 5 \cdot 2$	$= 5$
right side triangle	$\frac{1}{2} \cdot 4 \cdot 4$	$= 8$
trapezoid	$\frac{(4+2) \cdot 2}{2}$	$= 6$
total area		$= 56.5$ un^2

Problems

Find the area of each of the following figures. Assume that anything that looks like a right angle is a right angle.

1.



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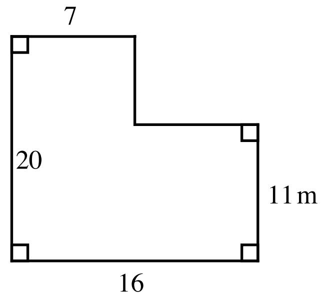

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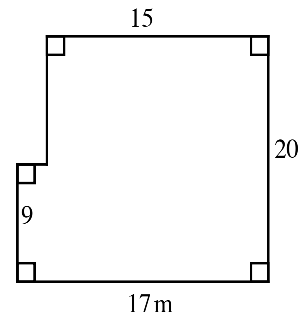

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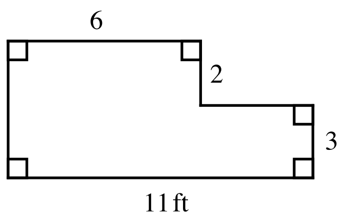

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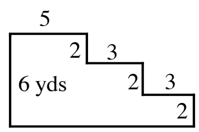

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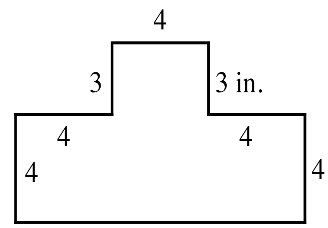

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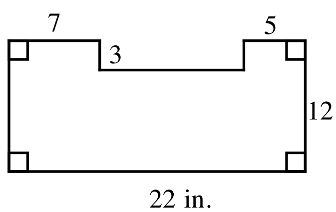

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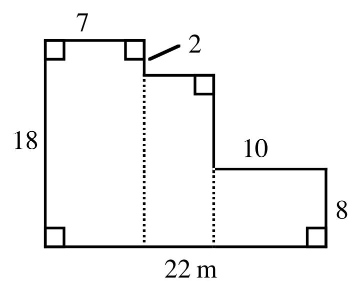

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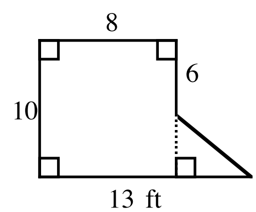

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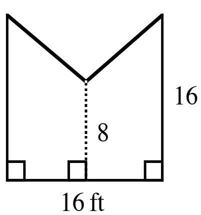
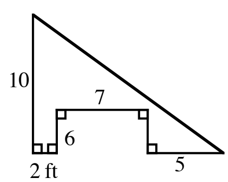


11.



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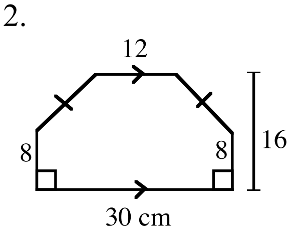

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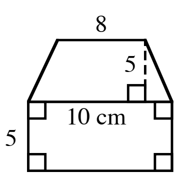

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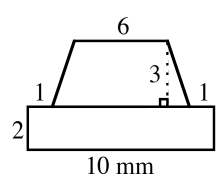

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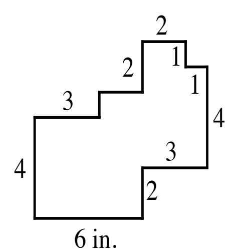

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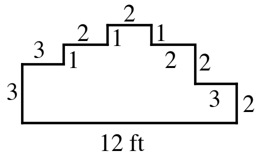

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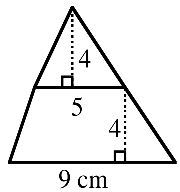

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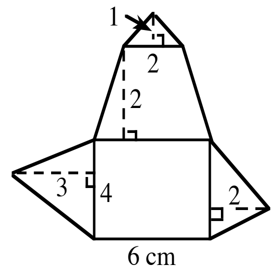

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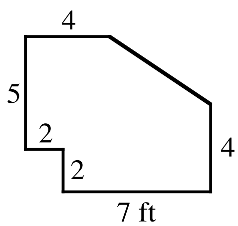

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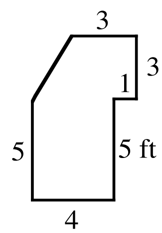
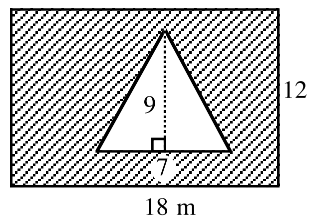
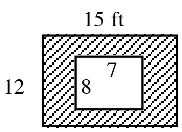


21. Find the area of the shaded region.

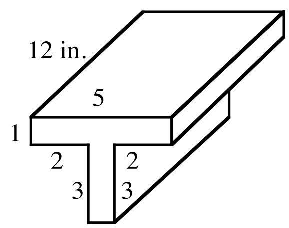


22. Find the area of the shaded region.



23 and 24: find the surface area of each polyhedron.

23.



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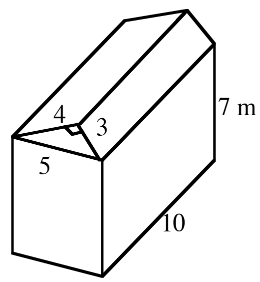


Answers

1. 168 m^2	2. 239 m^2	3. 318 m^2	4. 45 ft^2
5. 48 yds^2	6. 60 $in.^2$	7. 234 $in.^2$	8. 286 m^2
9. 90 ft^2	10. 192 ft^2	11. 28 ft^2	12. 408 cm^2
13. 95 cm^2	14. 40 mm^2	15. 40 $in.^2$	16. 41 ft^2
17. 38 cm^2	18. 43 cm^2	19. 51.5 ft^2	20. 32 ft^2
21. 184.5 m^2	22. 124 ft^2	23. 232 $in.^2$	24. 342 m^2

CIRCUMFERENCE #9

Circumference is the perimeter of a circle, that is, the distance around the circle.

$$\boldsymbol{C = \pi d} \quad \text{or} \quad \boldsymbol{C = 2\pi r}$$

d = diameter
r = radius
$\pi \approx 3.14$

Example 1

Find the circumference of a circle with a diameter of 15 inches.

d = 15 inches

$C = \pi d$
$= \pi(15)$ or $3.14(15)$
$= 47.1$ inches

Example 2

Find the circumference of a circle with a radius of 12 units.

$r = 12$, so $d = 2(12) = 24$

$C = 3.14(24)$
$= 75.36$ units

Example 3

Find the diameter of a circle with a circumference of 254.34 inches.

$C = \pi d$
$254.34 = \pi d$
$254.34 = 3.14d$
$d = \frac{254.34}{3.14} = 81$ inches

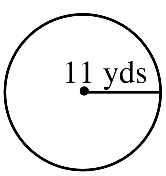
Problems

Find the circumference of each circle given the following diameter or radius lengths. Round your answer to the nearest hundredth. Use $\pi = 3.14$.

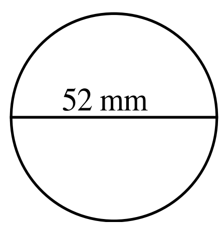
1. $d = 53$ ft
2. $d = 8.5$ ft
3. $r = 7.3$ m
4. $d = 63$ m
5. $r = 2.12$ cm

Find the circumference of each circle shown below. Round your answer to the nearest hundredth. Use $\pi = 3.14$.

6.



7.



Find the diameter of each circle given the circumference. Round your answer to the nearest tenth. Use $\pi = 3.14$.

8. $C = 54.636$ mm
9. $C = 135.02$ km
10. $C = 389.36$ km

Answers

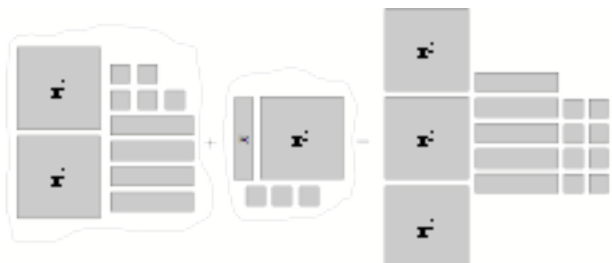
1.	166.42 ft	2.	26.69 ft	3.	45.84 m	4.	197.82 m
5.	13.31 cm	6.	69.08 yds	7.	163.28 mm	8.	17.4 mm
9.	43 km	10.	124 km				

COMBINING LIKE TERMS #10

LIKE TERMS are terms that are exactly the same except for their coefficients. Like terms can be combined into one quantity by adding and/or subtracting the coefficients of the terms. Terms are usually listed in the order of decreasing powers of the variable. Combining like terms using algebra tiles is shown in the first two examples.

Example 1

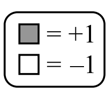
Simplify $(2x^2+4x+5)+(x^2+x+3)$ means combine $2x^2+4x+5$ with x^2+x+3.

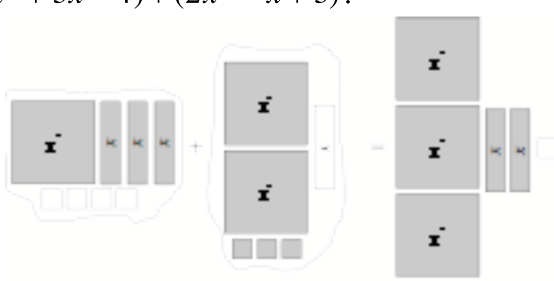


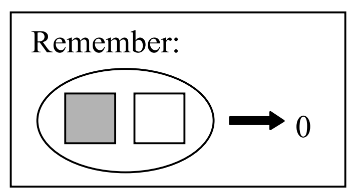
$(2x^2+4x+5)+(x^2+x+3) = 3x^2+5x+8$

Example 2

Simplify $(x^2+3x-4)+(2x^2-x+3)$.







$(x^2+3x-4) + (x^2+x+3) = 3x^2+2x-1$

Example 3

$(4x^2+3x-7)+(-2x^2-2x-3) = 4x^2+(-2x^2)+3x+(-2x)-7+(-3) = 2x^2+x-10$

Example 4

$(-3x^2-2x+5)-(-4x^2+7x-6) = -3x^2-(-4x^2)-2x-(7x)+5-(-6)$
$= -3x^2+4x^2-2x-7x+5+6 = x^2-9x+11$

Problems

Combine like terms for each expression below.

1. $(x^2 + 3x + 4) + (x^2 + 3x + 2)$
2. $(x^2 + 4x + 3) + (x^2 + 2x + 5)$
3. $(2x^2 + 2x + 1) + (x2 + 4x + 5)$
4. $(3x^2 + x + 7) + (3x^2 + 2x + 4)$
5. $(2x^2 + 4x + 3) + (x^2 + 3x + 5)$
6. $(4x^2 + 2x + 8) + (2x^2 + 5x + 1)$
7. $(4x^2 + 2x + 8) + (3x^2 + 5x + 3)$
8. $(3x^2 + 4x + 1) + (2x^2 + 4x + 5)$
9. $(5x^2 + 4x - 7) + (3x^2 + 2x + 3)$
10. $(3x^2 - 4x + 2) + (2x^2 + 2x + 4)$
11. $(3x^2 - x + 2) + (4x^2 + 3x - 1)$
12. $(2x^2 - 2x + 7) + (5x^2 + 4x - 3)$
13. $(2x^2 - 3x - 3) + (5x^2 - 4x + 4)$
14. $(3x^2 - 3x + 6) + (2x^2 - x - 4)$
15. $(-4x^2 + x + 2) + (6x^2 - 3x + 2)$
16. $(-3x^2 + 4x + 2) + (5x^2 - 6x - 1)$
17. $(x^2 - 4) + (-x^2 + x - 3)$
18. $(3x^2 + x) + (-2x^2 + 4)$
19. $(3x^2 + 4) + (x^2 - 2x + 3)$
20. $(-2x^2 - x) + (4x^2 - 3)$
21. $(7x^2 - 2x + 3) - (3x^2 - 4x + 7)$
22. $(x^2 - 3x - 2) - (4x^2 + 3x - 3)$
23. $(8x^2 + 4x - 7) - (-4x^2 + 3x - 4)$
24. $(-2x^2 + 14) - (3x^2 + 4x - 7)$

Answers

1. $2x^2 + 6x + 6$
2. $2x^2 + 6x + 8$
3. $3x^2 + 6x + 6$
4. $6x^2 + 3x + 11$
5. $3x^2 + 7x + 8$
6. $6x^2 + 7x + 9$
7. $7x^2 + 7x + 11$
8. $5x^2 + 8x + 6$
9. $8x^2 + 6x - 4$
10. $5x^2 - 2x + 6$
11. $7x^2 + 2x + 1$
12. $7x^2 + 2x + 4$
13. $7x^2 - 7x + 1$
14. $5x^2 - 4x + 2$
15. $2x^2 - 2x + 4$
16. $2x^2 - 2x + 1$
17. $x - 7$
18. $x^2 + x + 4$
19. $4x^2 - 2x + 7$
20. $2x^2 - x - 3$
21. $4x^2 + 2x - 4$
22. $-3x^2 - 6x + 1$
23. $12x^2 + x - 3$
24. $-5x^2 - 4x + 21$

DISTRIBUTIVE PROPERTY #11

The **DISTRIBUTIVE PROPERTY** shows how to express sums and products in two ways: $a(b+c)=ab+ac$. This can also be written $(b+c)a=ab+ac$.

Factored form	Distributed form	Simplified form
$\boldsymbol{a(b+c)}$	$\boldsymbol{a(b)+a(c)}$	$\boldsymbol{ab+ac}$

To simplify: Multiply each term on the inside of the parentheses by the term on the outside. Combine terms if possible.

Example 1

$$2(47)=2(40+7)$$
$$=(2\cdot 40)+(2\cdot 7)$$
$$=80+14=94$$

Example 2

$$3(x+4)=(3\cdot x)+(3\cdot 4)$$
$$=3x+12$$

Example 3

$$4(x+3y+1)=(4\cdot x)+(4\cdot 3y)+4(1)$$
$$=4x+12y+4$$

Problems

Simplify each expression below by applying the Distributive Property.

1. $6(9+4)$
2. $4(9+8)$
3. $7(8+6)$
4. $5(7+4)$
5. $3(27)=3(20+7)$
6. $6(46)=6(40+6)$
7. $8(43)$
8. $6(78)$
9. $3(x+6)$
10. $5(x+7)$
11. $8(x-4)$
12. $6(x-10)$
13. $(8+x)4$
14. $(2+x)5$
15. $-7(x+1)$
16. $-4(y+3)$
17. $-3(y-5)$
18. $-5(b-4)$
19. $-(x+6)$
20. $-(x+7)$
21. $-(x-4)$
22. $-(-x-3)$
23. $x(x+3)$
24. $4x(x+2)$
25. $-x(5x-7)$
26. $-x(2x-6)$

Answers

1. $(6\cdot 9)+(6\cdot 4)=54+24=78$
2. $(4\cdot 9)+(4\cdot 8)=36+32=68$
3. $56+42=98$
4. $35+20=55$
5. $60+21=81$
6. $240+36=276$
7. $320+24=344$
8. $420+48=468$
9. $3x+18$
10. $5x+35$
11. $8x-32$
12. $6x-60$
13. $4x+32$
14. $5x+10$
15. $-7x-7$
16. $-4y-12$
17. $-3y+15$
18. $-5b+20$
19. $-x-6$
20. $-x-7$
21. $-x+4$
22. $x+3$
23. x^2+3x
24. $4x^2+8x$
25. $-5x^2+7x$
26. $-2x^2+6x$

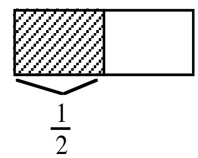
DIVISION BY FRACTIONS USING AN AREA MODEL #12

Fractions can be divided using a rectangular area model. The division problem $8 \div 2$ means, "In 8, how many groups of 2 are there?" Similarly, $\frac{1}{2} \div \frac{1}{4}$ means, "In $\frac{1}{2}$, how many fourths are there?"

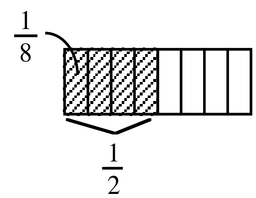
Example 1

Use the rectangular model to divide: $\frac{1}{2} \div \frac{1}{8}$, that is, "How many eighths are in $\frac{1}{2}$?"

Step 1: Using the rectangle, we first divide it into 2 equal pieces. Each piece represents $\frac{1}{2}$. Shade $\frac{1}{2}$ of it.



Step 2: Then divide the <u>original</u> rectangle into eight equal pieces. Each section represents $\frac{1}{8}$. In the shaded section, $\frac{1}{2}$, there are 4 eighths.

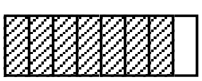


Step 3: Write the equation. $\frac{1}{2} \div \frac{1}{8} = 4$

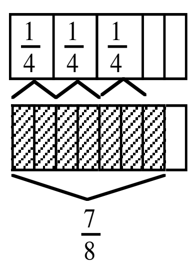
Example 2

In $\frac{7}{8}$, how many $\frac{1}{4}$'s are there?

That is, $\frac{7}{8} \div \frac{1}{4} = ?$



Start with $\frac{7}{8}$.



In $\frac{7}{8}$ there are three full $\frac{1}{4}$'s shaded and half of another one (that is, half of one-fourth).

So: $\frac{7}{8} \div \frac{1}{4} = 3\frac{1}{2}$
(three and one-half fourths)

Problems

Use the rectangular model to divide.

1. $1\frac{2}{3} \div \frac{1}{9}$
2. $\frac{3}{2} \div \frac{1}{4}$
3. $1 \div \frac{1}{5}$
4. $1\frac{1}{8} \div \frac{1}{2}$
5. $2\frac{1}{3} \div \frac{5}{6}$

Answers

1. 15
2. 6
3. 5
4. $\frac{9}{4}$ or $2\frac{1}{4}$
5. $\frac{14}{5}$ or $2\frac{4}{5}$

DIVISION BY FRACTIONS USING RECIPROCALS

Two numbers that have a product of 1 are reciprocals. For example, $\frac{1}{3} \cdot \frac{3}{1} = 1$, $\frac{1}{8} \cdot \frac{8}{1} = 1$, and $\frac{1}{5} \cdot \frac{5}{1} = 1$, so $\frac{1}{3}$ and $\frac{3}{1}$, $\frac{1}{8}$ and $\frac{8}{1}$, and $\frac{1}{5}$ and $\frac{5}{1}$ are all pairs of reciprocals.

There is another way to divide fractions: invert the divisor, that is, write its reciprocal, then proceed as you do with multiplication. (The divisor is the number after the division sign.) After inverting the divisor, change the division sign to a multiplication sign and multiply. Simplify if possible. This rule is a simplified way to divide that is based on using a Giant One as shown in examples 3 – 6.

Example 1

$\frac{3}{8} \div \frac{1}{2} \Rightarrow \frac{3}{8} \cdot \frac{2}{1} \Rightarrow \frac{6}{8} = \frac{3}{4}$

Example 2

$1\frac{1}{5} \div \frac{1}{6} \Rightarrow \frac{6}{5} \cdot \frac{6}{1} \Rightarrow \frac{36}{5} \Rightarrow 7\frac{1}{5}$

The examples above were written horizontally, but a division of fractions problem can also be written in the vertical form such as $\frac{\frac{1}{2}}{\frac{1}{3}}$, $\frac{\frac{1}{4}}{\frac{1}{2}}$, and $\frac{1\frac{1}{2}}{\frac{1}{6}}$. They still mean the same thing:

$\frac{\frac{1}{2}}{\frac{1}{3}}$ means, "In $\frac{1}{2}$, how many $\frac{1}{3}$'s are there?" $\frac{\frac{1}{4}}{\frac{1}{2}}$ means, "In $\frac{1}{4}$, how many $\frac{1}{2}$'s are there?"

$\frac{1\frac{1}{2}}{\frac{1}{6}}$ means, "In $1\frac{1}{2}$, how many $\frac{1}{6}$'s are there?"

You can use a Super Giant One to solve these vertical division problems. This Super Giant One uses the reciprocal of the divisor.

Example 3

$$\frac{\frac{1}{2}}{\frac{1}{3}} \cdot \frac{\frac{3}{1}}{\frac{3}{1}} = \frac{\frac{3}{2}}{1} = \frac{3}{2} = 1\frac{1}{2}$$

Example 4

$$\frac{\frac{1}{4}}{\frac{1}{2}} \cdot \frac{\frac{2}{1}}{\frac{2}{1}} = \frac{\frac{2}{4}}{1} = \frac{2}{4} = \frac{1}{2}$$

Example 5

$$\frac{1\frac{1}{2}}{\frac{1}{6}} = \frac{\frac{3}{2}}{\frac{1}{6}} \cdot \frac{\frac{6}{1}}{\frac{6}{1}} = \frac{\frac{18}{2}}{1} = \frac{18}{2} = 9$$

Example 6

$$\frac{2}{5} \div \frac{1}{3} = \frac{2}{5} \cdot \frac{3}{1} = \frac{6}{5} = 1\frac{1}{5}$$

Compared to:

$$\frac{\frac{2}{5}}{\frac{1}{3}} \cdot \frac{\frac{3}{1}}{\frac{3}{1}} = \frac{\frac{6}{5}}{1} = \frac{6}{5} = 1\frac{1}{5}$$

Problems

Solve these division problems. Use any method.

1. $\frac{3}{5} \div \frac{3}{8}$	2. $2\frac{1}{2} \div \frac{7}{8}$	3. $\frac{4}{5} \div \frac{2}{3}$	4. $1\frac{1}{5} \div \frac{3}{5}$	5. $\frac{6}{7} \div \frac{3}{8}$
6. $\frac{3}{10} \div \frac{5}{6}$	7. $1\frac{2}{7} \div \frac{1}{3}$	8. $7 \div \frac{1}{4}$	9. $1\frac{5}{9} \div \frac{2}{5}$	10. $3\frac{1}{3} \div \frac{5}{9}$
11. $2\frac{1}{3} \div \frac{1}{6}$	12. $2\frac{1}{2} \div \frac{3}{4}$	13. $\frac{7}{8} \div 1\frac{1}{4}$	14. $5\frac{1}{3} \div \frac{2}{9}$	15. $\frac{3}{5} \div 9$

Answers

1. $\frac{8}{5}$ or $1\frac{3}{5}$	2. $\frac{20}{7}$ or $2\frac{6}{7}$	3. $\frac{6}{5}$ or $1\frac{1}{5}$	4. 2	5. $\frac{16}{7}$ or $2\frac{2}{7}$
6. $\frac{9}{25}$	7. $\frac{27}{7}$ or $3\frac{6}{7}$	8. 28	9. $\frac{35}{9}$ or $3\frac{8}{9}$	10. 6
11. 14	12. $\frac{10}{3}$ or $3\frac{1}{3}$	13. $\frac{7}{10}$	14. 24	15. $\frac{1}{15}$

DRAWING A GRAPH FROM A TABLE #13

One way to organize the points needed to graph an equation is to place them in an xy-table. In this course, linear equations will usually be written in y-form, such as $y = mx + b$. Make a table with rows for the x- and y-values. Choose some values for x. Substitute each x-value in the rule (the $mx + b$ part or the expression that is equal to y), evaluate, and record the result as the corresponding y-value. Select an appropriate scale for your axes and plot the graph.

Example

Complete a table to graph $y = 5x - 8$, then graph the equation.

x	–3	–2	–1	0	1	2	3	4	5
y									

- Make a table with x-values
- Each y-value is found by:
  - Substituting the value for x.
  - Multiplying it by 5.
  - Then subtracting 8.

For example:

$$\begin{aligned} y &= 5(-3) - 8 \\ &= -15 - 8 \\ &= -23 \end{aligned}$$

The point $(-3, -23)$ is on the graph.

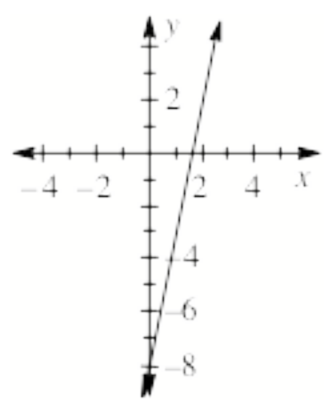
The completed table is shown below. Not all points are necessary to create a meaningful graph.

$y = 5x - 8$

x	–3	–2	–1	0	1	2	3	4	5
y	–23	–18	–13	–8	–3	2	7	12	17

- x-values may be referred to as inputs. The set of all input values is the **domain**.
- y-values may be referred to as outputs. The set of all output values is the **range**.

Use the pairs of xy-values in the table to graph the equation. A portion of the graph is shown at right.



Problems

Copy and complete each table. Graph each rule.

1. $y = 4x - 3$

x	–3	–2	–1	0	1	2	3
y							

2. $y = -x + 5$

x	–3	–2	–1	0	1	2	3
y							

3. $y = -2x - 3$

x	–3	–2	–1	0	1	2	3
y							

4. $y = -5x - 0.5$

x	–3	–2	–1	0	1	2	3
y							

5. $y = \frac{1}{2}x - 3$

x	–5	–1	0	1	2	3	6
y							

6. $y = \frac{3}{2}x - 4$

x	–4	–2	0	1	2	4	6
y							

7. $y = \frac{1}{5}x + 3$

x	–6	–5	–2	0	2	4	5
y							

8. $y = \frac{3}{4}x - 2$

x	–4	–2	0	1	2	3	4
y							

9. $y = -2x + 4$

x	–2	–1	0	1	2	3	4
y							

10. $y = -\frac{2}{3}x + 5$

x	–6	–3	0	1	3	6	9
y							

11. $y = x^2 - 5$

x	–3	–2	–1	0	1	2	3
y							

12. $y = -x^2 + 1$
(Careful! Square first, then change the sign.)

x	–3	–2	–1	0	1	2	3
y							

13. $y = x^2 - 3x - 2$

x	–2	–1	0	1	2	3	4	5
y								

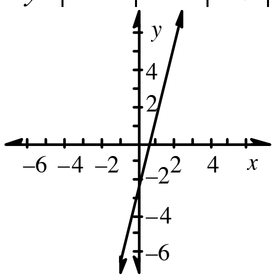
14. $y = x^2 - 4x - 2$

x	–1	0	1	2	3	4	5
y							

Answers

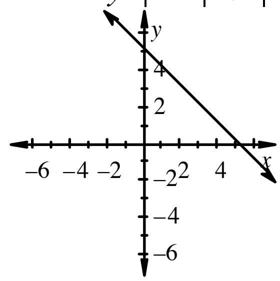
1. $y = 4x - 3$

x	–3	–2	–1	0	1	2	3
y	–15	–11	–7	–3	1	5	9



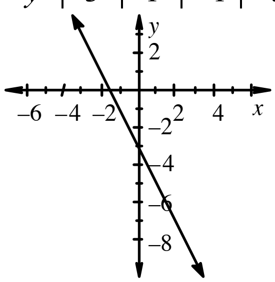
2. $y = -x + 5$

x	–3	–2	–1	0	1	2	3
y	8	7	6	5	4	3	2



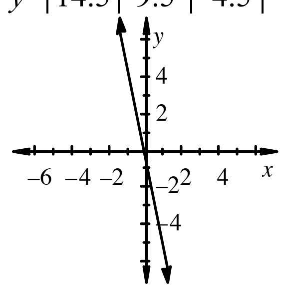
3. $y = -2x - 3$

x	–3	–2	–1	0	1	2	3
y	3	1	–1	–3	–5	–7	–9



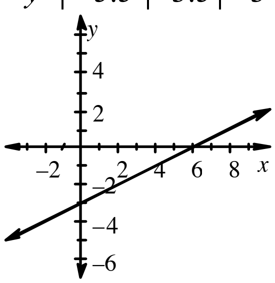
4. $y = -5x - 0.5$

x	–3	–2	–1	0	1	2	3
y	14.5	9.5	4.5	–0.5	–5.5	–10.5	–15.5



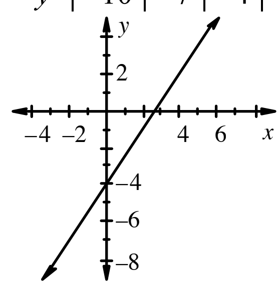
5. $y = \frac{1}{2}x - 3$

x	–5	–1	0	1	2	3	–6
y	–5.5	–3.5	–3	–2.5	–2	–1.5	0



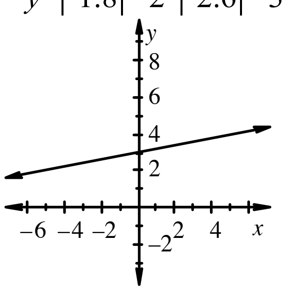
6. $y = \frac{3}{2}x - 4$

x	–4	–2	0	1	2	4	6
y	–10	–7	–4	–2.5	–1	2	5



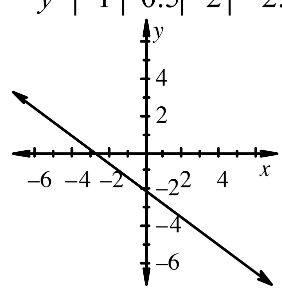
7. $y = \frac{1}{5}x + 3$

x	–6	–5	–2	0	2	4	5
y	1.8	2	2.6	3	3.4	3.8	4



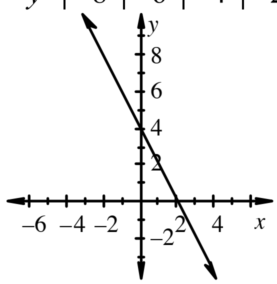
8. $y = -\frac{3}{4}x - 2$

x	–4	–2	0	1	2	3	4
y	1	–0.5	–2	–2.75	–3.5	–4.25	–5



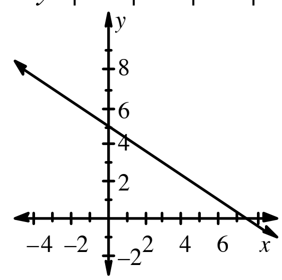
9. $y = -2x + 4$

x	-2	-1	0	1	2	3	4
y	8	6	4	2	0	-2	-4



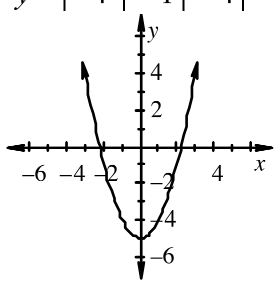
10. $y = -\frac{2}{3}x + 5$

x	-6	-3	0	1	3	6	9
y	9	7	5	4.33	3	1	-1



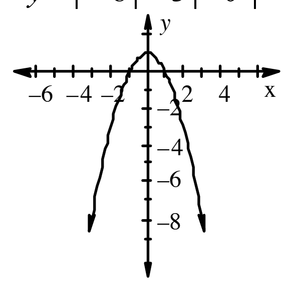
11. $y = x^2 - 5$

x	-3	-2	-1	0	1	2	3
y	4	-1	-4	-5	-4	-1	4



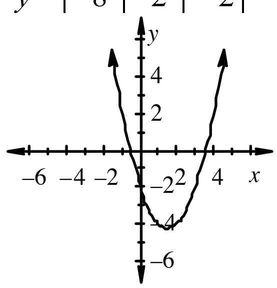
12. $y = -x^2 + 1$

x	-3	-2	-1	0	1	2	3
y	-8	-3	0	1	0	-3	-8



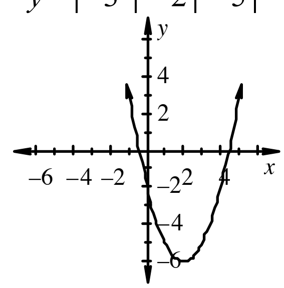
13. $y = x^2 - 3x - 2$

x	-2	-1	0	1	2	3	4	5
y	8	2	-2	-4	-4	-2	2	8



14. $y = x^2 - 4x - 2$

x	-1	0	1	2	3	4	5
y	3	-2	-5	-6	-5	-2	3



EQUIVALENT FRACTIONS #14

Fractions that name the same value are called equivalent fractions, such as $\frac{2}{3} = \frac{6}{9}$.
Three methods for finding equivalent fractions are using a ratio table, a rectangular area model, and the Identity Property of Multiplication (the Giant One). The ratio table method is discussed in the "Ratio" topic of the Extra Practice.

RECTANGULAR AREA MODEL

This method for finding equivalent fractions is based on the fact that the area of a rectangle is the same no matter how it is dissected (cut up). Draw, divide (using vertical lines), and shade a rectangle to represent the original fraction. Next, add horizontal lines to the rectangle to divide the area equally so that the rectangle has the same number of equal pieces as the number in the denominator of the second fraction. Note that each rectangle has the same amount of shaded area. Renaming the shaded area in terms of the new, smaller pieces gives the equivalent fraction.

Example 1

Use the rectangular area model to find three equivalent fractions for $\frac{1}{3}$.

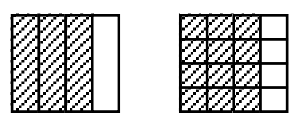
$\frac{1}{3}$ $\frac{2}{6}$ $\frac{3}{9}$ $\frac{4}{12}$

The horizontal line in the second figure creates two rows, which is the same as using a Giant One $\frac{2}{2}$. The same idea is used in the third and fourth figures, $\frac{3}{3}$ and $\frac{4}{4}$.

Example 2

Use the rectangular area model to find the specified equivalent fraction.

$$\frac{3}{4} \cdot \boxed{} = \frac{}{16}$$



$$\frac{3}{4} \cdot \frac{4}{4} = \frac{12}{16}$$

After drawing the fraction $\frac{3}{4}$, the diagram is divided into four horizontal rows because 16 ÷ 4 equals 4. The diagram now shows 12 shaded parts out of 16 total parts. This area model shows the equivalent fractions: $\frac{3}{4} = \frac{12}{16}$.

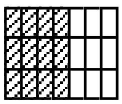
Problems

Draw rectangular models to find the specified equivalent fraction.

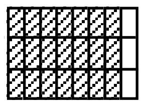
1. $\frac{4}{7} = \frac{\ \ }{21}$ 2. $\frac{7}{8} = \frac{\ \ }{24}$ 3. $\frac{4}{9} = \frac{\ \ }{36}$ 4. $\frac{5}{3} = \frac{\ \ }{9}$

Answers

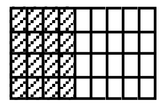
1. $\frac{12}{21}$



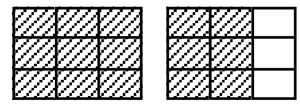
2. $\frac{21}{24}$



3. $\frac{16}{36}$



4. $\frac{15}{9}$



> **THE IDENTITY PROPERTY OF MULTIPLICATION (GIANT ONE)**
>
> Multiplying by 1 does not change the value of a number. The Giant One uses a fraction that has the same numerator and denominator, such as $\frac{2}{2}$, to find an equivalent fraction.

Example 1

Find three equivalent fractions for $\frac{1}{3}$.

$\frac{1}{3} \cdot \frac{2}{2} = \frac{2}{6}$ $\frac{1}{3} \cdot \frac{3}{3} = \frac{3}{9}$ $\frac{1}{3} \cdot \frac{4}{4} = \frac{4}{12}$

Example 2

Use the Giant One to find an equivalent fraction to $\frac{5}{12}$ using 48ths: $\frac{5}{12} \cdot \frac{\ }{\ } = \frac{?}{48}$

Since $48 \div 12 = 4$, the Giant One is $\frac{4}{4}$: $\frac{5}{12} \cdot \frac{4}{4} = \frac{20}{48}$

Problems

Use the Giant One to find the specified equivalent fraction. Your answer should include the Giant One you use and the equivalent numerator.

1. $\frac{5}{3} \cdot \boxed{} = \frac{?}{21}$

2. $\frac{7}{9} \cdot \boxed{} = \frac{?}{45}$

3. $\frac{9}{5} \cdot \boxed{} = \frac{?}{45}$

4. $\frac{3}{4} \cdot \boxed{} = \frac{?}{24}$

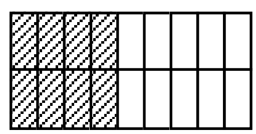
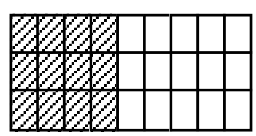
5. $\frac{7}{6} \cdot \boxed{} = \frac{?}{30}$

6. $\frac{8}{5} \cdot \boxed{} = \frac{?}{30}$

Answers

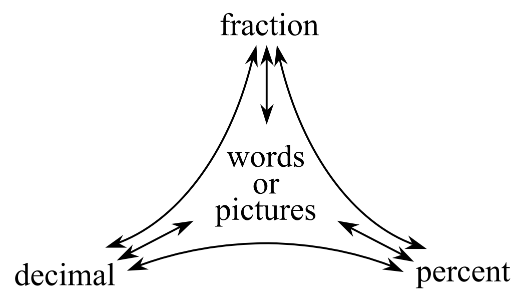
1. $\frac{7}{7}$, 35 2. $\frac{5}{5}$, 35 3. $\frac{9}{9}$, 81 4. $\frac{6}{6}$, 18 5. $\frac{5}{5}$, 35 6. $\frac{6}{6}$, 48

The following table summarizes the three methods for finding equivalent fractions.

Fraction	Ratio Table	Giant 1	Rectangular Model
$\frac{4}{9}$	$\begin{array}{c\|c\|c} 4 & 8 & 12 \\ \hline 9 & 18 & 27 \end{array}$	$\frac{4}{9} \cdot \boxed{\frac{2}{2}} = \frac{8}{18}$ $\frac{4}{9} \cdot \boxed{\frac{3}{3}} = \frac{12}{27}$	 $\frac{4}{9} = \frac{8}{18}$  $\frac{4}{9} = \frac{12}{27}$

FRACTION, DECIMAL, AND PERCENT EQUIVALENTS #15

Fractions, decimals, and percents are different ways to represent the same number.



Representations of a Portion

Examples

Decimal to percent:

Multiply the decimal by 100.

$(0.27)(100) = 27\%$

Percent to decimal:

Divide the percent by 100.

$47.3\% \div 100 = 0.473$

Fraction to percent:

Write a proportion to find an equivalent fraction using 100 as the denominator. The numerator is the percent.

$\frac{3}{5} = \frac{x}{100}$ so $\frac{3}{5} = \frac{60}{100} = 60\%$

Percent to fraction:

Use 100 as the denominator. Use the percent as the numerator. Simplify as needed.

$24\% = \frac{24}{100} = \frac{6}{25}$

Decimal to fraction:

Use the decimal as the numerator. Use the decimal place value name as the denominator. Simplify as needed.

a. $0.4 = \frac{4}{10} = \frac{2}{5}$ b. $0.37 = \frac{37}{100}$

Fraction to decimal:

Divide the numerator by the denominator.

$\frac{5}{8} = 5 \div 8 = 0.625$

Problems

Convert the fraction, decimal, or percent as indicated.

1. Change $\frac{1}{5}$ to a decimal.
2. Change 40% to a fraction in lowest terms.
3. Change 0.54 to a fraction in lowest terms.
4. Change 35% to a decimal.
5. Change 0.43 to a percent.
6. Change $\frac{1}{4}$ to a percent.
7. Change 0.7 to a fraction.
8. Change $\frac{3}{8}$ to a decimal.
9. Change $\frac{2}{3}$ to a decimal.
10. Change 0.07 to a percent.
11. Change 67% to a decimal.
12. Change $\frac{4}{5}$ to a percent.
13. Change 0.6 to a fraction in lowest terms.
14. Change 85% to a fraction in lowest terms.
15. Change $\frac{2}{9}$ to a decimal.
16. Change 135% to a fraction in lowest terms.
17. Change $\frac{9}{5}$ to a decimal.
18. Change 5.25 to a percent.
19. Change $\frac{1}{18}$ to a decimal, then change the decimal to a percent.
20. Change 47% to a fraction, then change the fraction to a decimal.
21. Change $\frac{3}{7}$ to a decimal.
22. Change 0.625 to a percent.
23. Change $\frac{5}{8}$ to a decimal, then change the decimal to a percent.
24. Change 45% to a decimal, then change the decimal to a fraction.

Answers

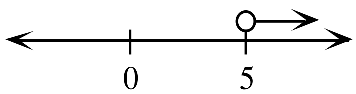
1. 0.20	2. $\frac{40}{100}=\frac{2}{5}$	3. $\frac{54}{100}=\frac{27}{50}$	4. 0.35
5. 43%	6. 25%	7. $\frac{7}{10}$	8. 0.325
9. $0.\overline{6}$	10. 7%	11. 0.67	12. 80%
13. $\frac{3}{5}$	14. $\frac{17}{50}$	15. $0.\overline{2}$	16. $\frac{27}{20}$
17. 1.8	18. 525%	19. $0.0\overline{5}$; 5.6%	20. $\frac{47}{100}$; 47%
21. 0.429	22. 62.5%	23 0.625; 62.5%	24. 0.45; $\frac{9}{20}$

GRAPHING INEQUALITIES #16

The solution(s) to an equation can be represented as a point (or points) on the number line. The solutions to inequalities are represented by rays or segments with solid or open endpoints. Solid endpoints indicate that the endpoint is included in the solution (≤ or ≥), while the open dot indicates that it is not part of the solution (< or >).

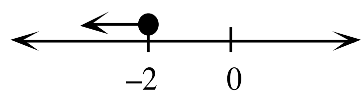
Example 1

$x > 5$



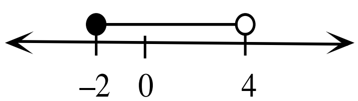
Example 2

$x \leq -2$



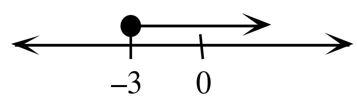
Example 3

$-2 \leq m < 4$



Example 4

$q \geq -3$

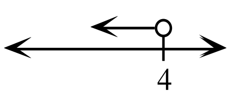


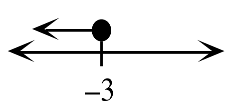
Problems

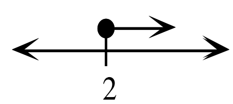
Graph each inequality on a number line.

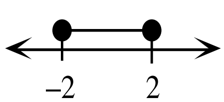
1. $m < 4$
2. $x \leq -3$
3. $y \geq 2$
4. $-2 \leq x \leq 2$
5. $-8 < x < -4$
6. $-2 < x \leq 3$
7. $m > -7$
8. $x \neq 4$
9. $x \leq 5$
10. $x \geq -2$

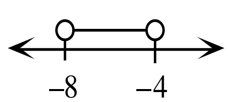
Answers

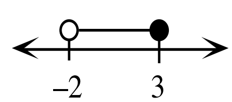
1. 

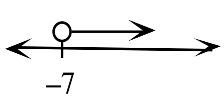
2. 

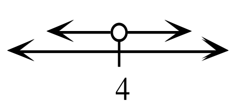
3. 

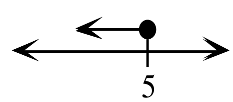
4. 

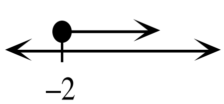
5. 

6. 

7. 

8. 

9. 

10. 

LAWS OF EXPONENTS #17

In the expression 5^3, 5 is the **base** and 3 is the **exponent**. For x^a, x is the base and a is the exponent. 5^3 means $5 \cdot 5 \cdot 5$. 5^4 means $5 \cdot 5 \cdot 5 \cdot 5$, so you can write $\frac{5^7}{5^4}$ which means $5^7 \div 5^4$ or you can write it as: $\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5 \cdot 5}$.

You can use the Giant One to find the matching pairs of numbers in the numerator and denominator. There are four Giant Ones, namely, $\frac{5}{5}$, four times, so $\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5 \cdot 5} = 5^3$ or 125. Writing 5^3 is usually sufficient.

When there is a variable, it is treated the same way. $\frac{x^6}{x^2}$ means $\frac{x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x}$.
The Giant One here is $\frac{x}{x}$, two of them. The answer is x^4.

$5^2 \cdot 5^3$ means $(5 \cdot 5)(5 \cdot 5 \cdot 5)$ which is 5^5.

$(5^2)^3$ means $(5^2)(5^2)(5^2)$ or $(5 \cdot 5)(5 \cdot 5)(5 \cdot 5)$ which is 5^6.

When the problems have variables such as $x^3 \cdot x^5$, you only need to add the exponents.
The answer is x^8. If the problem is $(x^3)^5$ (x^3 to the fifth power) it means $x^3 \cdot x^3 \cdot x^3 \cdot x^3 \cdot x^3$.
The answer is x^{15}. You multiply exponents in this case.

If the problem is $\frac{x^8}{x^3}$, you subtract the bottom exponent from the top exponent $(8 - 3)$.
The answer is x^5. You can also have problems like $\frac{x^8}{x^{-3}}$. You still subtract, $8 - (-3)$ is 11, and the answer is x^{11}.

You need to be sure the bases are the same to use these laws. For example, $x^4 \cdot y^5$ cannot be further simplified, since x and y are not the same base.

In general, the **LAWS OF EXPONENTS**, where $x \neq 0$, are:

$$x^a \cdot x^b = x^{(a+b)} \qquad (x^a)^b = x^{ab} \qquad \frac{x^a}{x^b} = x^{(a-b)}$$

$$x^0 = 1 \qquad (x^a y^b)^c = x^{ac} y^{bc}$$

Examples

a. $x^6 \cdot x^5 = x^{6+5} = x^{11}$

b. $\frac{x^{18}}{x^{12}} = x^{18-12} = x^6$

c. $(z^5)^3 = z^{5 \cdot 3} = z^{15}$

d. $(x^2y^3)^5 = x^{2 \cdot 5}y^{3 \cdot 5} = x^{10}y^{15}$

e. $\frac{x^5}{x^{-2}} = x^{5-(-2)} = x^7$

f. $(3x^2y^2)^2 = 3^2x^{2 \cdot 2}y^{2 \cdot 2} = 9x^4y^4$

g. $\left(3x^3y^{-2}\right)^3 = 3^3x^{3 \cdot 3}y^{-2 \cdot 3} = 27x^9y^{-6}$ or $\frac{27x^6}{y^6}$

h. $\frac{x^7y^5z^3}{x^3y^6z^{-2}} = x^{7-3}y^{5-6}z^{3-(-2)} = x^4y^{-1}z^5$ or $\frac{x^4z^5}{y}$

Problems

Simplify each expression.

1. $5^3 \cdot 5^4$
2. $x^3 \cdot x^5$
3. $\frac{5^{18}}{5^{14}}$
4. $\frac{x^{11}}{x^4}$
5. $(5^4)^3$
6. $(x^5)^3$
7. $(2x^3y^4)^4$
8. $\frac{5^4}{5^{-4}}$
9. $5^6 \cdot 5^{-3}$
10. $(y^2)^{-4}$
11. $(3a^4b^{-3})^3$
12. $\frac{x^7y^5z^2}{x^4y^3z^2}$
13. $\frac{x^8y^4z^4}{x^{-2}y^3z^{-1}}$
14. $5x^3 \cdot 3x^4$

Answers

1. 5^7
2. x^8
3. 5^4
4. x^7
5. 5^{12}
6. x^{15}
7. $16x^{12}y^{16}$
8. 5^8
9. 5^3
10. y^{-8} or $\frac{1}{y^8}$
11. $27a^{12}b^{-9}$ or $\frac{27a^{12}}{b^9}$
12. x^3y^2
13. $x^{10}yz^5$
14. $15x^7$

MEASURES OF CENTRAL TENDENCY #18

The **MEASURES OF CENTRAL TENDENCY** are numbers that locate or approximate the "center" of a set of data. Mean, median, and mode are the most common measures of central tendency.

The **MEAN** is the arithmetic average of a data set. Add all the values in a set and divide this sum by the number of values in the set.

The **MEDIAN** is the middle number in a set of data arranged in numerical order. If there are an even number of values, the median is the mean of the two middle numbers.

The **RANGE** of a set of data is the difference between the highest value and the lowest value.

Example 1

Find the mean of this set of data: 23, 31, 46, 23, 38, 47, 23

- Add the numbers: $23 + 31 + 46 + 23 + 38 + 47 + 23 = 231$
- Divide by the number of values: $231 \div 7 = 33$

The mean of this set of data is 33.

Example 2

Find the mean of this set of data: 82, 72, 70, 82, 68, 65, 85, 64

- Add the numbers: $82 + 72 + 70 + 82 + 68 + 65 + 85 + 64 = 588$
- Divide by the number of values: $588 \div 8 = 73.5$

The mean of this set of data is 73.5.

Example 3

Find the median of this set of data: 23, 34, 25, 37, 35, 22, 30

- Arrange the data in order: 22, 23, 25, 30, 34, 35, 37
- Find the middle value: 30, since there are the same number of values on either side.

Therefore, the median of this data set is 30.

Example 4

Find the median of this set of data: 52, 54, 58, 42, 53, 51, 25, 28

- Arrange the data in order: 25, 28, 42, 51, 52, 53, 54, 58.
- Find the middle value(s): 51 and 52, since there are three values on either side.
- Since there are two middle values, find their mean: $51 + 52 = 103$

  $103 \div 2 = 51.5$

The median of this data set is 51.5

Example 5

Find the range of this set of data: 15, 46, 13, 23, 32, 40, 38, 18, 27, 16

- The highest value is 46.
- The lowest value is 13.
- Subtract the lowest value from the highest: 46 – 13 = 33

The range of this set of data is 33.

Problems

Identify the mean, median, and range for each set of data.

1. 10, 8, 7, 9, 12, 14, 9, 9, 10
2. 21, 44, 32, 27, 38, 36, 32, 45, 47, 40, 23
3. 68, 55, 53, 55, 64, 60, 35, 42, 47, 46
4. 120, 88, 74, 82, 78, 80, 100, 110, 74, 78
5. 82, 83, 84, 82, 77, 82, 77, 70, 70, 77, 70, 85
6. 18, 32, 37, 42, 56, 78, 82, 95, 100, 7

Answers

1. Mean: 9.78 Median: 9
   Range: 7
2. Mean: 35 Median: 36
   Range: 26
3. Mean: 52.5 Median: 54
   Range: 33
4. Mean: 88.4 Median: 81
   Range: 46
5. Mean: 78.25 Median: 79.5
   Range: 15
6. Mean: 54.7 Median: 49
   Range: 93

MULTIPLICATION OF FRACTIONS WITH AN AREA MODEL #19

When multiplying fractions using a rectangular area model, lines that represent one fraction are drawn vertically and the correct number of parts are shaded. Then lines that represent the second fraction are drawn horizontally and part of the shaded region is darkened that represents the product of the two fractions.

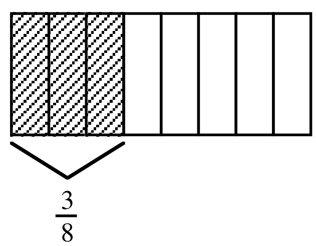
The rule for multiplying fractions derived from the area model is to multiply the numerators, then multiply the denominators. Simplify the product when possible. In general,

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

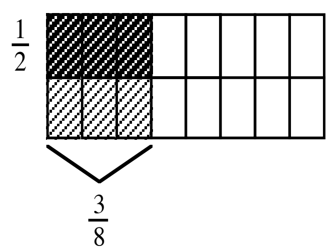
Example 1

$\frac{1}{2} \cdot \frac{3}{8}$ (that is, $\frac{1}{2}$ of $\frac{3}{8}$)

Step 1: Draw a unit rectangle and divide it into 8 pieces vertically. Lightly shade 3 of those pieces. Label it $\frac{3}{8}$.



Step 2: Use a horizontal line and divide the unit rectangle in half. Darkly shade $\frac{1}{2}$ of $\frac{5}{8}$ and label it.



Step 3: Write a number sentence. $\frac{1}{2} \cdot \frac{3}{8} = \frac{3}{16}$

Example 2

a. $\frac{2}{3} \cdot \frac{1}{5} \Rightarrow \frac{2 \cdot 1}{3 \cdot 5} \Rightarrow \frac{2}{15}$

b. $\frac{3}{5} \cdot \frac{5}{9} \Rightarrow \frac{3 \cdot 5}{5 \cdot 9} \Rightarrow \frac{15}{45} \Rightarrow \frac{1}{3}$

Problems

Draw an area model for each of the following multiplication problems and write the answer.

1. $\frac{1}{3} \cdot \frac{5}{6}$ 2. $\frac{3}{4} \cdot \frac{2}{5}$ 3. $\frac{1}{3} \cdot \frac{4}{9}$

Use the rule for multiplying fractions to find each product. Simplify when possible.

4. $\frac{2}{3} \cdot \frac{2}{7}$ 5. $\frac{2}{3} \cdot \frac{2}{5}$ 6. $\frac{3}{7} \cdot \frac{2}{5}$ 7. $\frac{4}{9} \cdot \frac{1}{3}$ 8. $\frac{2}{3} \cdot \frac{5}{8}$

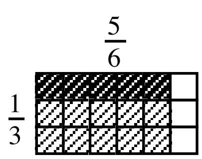
9. $\frac{5}{7} \cdot \frac{2}{5}$ 10. $\frac{4}{7} \cdot \frac{3}{4}$ 11. $\frac{5}{12} \cdot \frac{2}{3}$ 12. $\frac{4}{7} \cdot \frac{1}{2}$ 13. $\frac{5}{8} \cdot \frac{4}{5}$

14. $\frac{5}{9} \cdot \frac{3}{5}$ 15. $\frac{7}{10} \cdot \frac{2}{7}$ 16. $\frac{5}{12} \cdot \frac{6}{9}$ 17. $\frac{5}{6} \cdot \frac{3}{20}$ 18. $\frac{10}{13} \cdot \frac{3}{5}$

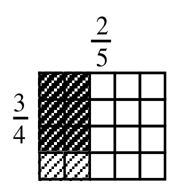
19. $\frac{7}{12} \cdot \frac{3}{7}$ 20. $\frac{7}{10} \cdot \frac{5}{14}$

Answers

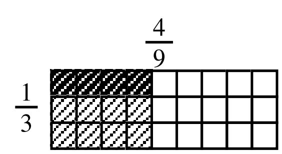
1. $\frac{5}{18}$



2. $\frac{6}{20} = \frac{3}{10}$



3. $\frac{4}{27}$



4. $\frac{4}{21}$ 5. $\frac{4}{15}$ 6. $\frac{6}{35}$ 7. $\frac{4}{27}$ 8. $\frac{10}{24} = \frac{5}{12}$

9. $\frac{10}{35} = \frac{2}{7}$ 10. $\frac{12}{28} = \frac{3}{7}$ 11. $\frac{10}{36} = \frac{5}{18}$ 12. $\frac{4}{14} = \frac{2}{7}$ 13. $\frac{20}{40} = \frac{1}{2}$

14. $\frac{15}{45} = \frac{1}{3}$ 15. $\frac{14}{70} = \frac{1}{5}$ 16. $\frac{30}{108} = \frac{5}{18}$ 17. $\frac{15}{120} = \frac{1}{8}$ 18. $\frac{30}{65} = \frac{6}{13}$

19. $\frac{21}{84} = \frac{1}{4}$ 20. $\frac{35}{140} = \frac{1}{4}$

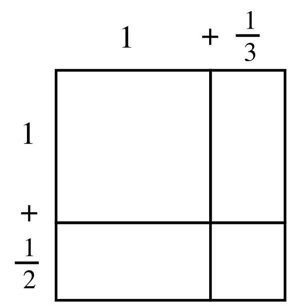
MULTIPLICATION OF MIXED NUMBERS

There are two ways to multiply mixed numbers. One is with generic rectangles. You can also multiply mixed numbers by changing them to fractions greater than 1, then multiplying the numerators and multiplying the denominators. Simplify if possible.

Example 1

Find the product: $1\frac{1}{2} \cdot 1\frac{1}{3}$

Step 1: Draw the generic rectangle. Label the top 1 plus $\frac{1}{3}$. Label the side 1 plus $\frac{1}{2}$.

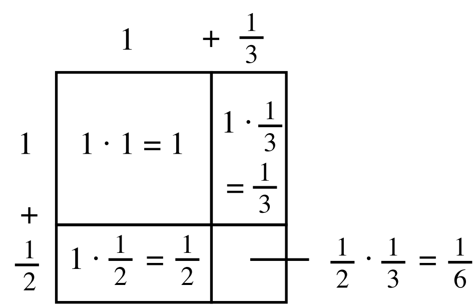


Step 2: Write the area of each smaller rectangle in each of the four parts of the drawing.

Find the total area:

$1+\frac{1}{3}+\frac{1}{2}+\frac{1}{6} \quad => \quad 1+\frac{2}{6}+\frac{3}{6}+\frac{1}{6}$

$=> \quad 1\frac{6}{6} \quad => \quad 2$

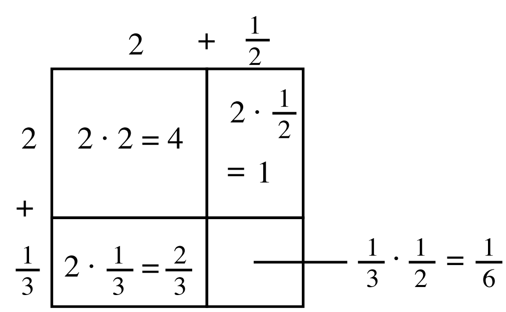


Step 3: Write a number sentence: $1\frac{1}{2} \cdot 1\frac{1}{3} = 2$

Example 2

Find the product: $2\frac{1}{3} \cdot 2\frac{1}{2}$

$4+1+\frac{2}{3}+\frac{1}{6} \quad => \quad 5+\frac{4}{6}+\frac{1}{6} \quad => \quad 5\frac{5}{6}$



Problems

Use a generic rectangle to find each product.

1. $2\frac{1}{4} \cdot 1\frac{1}{2}$ 2. $2\frac{1}{6} \cdot 2\frac{1}{3}$ 3. $2\frac{3}{4} \cdot 2\frac{1}{2}$ 4. $1\frac{1}{3} \cdot 2\frac{1}{6}$ 5. $2\frac{1}{2} \cdot 1\frac{1}{6}$

Answers

1. $\frac{27}{8}$ or $2\frac{3}{8}$ 2. $\frac{91}{18}$ or $5\frac{1}{18}$ 3. $\frac{55}{8}$ or $6\frac{7}{8}$ 4. $\frac{52}{18}$ or $2\frac{16}{18}$ or $2\frac{8}{9}$ 5. $\frac{35}{12}$ or $2\frac{11}{12}$

	2	$+ \frac{1}{4}$
1	2	$\frac{1}{4}$
$+ \frac{1}{2}$	1	$\frac{1}{8}$

	2	$+ \frac{1}{3}$
2	4	$\frac{2}{3}$
$+ \frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{18}$

	2	$+ \frac{1}{2}$
2	4	1
$+ \frac{3}{4}$	$\frac{6}{4}$	$\frac{3}{8}$

	2	$+ \frac{1}{6}$
1	2	$\frac{1}{6}$
$+ \frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{18}$

	1	$+ \frac{1}{6}$
2	2	$\frac{2}{6} = \frac{1}{3}$
$+ \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{12}$

Example 3

$$1\frac{1}{2} \cdot 1\frac{1}{3} \Rightarrow \frac{3}{2} \cdot \frac{4}{3} \Rightarrow \frac{3 \cdot 4}{2 \cdot 3} \Rightarrow \frac{12}{6} \Rightarrow 2$$

Problems

Find each product, using the method of your choice. Simplify when possible.

1. $2\frac{1}{4} \cdot 2\frac{5}{8}$ 2. $2\frac{1}{5} \cdot 2\frac{1}{5}$ 3. $1\frac{3}{8} \cdot 1\frac{5}{6}$ 4. $2\frac{5}{9} \cdot 2\frac{5}{6}$

5. $1\frac{2}{7} \cdot 1\frac{1}{7}$ 6. $2\frac{4}{7} \cdot 3\frac{8}{9}$ 7. $2\frac{3}{7} \cdot 1\frac{1}{12}$ 8. $2\frac{7}{9} \cdot 2\frac{4}{5}$

9. $2\frac{1}{3} \cdot 1\frac{2}{7}$ 10. $2\frac{2}{5} \cdot 2\frac{3}{10}$

Answers

1. $\frac{189}{32}$ or $5\frac{29}{32}$ 2. $\frac{121}{25}$ or $4\frac{21}{25}$ 3. $\frac{121}{48}$ or $2\frac{25}{48}$ 4. $\frac{391}{54}$ or $7\frac{13}{54}$

5. $\frac{72}{49}$ or $1\frac{23}{49}$ 6. 10 7. $\frac{221}{84}$ or $2\frac{53}{84}$ 8. $\frac{70}{9}$ or $7\frac{7}{9}$

9. 3 10. $\frac{138}{25}$ or $5\frac{13}{25}$

ORDER OF OPERATIONS #20

The **ORDER OF OPERATIONS** establishes the necessary rules so that expressions are evaluated in a consistent way by everyone.

A numerical term is a single number, or numbers multiplied together. A numerical expression is a combination of numbers and operation symbols such as $+$, $-$, $\cdot$, $\div$. Examples of numerical terms include 5, $6(3-1)$, $\frac{2+7}{4}$, 8^2, and $-\frac{1}{4}(2-5)$.

To evaluate an expression:

1. *Circle* the parentheses or any other grouped numbers, and then circle the terms inside the grouping. See note below.
2. *Simplify* each term in the parentheses.
3. *Circle* the terms in the expression.
4. *Simplify* each term until it is one number by:
   - Evaluating each exponential number (e.g., 7^2).
   - Multiplying and dividing from left to right.
5. Finally, *combine* terms by adding and subtracting left to right.

Note: In the example 4 below, $\frac{12-2}{5}$ is treated as $\frac{1}{5}(12-2)$, that is, a term with parentheses.

Example 1

- Circle the terms.
- Simplify each term until it is one number.
- Add the terms going from left to right.

$7+3\cdot 8$

$7 + 3\cdot 8$

$7+24$

31

Example 2

- Circle the terms.
- Simplify each term until it is one number.
  - Subtract 2 from 5.
  - Evaluate 2^2.
  - Multiply within each term, left to right.
  - Add the numbers.

$2^2\cdot 4+4(5-2)+7$

$2^2\cdot 4 + 4(5-2) + 7$

$4\cdot 4 + 4(3) + 7$

$16+12+7$

35

Example 3

- Circle the terms.
- Simplify each term until it is one number.
    - Evaluate 3^2 first.
    - Add 4 + 3 in the parentheses.
    - Multiply and divide left to right in each term.
    - Add and subtract the numbers from left to right.

$$7 - 9 \div 3^2 + 4(4+3) - 7$$
$$7 - 9 \div 3^2 + 4(4+3) - 7$$
$$7 - 9 \div 9 + 4(7) - 7$$
$$7 - 1 + 28 - 7$$
$$27$$

Example 4

- Circle the terms.
- Simplify each term until it is one number.
    - Subtract the numerator.
    - Evaluate 3^2.
    - Divide.
    - Add or subtract the numbers from left to right.

$$18 + \frac{12-2}{5} - 3^2 + 18 \div 6$$
$$18 + \frac{12-2}{5} - 3^2 + 18 \div 6$$
$$18 + \frac{10}{5} - 9 + 3$$
$$18 + 2 - 9 + 3$$
$$14$$

Problems

Circle the terms, then simplify each expression.

1. $7 \cdot 3 + 5$
2. $8 \div 4 + 3$
3. $2(12 - 4) + 4$
4. $4(9+3) + 10 \div 2$
5. $24 \div 3 + 7(9+1) - 4$
6. $\frac{12}{3} + 5 \cdot 4^2 - 2(12 - 5)$
7. $\frac{20}{3+2} + 9 \cdot 2 \div 3$
8. $\frac{4+24}{7} + 5^2 - 27 \div 9$
9. $3^2 + 8 - 16 \div 4^2 \cdot 2$
10. $16 - 4^2 + 4 - 2^2$
11. $5(19 - 3^2) + 5 \cdot 3 - 7$
12. $(6-2)^2 + (8+1)^2$
13. $4^2 + 8(2) \div 4 + (6-2)^2$
14. $\frac{16}{2^2} + \frac{7 \cdot 3}{7}$
15. $3(8 - 2)^2 + 10 \div 5 - 6 \cdot 5$
16. $18 \div 2 + 7 \cdot 8 \div 2 - (9-4)^2$
17. $\frac{24}{3} + 16 - 12 \div 3 - (3 + 5)^2$
18. $22 \cdot 2 \div 4 - (7 + 3)^2 + 3(7 - 2)^2$
19. $\left(\frac{22 + 3}{5}\right)^2 + 4^2 - (2 \cdot 3)^2$
20. $5^2 - \left(\frac{40 + 4}{4}\right)^2 + (3 \cdot 4)^2$

Answers

1.	26	2.	5	3.	20	4.	53
5.	74	6.	70	7.	10	8.	26
9.	9	10.	0	11.	58	12.	97
13.	36	14.	7	15.	80	16.	12
17.	–44	18.	–14	19.	5	20.	48

PERCENTS #21

PERCENTAGES AND PERCENT PROBLEMS are calculated and solved using a variety of methods depending on the situation. Mental math strategies, calculations with fractions or decimals, equivalent fractions (proportions), and linear models are all useful.

PERCENT OF A NUMBER

Students calculate a **percent of a number** using two methods. Knowing quick methods to calculate 10% of a number and 1% of a number will help you to calculate other percents **by composition**. Use the fact that $10\% = \frac{1}{10}$ and $1\% = \frac{1}{100}$. Direct computation is useful with more complicated expressions and they are calculated by first converting the percent into a decimal.

Example 1

To calculate 23% of 50, you can think of 2(10% of 50) + 3(1% of 50).

10% of 50 => $\frac{1}{10}$ of 50 = 5 and 1% of 50 => $\frac{1}{100}$ of 50 = 0.5 so
23% of 50 => 2(5) + 3(0.5) => 10 + 1.5 = 11.5

Example 2

To calculate 9% of 450, you can think of 10% of 450 – 1% of 450.

10% of 450 => $\frac{1}{10}$ of 450 = 45 and 1% of 450 => $\frac{1}{100}$ of 450 = 4.5 so
9% of 450 => 45 – 4.5 = 40.5

Other common percents such as $50\% = \frac{1}{2}$, $25\% = \frac{1}{4}$, $75\% = \frac{3}{4}$, $20\% = \frac{1}{5}$ may also be used.

Example 3

To calculate 17% of 123.4 convert the percent to a decimal and then direct computation is better.

17% of 123.4 => $\frac{17}{100}$(123.4) => 0.17(123.4) = 20.978

Example 4

To calculate 8.2% of $49.79 convert the percent to decimal and then direct computation is better.

8.2% of $49.70 => $\frac{8.2}{100}$(\$49.79) => 0.082(\$49.79) = \$4.08278 => \$4.08 (rounded)

Problems

Use composition to compute the values.

1.	12% of 23	2.	21% of 350	3.	13% of 45
4.	19% of 40	5.	49% of 320	6.	28% of 47
7.	32% of 120	8.	24% of 80	9.	19% of 4500

Compute by any method. Round to the nearest cent as appropriate.

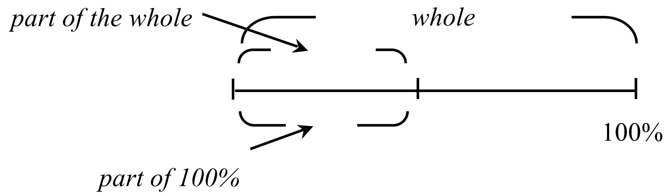
10.	17% of 125.8	11.	32% of 4.825	12.	125% of 49
13.	6.5% of 48	14.	5.2% of 8.7	15.	7% of $14.95
16.	9% of $32.95	17.	1.5% of $3200	18.	1.65% of $10,500
19.	120% of 67	20.	167% of 45.8	21.	15% of $24.96
22.	20% of 32	23.	2.5% of 1400	24.	3.6% of 2450

Answers

1.	2.76	2.	73.5	3.	5.85
4.	7.6	5.	156.8	6.	13.16
7.	38.4	8.	19.2	9.	855
10.	21.386	11.	1.544	12.	61.25
13.	3.12	14.	0.4524	15.	$1.05
16.	$2.97	17.	$48	18.	$173.25
19.	80.4	20.	76.486	21.	$3.74
22.	6.4	23.	35	24.	88.2

PERCENTAGE PROBLEMS DESCRIBED IN WORDS

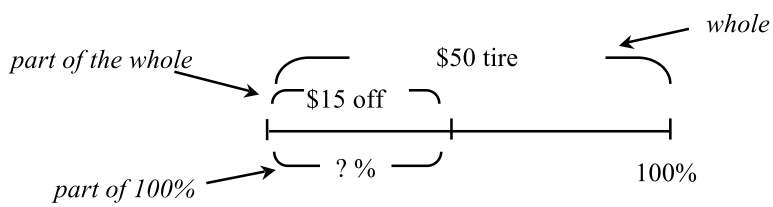
A variety of **percent problems described in words** involve the relationship between "the percent," "the part" and "the whole." When this is represented using a number line, solutions may be found using logical reasoning or equivalent fractions (proportions). These linear models might look like the diagram below:



Example 1

Sam's Discount Tires advertises a tire that originally cost \$50 on sale for \$35. What is the percent discount?

A possible diagram for this situation is shown at right:



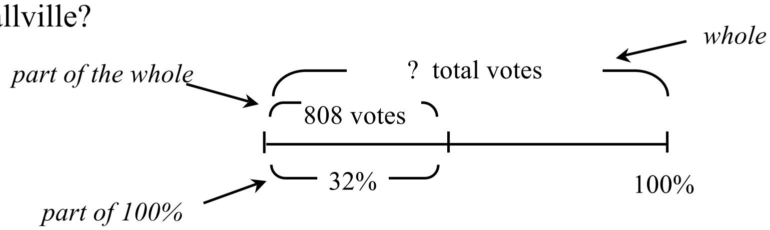
In this situation it is easy to reason that since the percent number total (100%) is twice the cost number total (\$50), the percent number saved is twice the cost number saved and is therefore a 30% discount. The problem could also be solved using a proportion:

$$\frac{15}{50} = \frac{x}{100} \Rightarrow 1500 = 50x \Rightarrow \frac{1500}{50} = x \Rightarrow x = 30$$

Example 2

Martin received 808 votes for mayor of Smallville. If this was 32% of the total votes cast, how many people voted for mayor of Smallville?

A possible diagram for this situation is shown at right:



In this case it is better to write a pair of equivalent fractions as a proportion: $\frac{808}{32} = \frac{x}{100}$

If using the Giant One, the multiplier is $\frac{100}{32} = 3.125$ so $\frac{808}{32} \cdot \frac{3.125}{3.125} = \frac{2525}{100}$.

A total of 2525 people voted for mayor of Smallville.

Note that the proportion in this problem could also be solved using cross-multiplication.

$$\frac{808}{32} = \frac{x}{100} \Rightarrow 808(100) = 32x \Rightarrow \frac{808(100)}{32} = x \Rightarrow 2525 = x$$

Problems

Use a diagram to solve each of the problems below.

1. Sarah's English test had 90 questions and she got 18 questions wrong. What percent of the questions did she get correct?
2. Cargo pants that regularly sell for $36 are now on sale for 30% off. How much is the discount?
3. The bill for a stay in a hotel was $188 including $15 tax. What percent of the bill was the tax?
4. Alicia got 60 questions correct on her science test. If she received a score of 75%, how many questions were on the test?
5. Basketball shoes are on sale for 22% off. What is the regular price if the sale price is $42?
6. Sergio got 80% on his math test. If he answered 24 questions correctly, how many questions were on the test
7. A $65 coat is now on sale for $52. What percent discount is given?
8. Ellen bought soccer shorts on sale for $6 off the regular price of $40. What percent did she save?
9. According to school rules, Carol has to convince 60% of her classmates to vote for her in order to be elected class president. There are 32 students in her class. How many students must she convince?
10. A sweater that regularly sold for $52 is now on sale at 30% off. What is the sale price?
11. Jody found an $88 pair of sandals marked 20% off. What is the dollar value of the discount?
12. Ly scored 90% on a test. If he answered 135 questions correctly, how many questions were on the test?
13. By the end of wrestling season, Mighty Max had lost seven pounds and now weighs 128 pounds. What was the percentage decrease from his starting weight?
14. George has 245 cards in his baseball card collection. Of these, 85 of the cards are pitchers. What percent of the cards are pitchers?
15. Julio bought soccer shoes at the 35% off sale and saved $42. What was the regular price of the shoes?

Answers

1. 80%	2. $10.80	3. about 8%
4. 80 questions	5. $53.85	6. 30 questions
7. 20%	8. 15%	9. 20 students
10. $36.40	11. $17.60	12. 150 questions
13. about 5%	14. about 35%	15. $120

PERCENT INCREASE OR DECREASE #22

Finding the percent increase or decrease can be done using proportions.

The proportion used to find percent is: $\frac{\%}{100} = \frac{\text{part}}{\text{whole}}$

For percent increase or decrease, the same concept is used. The proportion becomes: $\frac{\%}{100} = \frac{\text{change (increase or decrease)}}{\text{original amount}}$

Example 1

A town's population grew from 1,437 to 6,254 over five years. What was the percent increase?

- Subtract to find the change:

$$6{,}254 - 1437 = 4817$$

- Put the known numbers in the proportion:

$$\frac{\%}{100} = \frac{4817}{1437}\ \frac{\text{increase}}{\text{original}}$$

- The percent becomes x, the unknown:

$$\frac{x}{100} = \frac{4817}{1437}$$

- Cross multiply:

$$1437x = 481{,}700$$

- Divide each side by 1437:

$$x = 335.2\% \text{ increase}$$

Example 2

A sumo wrestler retired from sumo wrestling and went on a diet. When he retired he weighed 428 pounds. After two years he weighed 253 pounds. What was the percent decrease in his weight?

- Subtract to find the change:

$$428 - 253 = 175$$

- Put the known numbers in the proportion:

$$\frac{\%}{100} = \frac{175}{428}\ \frac{\text{decrease}}{\text{original}}$$

- The percent becomes x, the unknown:

$$\frac{x}{100} = \frac{175}{428}$$

- Cross multiply:

$$428x = 17{,}500$$

- Divide each side by 428:

$$x = 40.89\%.$$

His weight decreased by 40.89%.

Problems

Solve the following problems.

1. Forty years ago gasoline cost $0.30 per gallon. Today gasoline averages about $2.65 per gallon. What is the percent increase in the cost of gasoline?

2. When Spencer was 3, he was 26 inches tall. Today he is 5 feet 8 inches tall. What is the percent increase in Spencer's height?

3. The cars of the early 1900s cost $500. Today a new car costs an average of $28,500. What is the percent increase of the cost of an automobile?

4. The population of the U.S. at the first census in 1790 was 3,929,000 people. By 2000 the population had increased to 284,000,000! What is the percent increase in the population?

5. In 2002 the rate for a first class U.S. postage stamp increased to $0.37. This represents a $0.34 increase since 1917. What is the percent increase of postage since 1917?

6. In 1880, Americans consumed an average of 28.75 gallons of whole milk. By 1998 the average consumption was 8.32 gallons. What is the percent decrease in consumption of whole milk?

7. In 1980 there were 150 students for each computer in U.S. public schools. By 1998 there were 6.1 students for each computer. What is the percent decrease in the ratio of students to computers?

8. Sara bought a dress on sale for $28. She saved 40%. What was the original cost of the dress?

9. Pat was shopping and found a jacket with the original price of $120 on sale for $19.99. What was the percent decrease in the price of the jacket?

10. The price of a pair of pants decreased from $69.99 to $19.95. What was the percent decrease in the price of the pants?

Answers

1. $2.65 - 0.30 = 2.35$; $\frac{x}{100} = \frac{2.35}{0.30}$; $x = 743.3\%$

2. $68 - 26 = 42$; $\frac{x}{100} = \frac{42}{26}$; $x = 161.5\%$

3. $28{,}500 - 500 = 28{,}000$; $\frac{x}{100} = \frac{28000}{500}$; $x = 5600\%$

4. $284{,}000{,}000 - 3{,}929{,}000 = 280, 071{,}000$; $\frac{x}{100} = \frac{280071000}{3929000}$; $x = 7{,}128.3\%$

5. $0.37 - 0.34 = 0.03$; $\frac{x}{100} = \frac{0.34}{0.03}$; $x = 1133.3\%$

6. $28.75 - 8.32 = 20.43$; $\frac{x}{100} = \frac{20.43}{28.75}$; $x = 71.06\%$

7. $150 - 6.1 = 143.9$; $\frac{x}{100} = \frac{143.9}{150}$; $x = 95.93\%$

8. $100 - 40 = 60\%$; $\frac{60}{100} = \frac{28}{x}$; $x = \$46.67$

9. $120 - 19.99 = 100.01$; $\frac{x}{100} = \frac{100.01}{120}$; $x = 83.34\%$

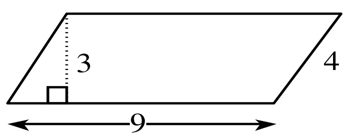
10. $69.99 - 19.95 = 50.04$; $\frac{x}{100} = \frac{50.04}{69.99}$; $x = 71.5\%$

PERIMETER #23

The **PERIMETER** of a polygon is the distance around the outside of the figure.
The perimeter is found by adding the lengths of all of the sides.

Example 1

Find the perimeter of the parallelogram below.

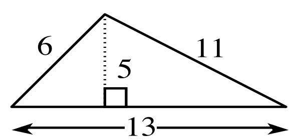


P = 9 + 4 + 9 + 4 = 26 units

(Parallelograms have opposite sides equal.)

Example 2

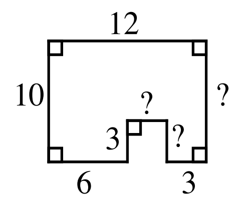
Find the perimeter of the triangle below.



P = 6 + 11 + 13 = 30 units

Example 3

Find the perimeter of the figure below.



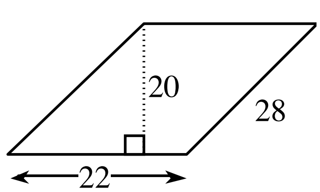
P = 10 + 12 + 10 + 3 + 3 + 3 + 3 + 6
= 50 units

(You need to look carefully to find the missing lengths of the sides.)

Problems

Find the perimeter of each shape.

1. A rectangle with base = 7 and height = 13
2. A square with sides of length 13
3. A parallelogram with base = 28 and side = 15
4. A triangle with sides 7, 10, and 12
5. A parallelogram



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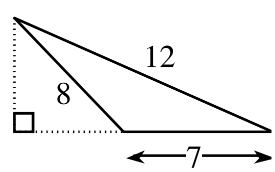

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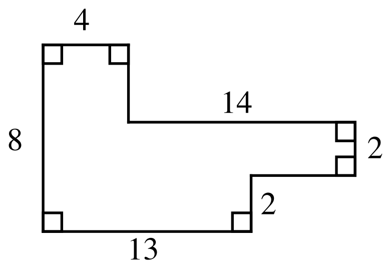

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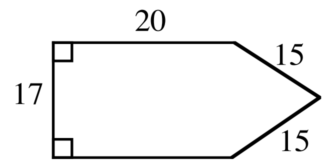

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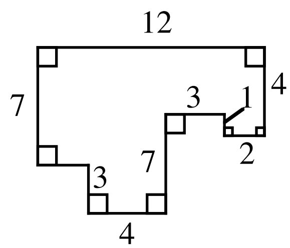
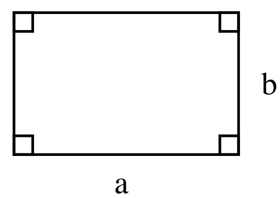


10.



Answers

1.	40 units	2.	52 units	3.	86 units	4.	29 units
5.	100 units	6.	27 units	7.	52 units	8.	87 units
9.	46 units	10.	a + b + a + b = 2a + 2b				

PROBABILITY #24

PROBABILITY is the likelihood that a specific outcome will occur, represented by a number between 0 and 1.

There are two categories of probability.

THEORETICAL PROBABILITY is calculated probability. If every outcome is equally likely, it is the ratio of outcomes in an event to all possible outcomes.

$$\text{Theoretical probability} = \frac{\text{number of outcomes in the specified event}}{\text{total number of possible outcomes}}$$

EXPERIMENTAL PROBABILITY is the probability based on data collected in experiments.

$$\text{Experimental probability} = \frac{\text{number of outcomes in the specified event}}{\text{total number of possible outcomes}}$$

Example 1

There are three pink pencils, two blue pencils, and one green pencil. If one pencil is picked randomly, what is the theoretical probability it will be blue?

- Find the total number of possible outcomes, that is, the total number of pencils. $3 + 2 + 1 = 6$
- Find the number of specified outcomes, that is, how many pencils are blue? 2
- Find the theoretical probability. P(blue pencil) = $\frac{2}{6} = \frac{1}{3}$ (You may reduce your answer.)

Example 2

Jayson rolled a die twelve times. He noticed that three of his rolls were fours.

a. What is the theoretical probability of rolling a four?

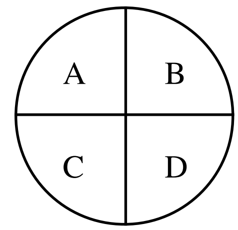
Because the six sides are equally likely and there is only one four, $P(4) = \frac{1}{6}$.

b. What is the experimental probability of rolling a four?

There were three fours in twelve rolls. The experimental probability is: $P(4) = \frac{3}{12} = \frac{1}{4}$.

Example 3

On the spinner, what is the probability of spinning an A or a B?

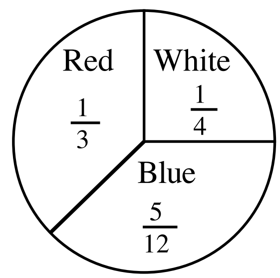


The probability of an A is $\frac{1}{4}$. The probability of a B is $\frac{1}{4}$. Add the two probabilities for the combined total.

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2} \quad P(\text{A or B}) = \frac{1}{2}$$

Example 4

What is the probability of spinning red or white?



- We know that $P(R) = \frac{1}{3}$ and $P(W) = \frac{1}{4}$.
- Add the probabilities together. $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

  $P(\text{R or W}) = \frac{7}{12}$

Problems

1. There are five balls in a bag: 2 red, 2 blue, and 1 white. What is the probability of randomly choosing a red ball?

2. In a standard deck of cards, what is the probability of drawing an ace?

3. A fair die numbered 1, 2, 3, 4, 5, 6 is rolled. What is the probability of rolling an odd number?

4. In the word "probability," what is the probability of selecting a vowel?

5. Anna has some coins in her purse: 5 quarters, 3 dimes, 2 nickels, and 4 pennies.

   a. What is the probability of selecting a quarter?

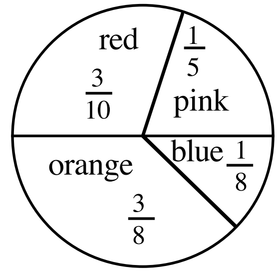
   b. What is the probability she will select a dime or a penny?

6. Tim has some gum drops in a bag: 20 red, 5 green, and 12 yellow.

   a. What is the probability of selecting a green?

   b. What is the probability of **not** selecting a red?

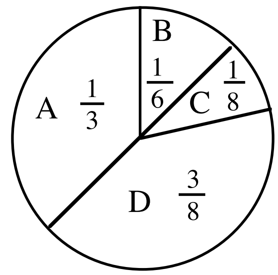
7. What is the probability of spinning:



a. Pink or blue? b. Orange or pink?

c. Red or orange? d. Red or blue?

8. What is the probability of spinning:



a. A or C? b. B or C?

c. A or D? d. B or D?

e. A, B, or C?

Answers

1. $\frac{2}{5}$

2. $\frac{4}{52} = \frac{1}{13}$

3. There are 3 odd numbers. $\frac{3}{6} = \frac{1}{2}$

4. $\frac{4}{11}$

5. a. $\frac{5}{14}$ b. $\frac{7}{14} = \frac{1}{2}$

6. a. $\frac{5}{37}$ b. $\frac{17}{37}$

7. a. $\frac{13}{40}$ b. $\frac{23}{40}$ c. $\frac{27}{40}$ d. $\frac{17}{40}$

8. a. $\frac{11}{24}$ b. $\frac{7}{24}$ c. $\frac{17}{24}$ d. $\frac{13}{24}$ e. $\frac{5}{8}$

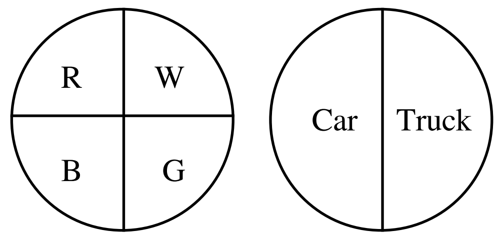
COMPOUND PROBABILITY

When multiple outcomes are specified, and either or several of them may happen but not both, find the probability of each specified outcome and add their probabilities.

If the desired outcome is a compound event, that is, it has more than one characteristic (as in Example 1, a red car), find the probability of each outcome (in Example 1, the probability of “red,” then the probability of “car”) and multiply the probabilities.

Example 1

To find the P(Red and Car):



- Find the probability of Red: $\frac{1}{4}$.
- Find the probability of Car: $\frac{1}{2}$.
- Multiply them together.
  $\frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$ P(Red and Car) = $\frac{1}{8}$

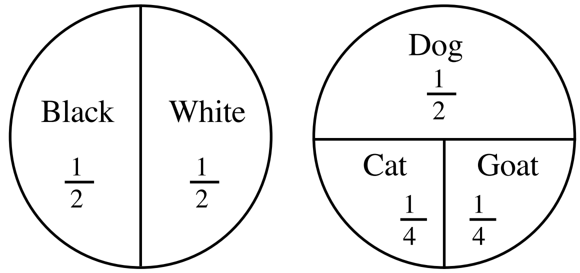
The solution can also be shown with a probability rectangle.

- There are four equally likely choices of color on the first spinner. The rectangle is divided vertically into four equal parts, each labeled with its probability and color.
- There are two equally likely choices of vehicle on the second spinner. The rectangle is divided horizontally into two equal parts, each labeled with its probability and vehicle.
- Write the probability of spinning each combination in its section of the rectangle, multiplying the probability to get the area of the rectangular subproblem as in a multiplication table.
- The P(Red and Car) = $\frac{1}{8}$.

	Red $\frac{1}{4}$	White $\frac{1}{4}$	Blue $\frac{1}{4}$	Green $\frac{1}{4}$
Car $\frac{1}{2}$	RC $\frac{1}{8}$	WC $\frac{1}{8}$	BC $\frac{1}{8}$	GC $\frac{1}{8}$
Truck $\frac{1}{2}$	RC $\frac{1}{8}$	WC $\frac{1}{8}$	BC $\frac{1}{8}$	GC $\frac{1}{8}$

Example 2

What is the probability of spinning a black dog or a black cat?



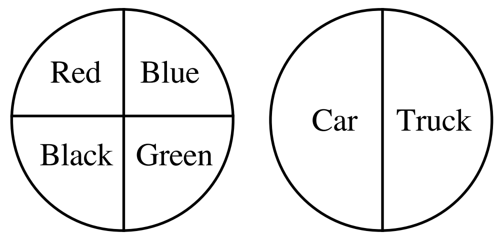
P(BCat) = $\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$

P(BDog) = $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

P(BCat or BDog) = $\frac{1}{8} + \frac{1}{4} = \frac{1}{8} + \frac{2}{8} = \frac{3}{8}$

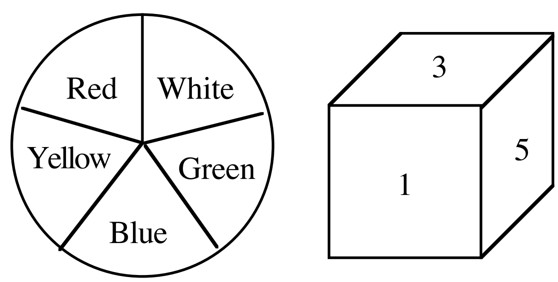
Problems

1. If each section in each spinner is the same size, what is the probability of getting a Black Truck?



2. Bipasha loves purple, pink, turquoise and black, and has a blouse in each color. She has two pairs of black pants and a pair of khaki pants. If she randomly chooses one blouse and one pair of pants, what is the probability she will wear a purple blouse with black pants?

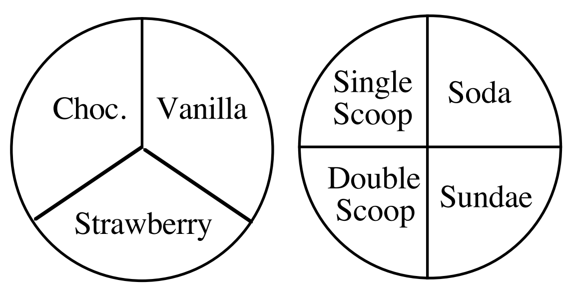
3. The spinner at right is divided into five equal regions. The die is numbered from 1 to 6. What is the probability of:



   a. Rolling a red 5?

   b. A white or blue even number?

4. What is the probability Joanne will win a:



   a. Chocolate double scoop?

   b. Chocolate or strawberry sundae?

   c. Chocolate double scoop or chocolate sundae?

5. Jay is looking in his closet, trying to decide what to wear. He has 2 red t-shirts, 2 black t-shirts, and 3 white t-shirts. He has 3 pairs of blue jeans and 2 pairs of black pants.

   a. What is the probability of his randomly choosing a red shirt with jeans?

   b. What is the probability of his randomly choosing an all black outfit?

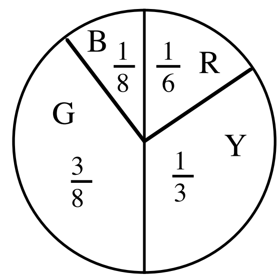
   c. Which combination out of all his possible choices has the greatest probability of being randomly picked? How can you tell?

6. What is the probability of spinning a:

   a. Red or Green?

   b. Blue or Yellow?

   c. Yellow or Green?



Answers

1. $\frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$

2. $\frac{1}{4} \cdot \frac{2}{3} = \frac{2}{12} = \frac{1}{6}$

3. a. $\frac{1}{5} \cdot \frac{1}{6} = \frac{1}{30}$

   b. $(\frac{1}{5} + \frac{1}{5}) \cdot \frac{3}{6} = \frac{1}{5}$

4. a. $\frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$

   b. $(\frac{1}{3} + \frac{1}{3}) \cdot \frac{1}{4} = \frac{1}{6}$

   c. $\frac{1}{3} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{6}$

5. a. $\frac{2}{7} \cdot \frac{3}{5} = \frac{6}{35}$

   b. $\frac{2}{7} \cdot \frac{2}{5} = \frac{4}{35}$

   c. White shirt, blue jeans; most selections in both categories

6. a. $\frac{13}{24}$ c. $\frac{17}{24}$

   b. $\frac{11}{24}$

PROBABILITY: DEPENDENT AND INDEPENDENT EVENTS

Two events are **DEPENDENT** if the outcome if the first event affects the outcome of the second event. For example, if you draw a card from a deck and do not replace it for the next draw, the two events – drawing one card without replacing it, then drawing a second card – are dependent.

Two events are **INDEPENDENT** if the outcome of the first event does not affect the outcome of the second event. For example, if you draw a card from a deck but replace it before you draw again, the two events are independent.

Example 1

Aiden pulls an ace from a deck of regular playing cards. He does not replace the card. What is the probability of pulling out a second ace?

First draw: $\frac{4}{52}$ Second draw: $\frac{3}{51}$ $\begin{array}{l}\text{aces left}\\ \text{cards left to pull from}\end{array}$

This is an example of a dependent event – the probability of the second draw has changed.

Example 2

Jayson was tossing coins. He tossed a head. What is the probability of tossing a second head?

It is still $\frac{1}{2}$. The probability for the second event has not changed. This is an independent event.

Problems

1. You throw a die twice. What is the probability of throwing a six and then a second six? Is this an independent or dependent event?

2. You have a bag of candy filled with pieces which are all the same size and shape. Four are gumballs and six are sweet and sours. You draw a gumball out, decide you don't like it, put it back, and select another piece of candy. What is the probability of selecting another gumball? Are these independent or dependent events?

3. Joey has a box of blocks with eight alphabet blocks and four plain red blocks. He gave an alphabet block to his sister. What is the probability his next selection will be another alphabet block? Are these independent or dependent events?

4. In your pocket you have three nickels and two dimes.

   a. What is the probability of selecting a nickel?

   b. What is the probability of selecting a dime?

   c. If you select a nickel and place it on a table, what is the probability the next coin selected is a dime? Is this an independent or dependent event?

   d. If all the coins are back in your pocket, what is the probability that the next coin you take out is a dime? Is this an independent or dependent event?

5. How do you tell the difference between dependent and independent events?

Answers

1. $\frac{1}{36}$, independent (the probability does not change)

2. $\frac{4}{10}$ or $\frac{2}{5}$, independent

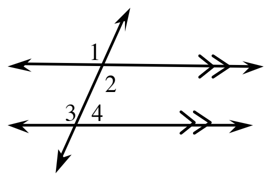
3. $\frac{7}{11}$, dependent

4. a. $\frac{3}{5}$ b. $\frac{2}{5}$ c. $\frac{2}{4}$, dependent d. $\frac{2}{5}$, independent

5. In dependent events, the second probability changes because there was no replacement.

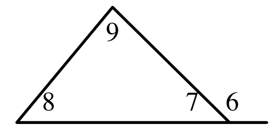
PROPERTIES OF ANGLES, LINES, AND TRIANGLES #25

Parallel lines



- corresponding angles are equal: $m\angle 1 = m\angle 3$
- alternate interior angles are equal: $m\angle 2 = m\angle 3$
- $m\angle 2 + m\angle 4 = 180^\circ$

Triangles

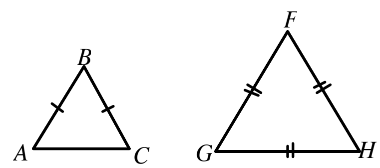


- $m\angle 7 + m\angle 8 + m\angle 9 = 180^\circ$
- $m\angle 6 = m\angle 8 + m\angle 9$
  (exterior angle = sum remote interior angles)
- $m\angle 10 + m\angle 11 = 90^\circ$
  (complementary angles)

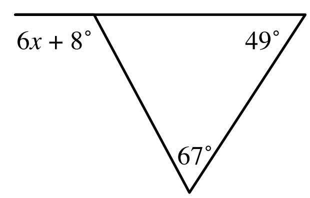
Also shown in the above figures:

- vertical angles are equal: $m\angle 1 = m\angle 2$
- linear pairs are supplementary: $m\angle 3 + m\angle 4 = 180^\circ$ and $m\angle 6 + m\angle 7 = 180^\circ$

In addition, an isosceles triangle, ΔABC, has $\overline{BA} = \overline{BC}$ and $m\angle A = m\angle C$. An equilateral triangle, ΔGFH, has $\overline{GF} = \overline{FH} = \overline{HG}$ and $m\angle \mathrm{G} = m\angle \mathrm{F} = m\angle \mathrm{H} = 60^\circ$.

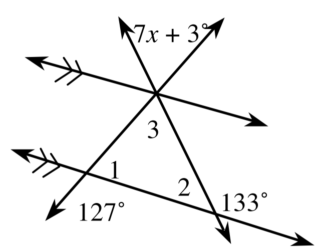


Example 1



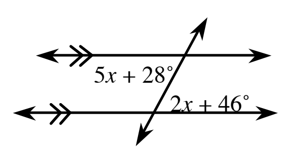
Solve for x.
Use the Exterior Angle Theorem: $6x + 8^\circ = 49^\circ + 67^\circ$
$6x^\circ = 108^\circ \Rightarrow x = \frac{108^\circ}{6} \Rightarrow x = 18^\circ$

Example 2



Solve for x.
There are a number of relationships in this diagram. First, Đ1 and the 127° angle are supplementary, so we know that $m\angle 1 + 127^\circ = 180^\circ$ so $m\angle 1 = 53^\circ$. Using the same idea, $m\angle 2 = 47^\circ$. Next, $m\angle 3 + 53^\circ + 47^\circ = 180^\circ$, so $m\angle 3 = 80^\circ$. Because angle 3 forms a vertical pair with the angle marked $7x + 3^\circ$, $80^\circ = 7x + 3^\circ$, so $x = 11^\circ$.

Example 3

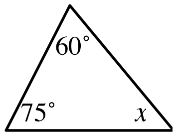


Find the measure of the acute alternate interior angles.
Parallel lines mean that alternate interior angles are equal, so $5x + 28^\circ = 2x + 46^\circ \Rightarrow 3x = 18^\circ \Rightarrow x = 6^\circ$. Use either algebraic angle measure: $2(6^\circ) + 46^\circ = 58^\circ$ for the measure of the acute angle.

Problems

Use the geometric properties you have learned to solve for x in each diagram and write the property you use in each case.

1.



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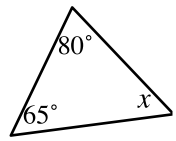

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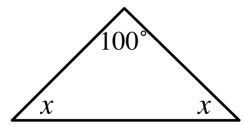

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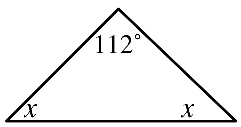

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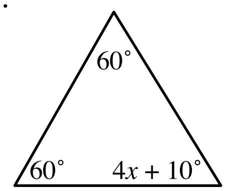

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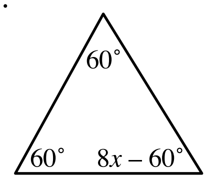

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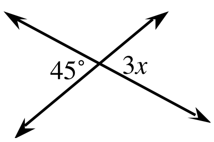

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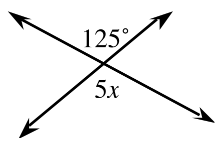

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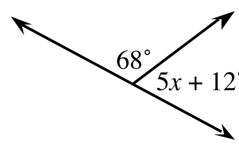

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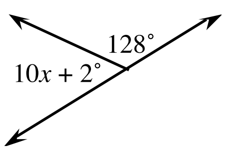

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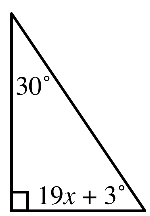

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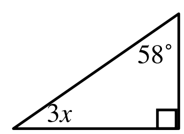

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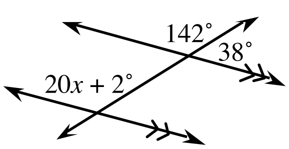

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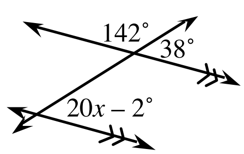

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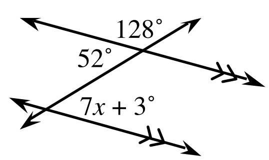

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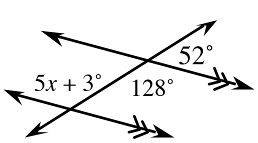

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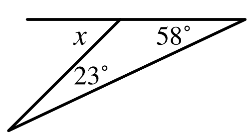

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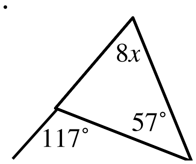

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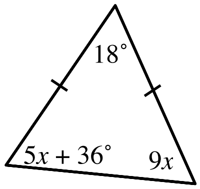

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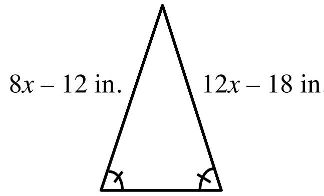

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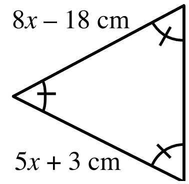

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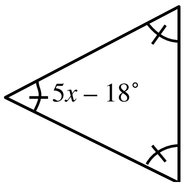

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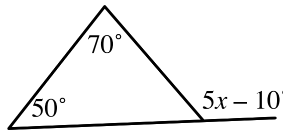

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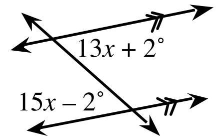

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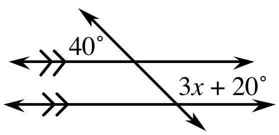

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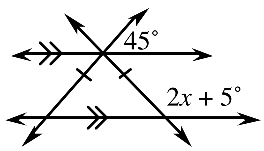

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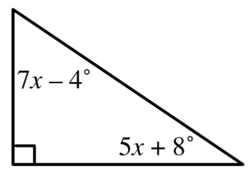

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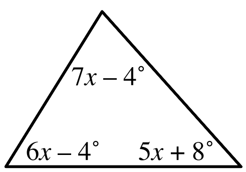


Answers

1.	45°	2.	35°	3.	40°	4.	34°	5.	12.5°	6.	15°
7.	15°	8.	25°	9.	20°	10.	5°	11.	3°	12.	$10\frac{2}{3}$°
13.	7°	14.	2°	15.	7°	16.	25°	17.	81°	18.	7.5°
19.	9°	20.	7.5°	21.	7°	22.	15.6°	23.	26°	24.	2°
25.	40°	26.	65°	27.	$7\frac{1}{6}$°	28.	10°				

PROPORTIONS WEB:
RATES, UNIT RATES, AND PROPORTIONAL GROWTH # 26

A rate is a ratio comparing two quantities and a unit rate has a denominator of one after simplifying. Unit rates or proportions (ratio equations) may be used to solve ratio problems. Situations of proportional growth are represented as lines passing through the origin. Solutions may be approximated by looking at the graph or by using the graph to calculate the rate and write an equation that represents the situation.

Example 1

The average cost of a pair of designer jeans has increased \$15 inches in 4 years. What is the unit growth rate (dollars per year)?

Solution: The growth rate is $\frac{15 \text{ dollars}}{4 \text{ years}}$. To create a unit rate we need a denominator of "one."
$\frac{15 \text{ dollars}}{4 \text{ years}} = \frac{x \text{ dollars}}{1 \text{ year}}$. Using a Giant One: $\frac{15 \text{ dollars}}{4 \text{ years}} = \frac{4}{4} \cdot \frac{x \text{ dollars}}{1 \text{ year}} \Rightarrow 3.75 \frac{\text{dollars}}{\text{year}}$.

Example 2

Ryan's famous chili recipe use 3 tablespoons of chili powder for 5 servings. How many tablespoons are needed for the family reunion needing 40 servings?

Solution: The rate is $\frac{3 \text{ tablespoons}}{5 \text{ servings}}$ so the problem may be written as a proportion: $\frac{3}{5} = \frac{t}{40}$.

One method of solving the proportion is to use the Giant One:

$$\frac{3}{5} = \frac{t}{40} \Rightarrow \frac{3}{5} \cdot \frac{8}{8} = \frac{24}{40} \Rightarrow t = 24$$

Another method is to use **cross multiplication:**

$$\frac{3}{5} = \frac{t}{40}$$
$$\frac{3}{5} = \frac{t}{40}$$
$$5 \cdot t = 3 \cdot 40$$
$$5t = 120$$
$$t = 24$$

Finally, since the unit rate is $\frac{3}{5}$ tablespoon per serving, the equation $t = \frac{3}{5}s$ represents the general proportional situation and one could substitute the number of servings needed into the equation: $t = \frac{3}{5} \cdot 40 = 24$. Using any method the answer is 24 tablespoons.

Example 3

Based on the table, what is the unit growth rate (meters per year)?

+2

height (m)	15	17
years	75	85

+10

Solution:

$$\frac{2\ \text{meters}}{10\ \text{years}} = \frac{2\ \text{meters}\cdot\frac{1}{10}}{10\ \text{years}\cdot\frac{1}{10}} = \frac{\frac{1}{5}\ \text{meter}}{1\ \text{year}} = \frac{1}{5}\ \frac{\text{meter}}{\text{year}}$$

Problems

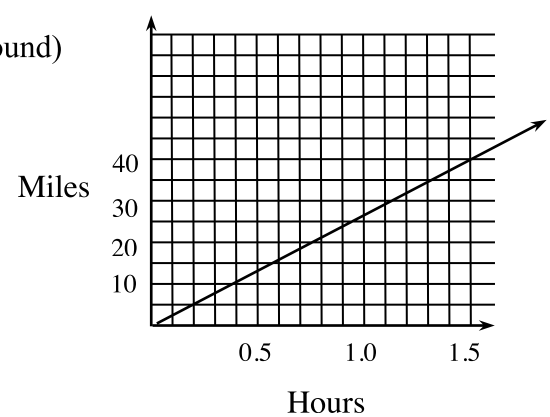
In problems 1 through 10 find the unit rate and in problems 11 through 25, solve each problem.

1. Typing 731 words in 17 minutes (words per minute)
2. Reading 258 pages in 86 minutes (pages per minute)
3. Buying 15 boxes of cereal for \$43.35 (cost per box)
4. Scoring 98 points in a 40 minute game (points per minute)
5. Buying $2\frac{1}{4}$ pounds of bananas cost \$1.89 (cost per pound)
6. Buying $\frac{2}{3}$ pounds of peanuts for \$2.25 (cost per pound)
7. Mowing $1\frac{1}{2}$ acres of lawn in $\frac{3}{4}$ of a hour (acres per hour)
8. Paying \$3.89 for 1.7 pounds of chicken (cost per pound)
9.

weight (g)	6	8	12	20
length (cm)	15	20	30	50

What is the weight per cm?

10. From the graph, what is the rate in miles per hour?



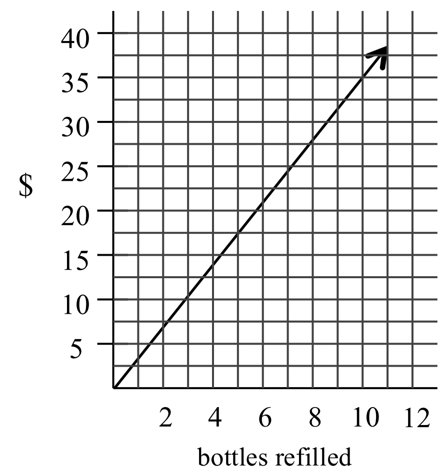
11. If a box of 100 pencils costs \$4.75, what should you expect to pay for 225 pencils?
12. When Amber does her math homework, she finishes 10 problems every 7 minutes. How long will it take for her to complete 35 problems?

13. Ben and his friends are having a TV marathon, and after 4 hours they have watched 5 episodes of the show. About how long will it take to complete the season, which has 24 episodes?

14. The tax on a $600 vase is $54. What should be the tax on a $1700 vase?

15. Use the table below to determine how it will take the Spirit club to wax 60 cars.

cars waxed	8	16	32
hours	3	6	12



16. While baking, Evan discovered a recipe that required $\frac{1}{2}$ cups of walnuts for every $2\frac{1}{4}$ cups of flour. How many cups of walnuts will he need for 4 cups of flour?

17. Based on the graph, what would the cost to refill 50 bottles?

18. Sam grew $1\frac{3}{4}$ inches in $4\frac{1}{2}$ months. How much should he grow in one year?

19. On his afternoon jog, Chris took 42 minutes to run $3\frac{3}{4}$ miles. How many miles can he run in 60 minutes?

20. If Caitlin needs $1\frac{1}{3}$ cans of paint for each room in her house, how many cans of paint will she need to paint the 7-room house?

21. Stephen receives 20 minutes of video game time every 45 minutes of dog walking he does. If he wants 90 minutes of game time, how many hours will he need to work?

22. Sarah's grape vine grew 15 inches in six weeks, write an equation to represent its growth after t weeks.

23. On average Max makes 45 out of 60 shots with the basketball, write an equation to represent the average number of shots made out of x attempts.

24. Write an equation to represent the situation in problem 14 above.

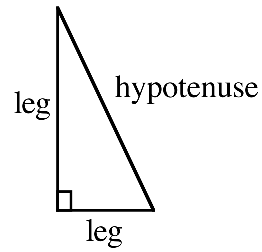
25. Write an equation to represent the situation in problem 17 above.

Answers

1. $43 \frac{\text{words}}{\text{minute}}$
2. $3 \frac{\text{pages}}{\text{minute}}$
3. $2.89 \frac{\$}{\text{box}}$
4. $2.45 \frac{\text{points}}{\text{minute}}$
5. $0.84 \frac{\$}{\text{pound}}$
6. $3.38 \frac{\$}{\text{pound}}$
7. $2 \frac{\text{acre}}{\text{hour}}$
8. $2.89 \frac{\$}{\text{pound}}$
9. $\frac{3}{5} \frac{\text{grams}}{\text{centimeter}}$
10. $\approx 27 \frac{\text{miles}}{\text{hour}}$
11. $10.69
12. 24.5 minutes
13. 19.2 hours
14. $153
15. 22.5 hours
16. $\frac{8}{9}$ cup
17. $175
18. $4\frac{2}{3}$ inches
19. ≈ 5.36 miles
20. $9\frac{1}{3}$ cans
21. $3\frac{3}{8}$ hours
22. $g = \frac{5}{2} t$
23. $s = \frac{3}{4} x$
24. $t = 0.09c$
25. $c = 3.5b$

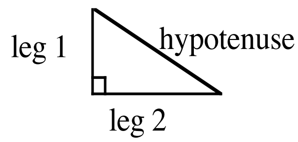
THE PYTHAGOREAN THEOREM	#27

A **right triangle** is a triangle in which the two shorter sides form a right angle. The shorter sides are called **legs**. Opposite the right angle is the third and longest side called the **hypotenuse**.



The **Pythagorean theorem** states that for any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

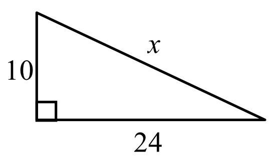
(leg 1)^2 + (leg 2)^2 = (hypotenuse)^2



Example 1

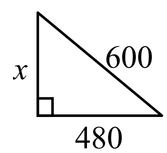
Use the Pythagorean theorem to find x.

a.



$$10^2 + 24^2 = x^2$$
$$100 + 576 = x^2$$
$$676 = x^2$$
$$26 = x$$

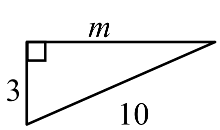
b.



$$x^2 + 480^2 = 600^2$$
$$x^2 + 230,400 = 360,000$$
$$x^2 = 129,600$$
$$x = 360$$

Example 2

Not all problems will have exact answers. Use square root notation and your calculator. Round your answers to the nearest hundredth.



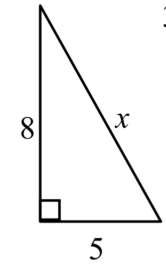
$$3^2 + m^2 = 10^2$$
$$9 + m^2 = 100$$
$$m^2 = 91$$
$$m = \sqrt{91} \approx 9.54$$

Example 3

A guy wire is needed to support a tower. The wire is attached to the ground five meters from the base of the tower. How long is the wire if the tower is 8 meters tall?

1. First draw a diagram to model the problem, then write an equation using the Pythagorean theorem and solve it.

2.

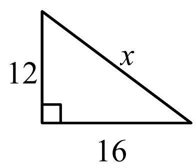


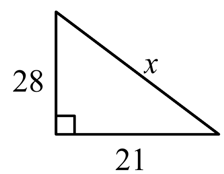
3.

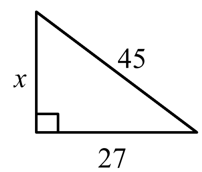
$$x^2 = 8^2 + 5^2$$
$$x^2 = 64 + 25$$
$$x^2 = 89$$
$$x = \sqrt{89} \approx 9.43$$

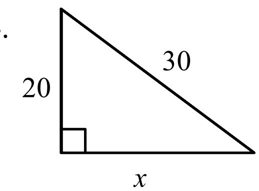
Problems

Write an equation and solve for each unknown side. Round to the nearest hundredth.

1.


2.


3.


4.


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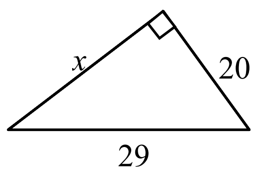
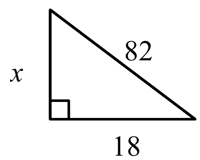
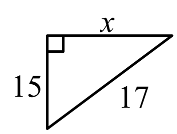


6.


7.


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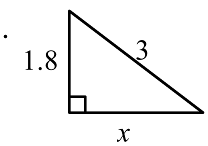
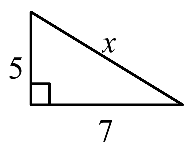
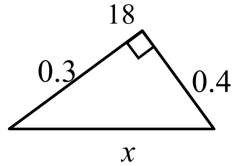
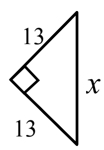
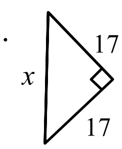
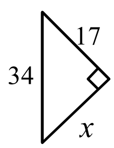


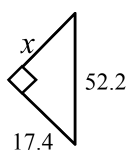
9.


10.


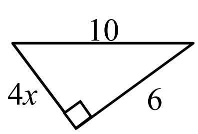
11.


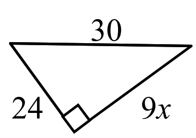
12.


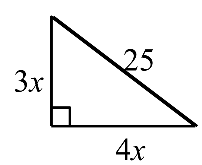
13.


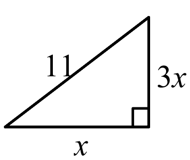
14.


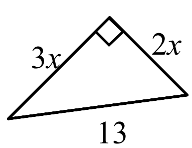
Be careful! Remember to square the whole side. For example, $(2x)^2 = (2x)(2x) = 4x^2$.

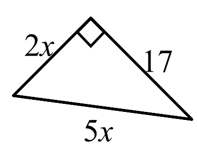
15.


16.


17.


18.


19.


20.


For each of the following problems draw and label a diagram. Then write an equation using the Pythagorean theorem and solve for the unknown. Round your answers to the nearest hundredth.

21. In a right triangle, the length of the hypotenuse is 13 inches. The length of one leg is 5 inches. Find the length of the other leg.

22. The length of the hypotenuse of a right triangle is 9 cm. The length of one leg is four cm. Find the length of the other leg.

23. Find the diagonal length of a television screen 20 inches wide by 17 inches long.

24. Find the length of a path that runs diagonally across a 75-yard by 120-yard soccer field.

25. A surveyor walked two miles north, then three miles west. How far was she from her starting point? Note: This question is different from the question, "How far did she walk?"

26. A 2.8 meter ladder is one meter from the base of a building. How high up the side of the building will the ladder reach?

27. What is the longest line you can draw on a paper that is 8.5 inches by 11 inches?

28. What is the longest length of an umbrella that will lay flat in the bottom of a backpack that is 12 inches by 17 inches?

29. Find the diagonal distance from one corner of a square classroom floor to the other corner of the floor if the length of the floor is 31 feet.

30. Mary can turn off her car alarm from up to15 yards away. Will she be able to do it from the far corner of a 15-yard by 12-yard parking lot if her car is parked in the diagonally opposite corner from where she is standing?

Answers

1. $x = 20$	2. $x = 35$	3. $x = 36$	4. $x \approx 22.36$
5. $x = 21$	6. $x = 80$	7. $x = 8$	8. $x = 2.4$
9. $x \approx 8.60$	10. $x = 0.5$	11. $x \approx 18.38$	12. $x \approx 24.04$
13. $x \approx 29.44$	14. $x \approx 49.21$	15. $x = 2$	16. $x = 2$
17. $x = 5$	18. $x \approx 3.48$	19. $x \approx 3.61$	20. $x \approx 3.71$
21. 12 inches	22. 8.06 cm	23. 26.25 inches	24. 141.51 yards
25. 3.61 mi	26. 2.62 m	27. 13.90 inches	28. 20.81 inches
29. 43.84 feet	30. The corner is 19.21 yards away, so no!		

RATIO #28

A **RATIO** is a comparison of two amounts. It can be written in several ways:

$\frac{65 \text{ miles}}{1 \text{ hour}}$, 65 miles : 1 hour, or 65 miles to 1 hour.

Both quantities of a ratio (numerator and denominator) can be multiplied by the same number. We can use a **ratio table** to organize the multiples. Each ratio in the table will be equivalent to the others. Patterns in the ratio table can be used in problem solving.

Example 1

130 cups of coffee can be made from one pound of coffee beans. Doubling the amount of coffee beans will double the number of cups of coffee that can be made. Use the doubling pattern to complete the ratio table for different weights of coffee beans.

Pounds of coffee beans	1	2	4	8
Cups of coffee	130	260		

Doubling two pounds of beans doubles the number of cups of coffee made, so for 4 pounds of beans, $2 \cdot 260 = 520$ cups of coffee are made. Since 4 pounds of beans make 520 cups of coffee, 8 pounds of beans make $2 \cdot 520 = 1040$ cups of coffee.

Example 2

You can use the ratio table from Example 1 to determine how many cups of coffee you could make from 6 pounds of beans. Add another column to your ratio table.

Pounds of coffee beans	1	2	4	8	6
Cups of coffee	130	260	520	1040	

You know that the value of 6 is halfway between the values of 4 and 8. The value halfway between 520 and 1040 is 780, so 6 pounds of beans should make 780 cups of coffee.

Example 3

Jane's cookie recipe uses $3\frac{1}{2}$ cups of flour and 2 eggs. She needs to know how much flour she will need if she uses 9 eggs. Jane started a ratio table but discovered that she could not simply keep doubling because 9 is not a multiple of 2. She used other patterns to find her answer. Study the top row of Jane's table to find the patterns she used, then complete the table.

Number of eggs	2	4	12	36	9
Cups of flour	$3\frac{1}{2}$				

Example continues on next page. →

Example continued from previous page.

Jane doubled to get 4, tripled 4 to get 12, and tripled again to get 36. Then she divided 36 by four to get to 9. She followed the same steps to complete the "cups of flour" row.
Her last step was to divide 63 by 4. Jane will need $15\frac{3}{4}$ cups of flour.

Number of eggs	2	4	12	36	9
Cups of flour	$3\frac{1}{2}$	7	21	63	$15\frac{3}{4}$

In the examples above, the ratio tables have the ratios listed in the order in which they were calculated. When a row of a ratio table is provided, it is acceptable to complete a ratio table in the order that fits the patterns you see and use. In most cases it is easier to see patterns by listing the values in the first (or top) row in numerical order.

Problems

Complete each ratio table.

1.

2	4	6	8	10	12	16	20
6	12						

2.

3	6	9	12	15	18	21	30
5	10						

3.

5			20	30	45	60	100
3	6	9					

4.

7	14	35		49		700	
2			20		50		500

5.

4	8	16	12		32		80
11				66		99	

6.

6	12	18	30			360	
4.5				45	90		540

7.

9	36	45			180	900	300
34			85	340			

8.

10	15	20	25		35		4000
6.5				26		260	

Answers

1.

2	4	6	8	10	12	16	20
6	12	18	24	30	36	48	60

2.

3	6	9	12	15	18	21	30
5	10	15	20	25	30	35	50

3.

5	10	15	20	30	45	60	100
3	6	9	12	18	27	36	60

4.

7	14	35	70	49	175	700	1750
2	4	10	20	14	50	200	500

5.

4	8	16	12	24	32	36	80
11	22	44	33	66	88	99	220

6.

6	12	18	30	60	120	360	720
4.5	9	13.5	22.5	45	90	270	540

7.

9	36	45	22.5	90	180	900	300
34	136	170	85	340	680	3400	1133.3

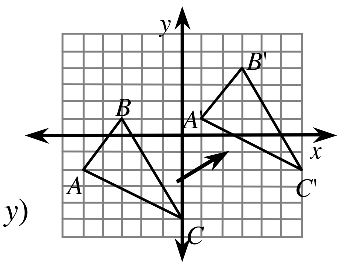
8.

10	15	20	25	40	35	400	4000
6.5	9.75	13	16.25	26	22.75	260	2600

RIGID TRANSFORMATIONS **#29**

A **RIGID TRANSFOMATION** is a transformation of a figure that preserves size and shape. The three kinds of rigid transformations studied are translation (slide), reflection (flip), and rotation (turn). These transformations may also be combined together. The original figure is called the pre-image and the new figure is called the image. For ΔABC it is common to say that the original points A, B, and C are mapped to the new points A', B', and C' respectively.

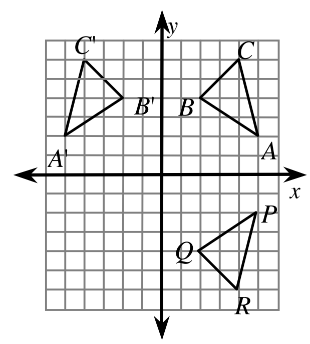
Example 1



Translate (slide) ΔABC right six units and up three units. Give the coordinates of the image triangle.

The original vertices are $A(-5,-2)$, $B(-3,1)$, and $C(0,-5)$. The new vertices are $A'\ (1,1)$, $B'(3,4)$, and $C'(6,-2)$. Notice that each point (x, y) is mapped to $(x+6, y+3)$.

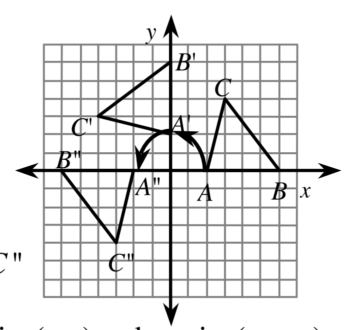
Example 2



Reflect (flip) ΔABC with coordinates $A(5,2)$, $B(2,4)$, and $C(4,6)$ across the y-axis to get $\Delta A'B'C'$. The key is that the reflection is the same distance from the axis as the original figure. The new points are $A'(-5,2)$, $B'(-2,4)$, and $C'(-4,6)$. Notice that in reflecting across the y-axis, each point (x, y) is mapped to the point $(-x, y)$.

If you reflect ΔABC across the x-axis to get ΔPQR, then the new points are $P(5,-2)$, $Q(2,-4)$, and $R(4,-6)$. In this case, reflecting across the x-axis, each point (x, y) is mapped to the point $(x,-y)$.

Example 3



Rotate (turn) ΔABC with coordinates $A(2,0)$, $B(6,0)$, and $C(3,4)$ 90º counterclockwise about the origin $(0,0)$ to get $\Delta A'B'C'$ with coordinates $A'(0,2)$, $B'(0,6)$, and $C'(-4,3)$. Notice that this 90° counterclockwise rotation about the origin maps each point (x, y) to the point $(-y, x)$.

Rotating another 90° (180° from the starting location) yields $\Delta A''B''C''$ with coordinates $A''(-2,0)$, $B''\ (-6,0)$, and C'' $(-3,-4)$. This 180° counterclockwise rotation about the origin maps each point (x, y) to the point $(-x,-y)$. Similarly a 270° counterclockwise or 90° clockwise rotation about the origin maps each point (x, y) to the point $(y,-x)$.

For the following problems, refer to the figures below:

Figure A

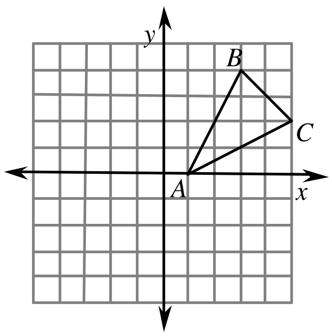


Figure B

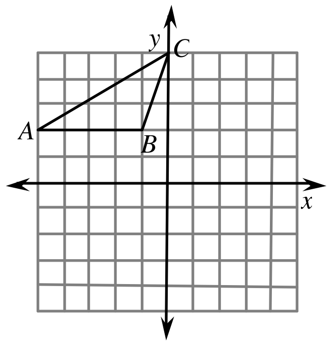
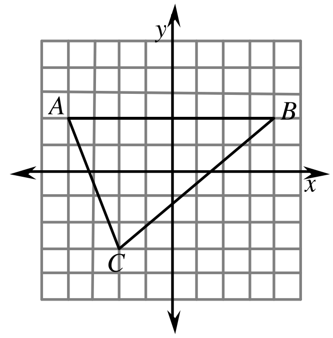


Figure C



Tell the new coordinates after each transformation.

1. Slide figure A left 2 units and down 3 units.
2. Slide figure B right 3 units and down 5 units.
3. Slide figure C left 1 unit and up 2 units.
4. Flip figure A across the x-axis.
5. Flip figure B across the x-axis.
6. Flip figure C across the x-axis.
7. Flip figure A across the y-axis.
8. Flip figure B across the y-axis.
9. Flip figure C across the y-axis.
10. Rotate figure A $90°$ counterclockwise about the origin.
11. Rotate figure B $90°$ counterclockwise about the origin.
12. Rotate figure C $90°$ counterclockwise about the origin.
13. Rotate figure A $180°$ counterclockwise about the origin.
14. Rotate figure C $180°$ counterclockwise about the origin.
15. Rotate figure B $270°$ counterclockwise about the origin.
16. Rotate figure C $90°$ clockwise about the origin.

Answers (given in order A', B', C')

1. (–1, –3) (1, 2) (3, –1)
2. (–2, –3) (2, –3) (3, 0)
3. (–5, 4) (3, 4) (–3, –1)
4. (1, 0) (3, –4) (5, –2)
5. (–5, –2) (–1, –2) (0, –5)
6. (–4, –2) (4, –2) (–2, 3)
7. (–1, 0) (–3, 4) (–5, 2)
8. (5, 2) (1, 2) (0, 5)
9. (4, 2) (–4, 2) (2, –3)
10. (0, 1) (–4, 3) (–2, 5)
11. (–2, –5) (–5, 0) (–2, –1)
12. (–2, –4) (–2, 4) (3, –2)
13. (–1, 0) (–3, –4) (–5, –2)
14. (4, –2) (–4, –2) (2, 3)
15. (2, 5) (2, 1) (5, 0)
16. (2, 4) (2, –4) (–3, 2)

SCIENTIFIC NOTATION #30

SCIENTIFIC NOTATION is a way of writing very large and very small numbers compactly. A number is said to be in scientific notation when it is written as the product of two factors as described below.

- The first factor is less than 10 and greater than or equal to 1.
- The second factor has a base of 10 and an integer exponent (power of 10).
- The factors are separated by a multiplication sign.
- A positive exponent indicates a number whose absolute value is greater than one.
- A negative exponent indicates a number whose absolute value is less than one.

Scientific Notation	Standard Form
3.51×10^{10}	35,100,000,000
4.73×10^{-13}	0.000000000000473

It is important to note that the exponent does not necessarily mean to use that number of zeros.

The number 3.51×10^{10} means $3.51 \times 10{,}000{,}000{,}000$. Thus, two of the 10 places in the standard form of the number are the 5 and the 1 in 3.51. Standard form in this case is 35,100,000,000. In this example you are moving the decimal point to the right 10 places to find standard form.

The number 4.73×10^{-13} means $4.73 \cdot 0.0000000000001$. You are moving the decimal point to the left 13 places to find standard form. Here the standard form is 0.000000000000473.

Example 1

Write each number in standard form.

$4.39 \times 10^{9} \Rightarrow 4{,}390{,}000{,}000$ and $5.17 \times 10^{-7} \Rightarrow 0.000000517$

When taking a number in standard form and writing it in scientific notation, remember there is only <u>one</u> digit before the decimal point, that is, the number must be between 1 and 9, inclusive.

Example 2

$43{,}207{,}000 \Rightarrow 4.3207 \times 10^{7}$ and $0.0000857 \Rightarrow 8.57 \times 10^{-5}$

The exponent denotes the number of places you have moved the decimal point in the standard form. In Example 2 above left, the decimal point is at the end of the number and it was moved 7 places. In the second part of the example above right, the exponent is negative because the original number is very small, that is, less than one.

Problems

Write each number in standard form.

1. 9.43×10^7
2. 1.347×10^{13}
3. 4.38762×10^4
4. 5.10327×10^2
5. 6.84×10^{-5}
6. 7.03×10^{-6}

Write each number in scientific notation.

7. 723,000,000
8. 123,780,000
9. 0.0000576
10. 0.000000702
11. 48.68
12. 354.3
13. 8,500,000
14. 437,000,000
15. 0.0000004
16. 0.0000057
17. 0.00473
18. 0.00032
19. 12.37
20. 13,980,000

Note: On your scientific calculator, numbers like 4.477^{10} and 3.45^{-4} are expressed in scientific notation. The first number means 4.477×10^{10} and the second means 3.45×10^{-4}. The calculator does this because there is not enough room in its display window to show the whole number.

Answers

1. 94,300,000
2. 13,470,000,000,000
3. 43,876.2
4. 510.327
5. 0.0000684
6. 0.00000703
7. 7.23×10^8
8. 1.2378×10^8
9. 5.76×10^{-5}
10. 7.02×10^{-7}
11. 4.868×10^1
12. 3.543×10^2
13. 8.5×10^6
14. 4.37×10^8
15. 4×10^{-7}
16. 5.7×10^{-6}
17. 4.73×10^{-3}
18. 3.2×10^{-4}
19. 1.237×10^1
20. 1.398×10^7

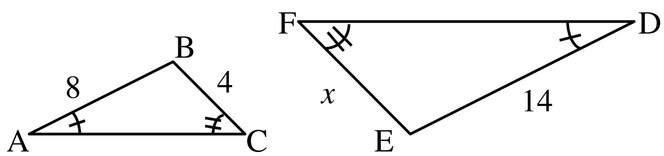
SIMILARITY **#31**

Two-dimensional shapes are similar if they have the same shape but not necessarily the same size. When shapes are known to be similar then the corresponding angles have the same size and the ratios of corresponding sides are equal.

Similarly, to determine if two figures are similar, check if the ratios of corresponding sides are equal or check if all of the corresponding angles are the same size. For triangles, it is sufficient to know that two angles are the same size. This is called angle, angle, similarity ($AA\sim$).

Example 1

ΔABC is similar to ΔDEF. Use ratios to find x.

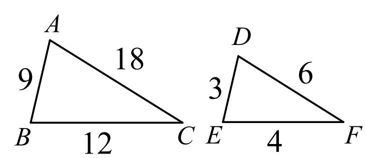


Since the triangles are similar, the ratios of the corresponding sides are equal.

$\frac{8}{14} = \frac{4}{x} \quad \Rightarrow \quad 8x = 56 \quad \Rightarrow \quad x = 7$

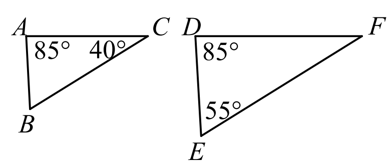
Example 2

The two triangles shown at right are similar based on the sides. The ratios of corresponding sides are equal: $\frac{9}{3} = \frac{18}{6} = \frac{12}{4}$. Therefore $\triangle ABC \sim \triangle DEF$.



Example 3

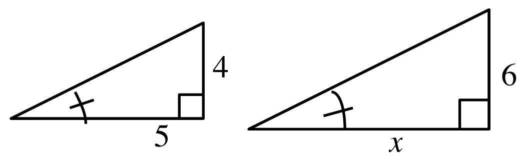
AA ~ is demonstrated by the two triangles at right. Since the sum of the angles of any triangle equal 180 degrees, m$\angle$F = 40°. The measures of two angles in the first triangle are equal to the measures of two angles in the second triangle. Therefore $\triangle ABC \sim \triangle DEF$ by AA ~ .



Problems

Each pair of figures is similar. Solve for the variable.

1.



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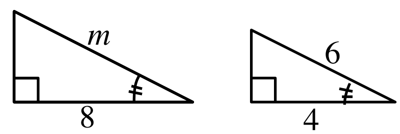

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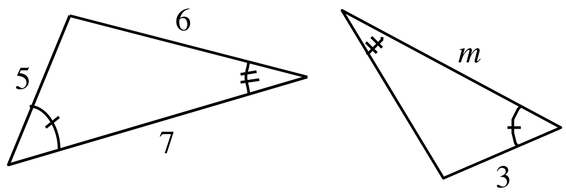

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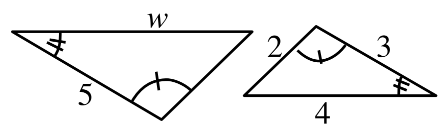

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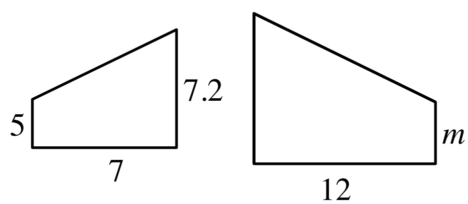

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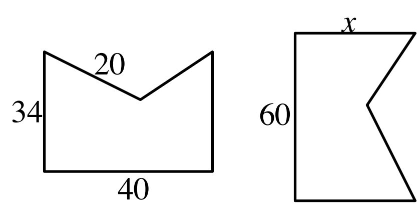

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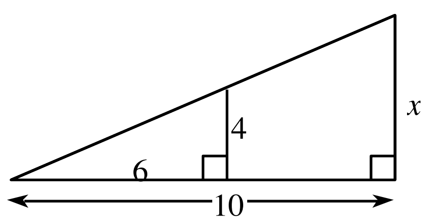

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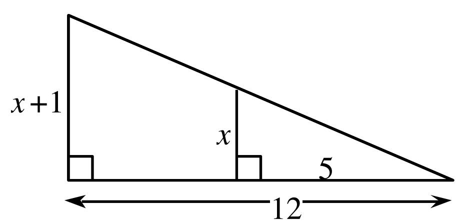
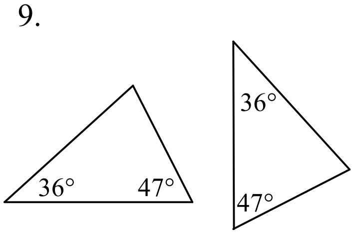


Determine if each pair of triangles is similar. If they are similar, justify your answer.

9.



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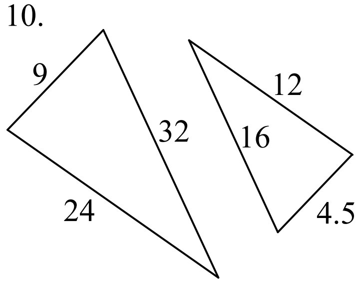

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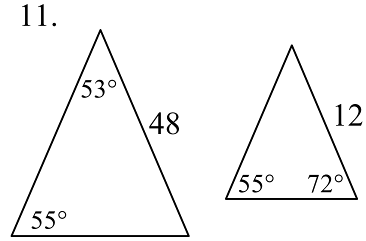

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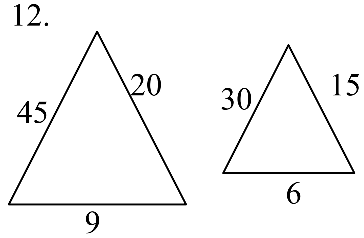

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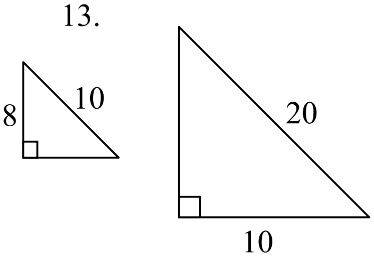

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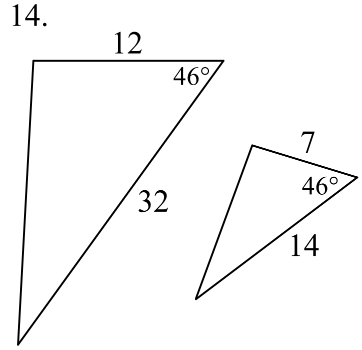

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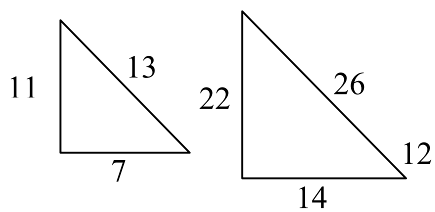

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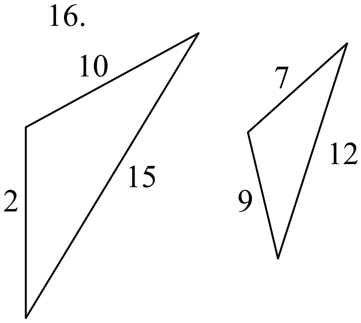

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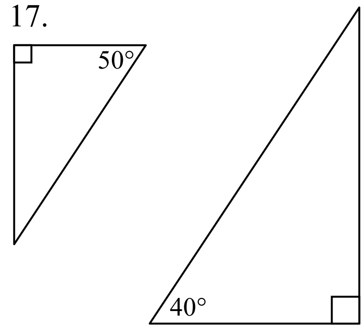

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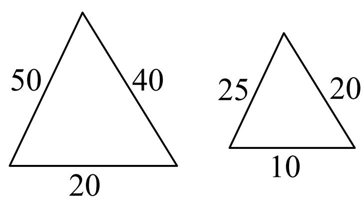

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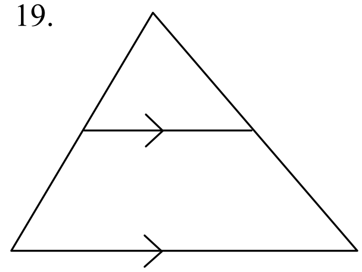

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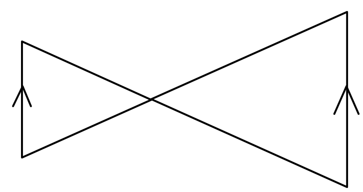


Answers

1. $\frac{30}{4} = 7\frac{1}{2}$	2. 12	3. $\frac{21}{5} = 4\frac{1}{5}$	4. $\frac{20}{3} = 6\frac{2}{3}$
5. $\frac{60}{7} = 8\frac{4}{7}$	6. 51	7. $\frac{20}{3} = 6\frac{2}{3}$	8. $\frac{5}{7}$
9. *AA* ~	10. ratio of sides ~	11. *AA* ~	12. not similar
13. not similar	14. not similar	15. ratio of sides ~	16. not similar
17. *AA* ~	18. ratio of sides ~	19. *AA* ~	20. *AA* ~

SIMPLE AND COMPOUND INTEREST #32

SIMPLE INTEREST is interest paid only on the original amount of the principal at each specified interval (such as annually). The formula is $I = Prt$, where I = Interest, P = Principal (money at the start), r = interest rate, and t = time.

COMPOUND INTEREST is interest paid on both the original principal and the interest earned previously. The formula for compound interest is $A = P(1 + r)^t$, where A = total amount including previous interest earned, P = principal, r = interest rate, and t = time.

Example 1

Wayne earns 4.7% simple interest for 5 years on $4500. How much interest does he earn?

Put the numbers in the formula $I = Prt$.	$I = 4500(4.7\%)5$
Change the percent to a decimal.	$= 4500(0.047)5$
Multiply.	$= 1057.50$ Wayne would earn $1057.50 interest.

Example 2

Use the numbers in Example 1 to find how much money Wayne would have if he earned 4.7% interest compounded annually.

Put the numbers in the formula $A = P(1 + r)^t$.	$A = 4500(1 + 4.7\%)^5$
Change the percent to a decimal.	$= 4500(1 + 0.047)^5$
Multiply.	$= 5661.69$ Wayne would have $5661.69.

Subtract the results of the two examples to compare the difference in earnings when Wayne is earning compound interest instead of simple interest. Wayne would earn $1161.69 in compound interest: $5661.69 – $4500 = $1161.69. $1161.69 – $1057.50 = $104.19, so Wayne would have $104.19 more with compound interest than he would have with simple interest.

Problems

Solve the following problems.

1. Tong loaned Jody \$70 for a month. He charged 5% simple interest for the month. How much did Jody have to pay Tong?

2. Jessica's grandparents gave her \$5000 for college to put in a savings account until she starts college in four years. Her grandparents agreed to pay her an additional 6.5% simple interest on the \$5000 for every year. How much extra money will her grandparents give her at the end of four years?

3. David read an ad offering $7\frac{1}{4}$ % simple interest on accounts over \$1500 left for a minimum of 5 years. He has \$1500 and thinks this sounds like a great deal. How much money will he earn in the 5 years?

4. Javier's parents set an amount of money aside when he was born. They earned 5.25% simple interest on that money each year. When Javier was 15, the account had a total of \$1181.25 interest paid on it. How much did Javier's parents set aside when he was born?

5. Kristina received \$125 for her birthday. Her parents offered to pay her 4.25% simple interest per year if she would save it for at least one year. How much interest could Kristina earn in a year?

6. Kristina decided she would do better if she put her money in the bank for one year, which paid 3.75% interest compounded annually. Was she right?

7. Suppose Jessica (from problem 2) had put her \$5000 in the bank at 2.85% interest compounded annually. How much money would she have earned there at the end of the 4 years?

8. Mai put \$3750 in the bank at 3.87% interest compounded annually. How much was in her account after 7 years?

9. What is the difference in the amount of money in the bank after five years if \$3500 is invested at 3.4% interest compounded annually or at 2.8% interest compounded annually?

10. Elizabeth was listening to her parents talking about what a good deal compounded interest was for a retirement account. She wondered how much money she would have if she invested \$2000 at age 20 at 2.8% interest compounded quarterly (four times each year) and left it until she reached age 65. Determine what the value of the \$2000 would become.

11. Elizabeth decided to try another problem. If she invested \$2500 at age 25 at 2.65% interest compounded quarterly, how much would it be worth when she reached age 65?

12. Compare your answers for problems 10 and 11. What are your conclusions?

Answers

1. $I = 70(0.05)1 = \$3.50$; Jody paid back \$73.50.
2. $I = 5000(0.065)4 = \$1300$
3. $I = \$1500(0.0725)5 = \$543.75$
4. $\$1181.25 = x(0.0525)15$; $x = \$1500$
5. $I = 125(0.0425)1 = \$5.31$
6. $A = 125(1 + 0.0375)^1 = \$129.69$; No, for one year she needs to take the higher interest rate if the compounding is done annually. Only after one year will compounding outstrip simple interest.
7. $A = 5000(1 + 0.0285)^4 = \$5594.83$; Jessica would have earned \$594.83.
8. $A = 3750(1 + 0.0387)^7 = \$4891.73$
9. $A = 3500(1 + 0.034)^5 - 3500(1 + 0.028)^5 = \$4136.86 - \$4018.22 = \$118.64$
10. $A = 2000(1 + 0.028)^{180}$ (because $45 \cdot 4 = 180$ quarters) $= \$288{,}264.15$
11. $A = 2500(1 + 0.0265)^{160}$ (because $40 \cdot 4 = 160$ quarters) $= \$164{,}199.79$
12. You should see that the number of years is an important factor when interest is compounded.

SOLVING EQUATIONS WITH FRACTIONS #33

Fractions that appear in algebraic equations can usually be eliminated in one step by multiplying each term on both sides of the equation by the common denominator for all of the fractions. If you cannot determine the common denominator, use the product of all the denominators. Multiply, simplify each term as usual, then solve the remaining equation. In this course we call this method for eliminating fractions in equations "fraction busting."

Example 1

Solve for x: $\frac{x}{9} + \frac{2x}{5} = 3$

$45\left(\frac{x}{9} + \frac{2x}{5}\right) = 45(3)$

$45\left(\frac{x}{9}\right) + 45\left(\frac{2x}{5}\right) = 135$

$5x + 18 = 135$

$23x = 135$

$x = \frac{135}{23}$

Example 2

Solve for x: $\frac{5}{2x} + \frac{1}{6} = 8$

$6x\left(\frac{5}{2x} + \frac{1}{6}\right) = 6x(8)$

$6x\left(\frac{5}{2x}\right) + 6x\left(\frac{1}{6}\right) = 48x$

$15 + x = 48$

$15 = 47x$

$x = \frac{15}{47}$

Problems

Solve the following equations using the fraction busters method.

1. $\frac{x}{6} + \frac{2x}{3} = 5$
2. $\frac{x}{3} + \frac{x}{2} = 1$
3. $\frac{16}{x} + \frac{16}{40} = 1$
4. $\frac{5}{x} + \frac{5}{3x} = 1$
5. $\frac{x}{2} - \frac{x}{5} = 9$
6. $\frac{x}{3} - \frac{x}{5} = \frac{2}{3}$
7. $\frac{x}{2} - 4 = \frac{x}{3}$
8. $\frac{x}{8} = \frac{x}{12} + \frac{1}{3}$
9. $5 - \frac{7x}{6} = \frac{3}{2}$
10. $\frac{2x}{3} - x = 4$
11. $\frac{x}{8} = \frac{x}{5} - \frac{1}{3}$
12. $\frac{2x}{3} - \frac{3x}{5} = 2$
13. $\frac{4}{x} + \frac{2}{x} = 1$
14. $\frac{3}{x} + 2 = 4$
15. $\frac{5}{x} + 6 = \frac{17}{x}$
16. $\frac{2}{x} - \frac{4}{3x} = \frac{2}{9}$
17. $\frac{x+2}{3} + \frac{x-1}{6} = 5$
18. $\frac{x}{4} + \frac{x+5}{3} = 4$
19. $\frac{x-1}{2x} + \frac{x+3}{4x} = \frac{5}{8}$
20. $\frac{2-x}{x} - \frac{x+3}{3x} = \frac{-1}{3}$

Answers

1. $x = 6$
2. $x = \frac{6}{5}$
3. $x = 26\frac{2}{3}$
4. $x = 6\frac{2}{3}$
5. $x = 30$
6. $x = 5$
7. $x = 24$
8. $x = 8$
9. $x = 3$
10. $x = -12$
11. $x = \frac{40}{9}$
12. $x = 30$
13. $x = 6$
14. $x = 1.5$
15. $x = 2$
16. $x = 3$
17. $x = 9$
18. $x = 4$
19. $x = -2$
20. $x = 1$

SOLVING INEQUALITIES #34

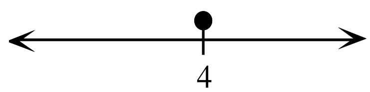
To solve an inequality, first solve it as you would an equation. Use the solution as a **dividing point** of the line. Then test a value from each side of the dividing point on the number line. If the test number is true, then that part of the number line is part of the solution. In addition, if the inequality is $\leq$ or $\leq$, then the dividing point is part of the solution and is indicated by a solid dot. If the inequality is $>$ or $<$, then the dividing point is not part of the solution, indicated by an open dot.

Example 1 $7 \geq m+3$

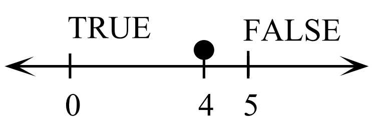
Solve the equation: $7 = m+3$

$4 = m$

Draw a number line. Put a solid dot at 4.

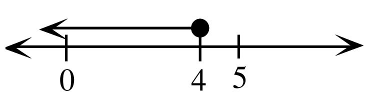


Test a number on each side of 4 in the original inequality. We use 0 and 5.



$m = 0$	$m = 5$
$7 > 0+3$	$7 > 5+3$
$7 > 3$	$7 > 8$
TRUE	FALSE

The solution is $m \leq 4$.



Example 2 $-3x - 4 = x + 8$

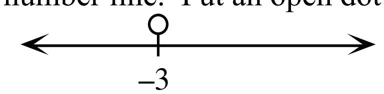
Solve the equation: $-3x - 4 = x + 8$

$-3x = x + 12$

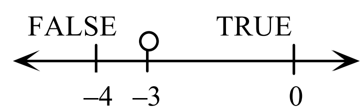
$-4x = 12$

$x = -3$

Draw a number line. Put an open dot at –3.

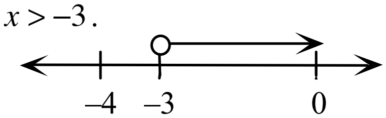


Test –4 and 0 in the original inequality.



	$x = 0$
$-3(-4) - 4 < 4 - 4 + 8$	$-3(0) - 4 < 0 + 8$
$12 - 4 < 4$	
$8 < 4$	TRUE
FALSE	

The solution is $x > -3$.



Problems

Solve each inequality.

1. $x + 5 > 2$
2. $y - 5 \leq 8$
3. $-2x \leq -8$
4. $3m + 3 \geq -9$
5. $-5 < -2y + 3$
6. $7 \geq 3m + 4$
7. $3x - 2 < -x + 6$
8. $3(m+2) \geq m - 6$
9. $2m + 3 \leq m + 6$
10. $4x + 3 \leq 2x - 7$

Answers

1. $x > -3$
2. $y \leq 13$
3. $x \geq 4$
4. $m \geq -4$
5. $y < 4$
6. $m \geq -1$
7. $x < 2$
8. $m \geq -6$
9. $m \leq 3$
10. $x \leq -5$

SOLVING LINEAR EQUATIONS #35

Solving equations involves "undoing" what has been done to create the equation. In this sense, solving an equation can be described as "working backward," generally known as using inverse (or opposite) operations. For example, to undo $x + 2 = 5$, that is, adding 2 to x, subtract 2 from both sides of the equation. The result is $x = 3$, which makes $x + 2 = 5$ true. For $2x = 17$, x is multiplied by 2, so divide both sides by 2 and the result is $x = 8.5$. For equations like those in the examples and exercises below, apply the idea of inverse (opposite) operations several times. Always follow the correct order of operations. Also see the textbook.

Example 1

Solve for x: $2(2x-1)-6=-x+2$

First distribute to remove the parentheses, then combine like terms.

$$\begin{aligned} 4x-2-6&=-x+2 \\ 4x-8&=-x+2 \end{aligned}$$

Next, move variables and constants by addition of opposites to get the variable term on one side of the equation.

$$\begin{array}{rcl} 4x-8 &=& -x+2 \\ +x & & +x \\ \hline 5x-8 &=& 2 \end{array}$$

$$\begin{array}{rcl} 5x-8 &=& 2 \\ +8 & & +8 \\ \hline 5x &=& 10 \end{array}$$

Now, divide by 5 to get the value of x.

$$\frac{5x}{5}=\frac{10}{5} \quad \Rightarrow \quad x=2$$

Finally, check that your answer is correct.

$$\begin{aligned} 2(2(2)-1)-6&=-(2)+2 \\ 2(4-1)-6&=0 \\ 2(3)-6&=0 \\ 6-6&=0 \text{ checks} \end{aligned}$$

Example 2

Solve for y: $2x+3y-9=0$

This equation has two variables, but the instruction says to isolate y. First move the terms without y to the other side of the equation by adding their opposites to both sides of the equation.

$$\begin{array}{rcl} 2x+3y-9 &=& 0 \\ +9 & & +9 \\ \hline 2x+3y &=& +9 \end{array}$$

$$\begin{array}{rcl} 2x+3y &=& 9 \\ -2x & & -2x \\ \hline 3y &=& -2x+9 \end{array}$$

Divide by 3 to isolate y. Be careful to divide every term on the right by 3.

$$\frac{3y}{3}=\frac{-2x+9}{3} \quad \Rightarrow \quad y=-\frac{2}{3}x+3$$

Problems

Solve each equation below.

1. $5x + 2 = -x + 14$
2. $3x - 2 = x + 10$
3. $6x + 4x - 2 = 15$
4. $6x - 3x + 2 = -10$
5. $\frac{2}{3}y - 6 = 12$
6. $\frac{3}{4}x + 2 = -7$
7. $2(x + 2) = 3(x - 5)$
8. $3(m - 2) = -2(m - 7)$
9. $3(2x + 2) + 2(x - 7) = x + 3$
10. $2(x + 3) + 5(x - 2) = -x + 10$
11. $4 - 6(w + 2) = 10$
12. $6 - 2(x - 3) = 12$
13. $3(2z - 7) = 5z + 17 + z$
14. $-3(2z - 7) = -3z + 21 - 3z$

Solve for the named variable.

15. $2x + b = c$ (for x)
16. $3x - d = m$ (for x)
17. $3x + 2y = 6$ (for y)
18. $-3x + 5y = -10$ (for y)
19. $y = mx + b$ (for b)
20. $y = mx + b$ (for x)

Answers

1. 2	2. 6	3. $\frac{17}{10}$	4. –4
5. 27	6. –12	7. 19	8. 4
9. $\frac{11}{7}$	10. $\frac{7}{4}$	11. –3	12. 0
13. no solution	14. all numbers	15. $x = \frac{c-b}{2}$	16. $x = \frac{m+d}{3}$
17. $y = -\frac{3}{2}x + 3$	18. $y = \frac{3}{5}x - 2$	19. $b = y - mx$	20. $x = \frac{y-b}{m}$

SOLVING PROPORTIONS #36

A **proportion** is an equation stating that two ratios (fractions) are equal. To solve a proportion, use cross multiplication to remove the fractions, then solve in the usual way. Cross multiplication may only be used to solve proportions, that is, when each side of the equation is a ratio. Multiply the numerator of the left ratio by the denominator of the right ratio, write an equal sign, then multiply the denominator of the left ratio by the numerator of the right ratio. Note that cross multiplication is a shortened equation solving process. It is the equivalent of multiplying both sides of the proportion by both denominators, then simplifying. To finish solving the resulting equation, divide by the remaining coefficient of the variable.

Example 1

$$\frac{m}{6} = \frac{12}{5}$$
$$5 \cdot m = 6 \cdot 12$$
$$5m = 72$$
$$m = 14.4$$

Example 2

$$\frac{6}{\$9.75} = \frac{6.5}{w}$$
$$6 \cdot w = \$9.75 \cdot 6.5$$
$$6w = \$63.375$$
$$w = \$10.56$$

Example 3

$$\frac{x+2}{3} = \frac{x-2}{7}$$
$$7(x+2) = 3(x-2)$$
$$7x + 14 = 3x - 6$$
$$4x = -20$$
$$x = -5$$

Problems

Solve these equations. Remember to check your answers.

1. $\frac{4}{5} = \frac{y}{15}$
2. $\frac{x}{32} = \frac{4}{8}$
3. $\frac{3}{200} = \frac{6}{m}$
4. $\frac{3}{8} = \frac{x}{50}$
5. $\frac{8}{18} = \frac{3}{m}$
6. $\frac{8}{500} = \frac{m}{1500}$
7. $\frac{6}{\$10.75} = \frac{x}{\$24.95}$
8. $\frac{6}{\$4.35} = \frac{7}{x}$
9. $\frac{20}{40} = \frac{45}{x}$
10. $\frac{40}{70} = \frac{35}{x}$
11. $\frac{20}{40} = \frac{45}{x}$
12. $\frac{2y}{5} = \frac{16}{10}$
13. $\frac{x}{x+3} = \frac{3}{4}$
14. $\frac{2}{3x} = \frac{5}{x+1}$
15. $\frac{3}{2y} = \frac{8}{y-2}$
16. $\frac{2x}{5} = \frac{x-2}{76}$
17. $\frac{4x}{5} = \frac{x+2}{6}$
18. $\frac{7+x}{5} = \frac{12}{3}$
19. $\frac{6-y}{4} = \frac{3y}{4}$
20. $\frac{x-2}{3} = \frac{8}{9}$
21. $\frac{1}{2x} = \frac{7}{x+1}$
22. $\frac{4}{2y} = \frac{5}{y-3}$
23. $\frac{7}{2x+3} = \frac{3}{x}$
24. $\frac{3y-4}{12} = \frac{2y}{4}$

Answers

1. $y = 12$	2. $x = 16$	3. $m = 400$	4. $x = 18.75$	5. $x = 6.75$
6. $m = 24$	7. $x = 13.93$	8. $m = \$5.08$	9. $x = 90$	10. $x = 61.25$
11. $x = 90$	12. $y = 4$	13. $x = 9$	14. $x = 0.15$	15. $y \approx -0.46$
16. $x = -\frac{10}{7} = -1\frac{3}{7}$		17. $x = \frac{10}{19}$	18. $x = 13$	19. $x = \frac{3}{2} = 1\frac{1}{2}$
20. $x = \frac{14}{3} = 4\frac{2}{3}$	21. $x = \frac{1}{13}$	22. $y = -2$	23. $x = 9$	24. $y = -\frac{4}{3} = -1\frac{1}{3}$

SOLVING SYSTEMS OF EQUATIONS: USING THE EQUAL VALUES METHOD OR SUBSTITUTION # 37

A system of equations in two variables may be represented by the graph of the two lines. To solve the system is equivalent to finding the coordinates of the point where two lines intersect. We could graph them, but there is a faster, more accurate algebraic method called the equal values method or substitution method. This method may also be used to solve systems of equations in word problems. Also see the Course 3 textbook.

Example 1

Solve the system: $y = -2x + 5$

$y = x - 1$

Take the two expressions that are equal to y and set them equal to each other. Then solve this new equation to find x.

$$\begin{aligned} -2x + 5 &= x - 1 \\ -3x &= -6 \\ x &= 2 \end{aligned}$$

The x-coordinate of the point of intersection is $x = 2$. To find the y-coordinate, substitute the value of x into either original equation. Solve for y, then write the solution as an ordered pair. The point $(2,1)$ is the point of intersection of the graphs and the solution to the system of equations.

$$\begin{aligned} y &= -2x + 5 \\ y &= -2(2) + 5 \\ y &= 1 \end{aligned}$$

Check that the point works in both equations.

Check: $1 = -2(2) + 5\ \checkmark$ and $1 = 2 - 1\ \checkmark$.

Example 2

The sales of Gizmo Sports Drink at the local supermarket are currently 6,500 bottles per month. Since New Age Refreshers were introduced, sales of Gizmo have been declining by 55 bottles per month. New Age currently sells 2,200 bottles per month and its sales are increasing by 250 bottles per month. If these rates of change remain the same, in about how many months will the sales for both companies be the same? How many bottles will each company be selling at that time?

Let x = months from now and y = total monthly sales.

For Gizmo: $y = 6500 - 55x$; for New Age: $y = 2200 + 250x$.

Substituting equal parts: $6500 - 55x = 2200 + 250x \Rightarrow 4300 = 305x \Rightarrow 14.10 \approx x$.

Use either equation to find y: $y = 2200 + 250(14.10) \approx 5725$ or $y = 6500 - 55(14.10) \approx 5725$.

The solution is (14.10, 5725). This means that in about 14 months, both drink companies will be selling 5,725 bottles of the sports drinks.

Problems

Find the point of intersection (x, y) for each pair of lines by using the equal values method.

1. $y = x + 2$
   $y = 2x - 1$
2. $y = 3x + 5$
   $y = 4x + 8$
3. $y = 11 - 2x$
   $y = x + 2$
4. $y = 3 - 2x$
   $y = 1 + 2x$
5. $y = 3x - 4$
   $y = \frac{1}{2}x + 7$
6. $y = -\frac{2}{3}x + 4$
   $y = \frac{1}{3}x - 2$
7. $y = 4.5 - x$
   $y = -2x + 6$
8. $y = 4x$
   $y = x + 1$

For each problem, define your variables, write a system of equations, and solve them by using the equal values method.

9. Janelle has $20 and is saving $6 per week. April has $150 and is spending $4 per week. When will they both have the same amount of money?
10. Sam and Hector are gaining weight for football season. Sam weighs 205 pounds and is gaining two pounds per week. Hector weighs 195 pounds but is gaining three pounds per week. In how many weeks will they both weigh the same amount?
11. PhotosFast charges a fee of $2.50 plus $0.05 for each picture developed. PhotosQuick charges a fee of $3.70 plus $0.03 for each picture developed. For how many pictures will the total cost be the same at each shop?
12. Playland Park charges $7 admission plus 75¢ per ride. Funland Park charges $12.50 admission plus 50¢ per ride. For what number of rides is the total cost the same at both parks?

Change one or both equations to $y = mx + b$ form if necessary and solve by the substitution method.

13. $y = 2x - 3$
    $x + y = 15$
14. $y = 3x + 11$
    $x + y = 3$
15. $x + y = 5$
    $2y - x = -2$
16. $x + 2y = 10$
    $3x - 2y = -2$
17. $x + y = 3$
    $2x - y = -9$
18. $y = 2x - 3$
    $x - y = -4$
19. $x + 2y = 4$
    $x + 2y = 6$
20. $3x = y - 2$
    $6x + 4 = 2y$

Answers

1. (3, 5)	2. (–3, –4)	3. (3, 5)	4. $\left(\frac{1}{2}, 2\right)$
5. (4.4, 9.2)	6. (6, 0)	7. (1.5, 3)	8. $\left(\frac{1}{3}, \frac{4}{3}\right)$
9. 13 weeks, $98	10. 10 wks, 225 lbs	11. 60 pictures, $5.50	12. 22 rides, $23.50
13. (6, 9)	14. (–2, 5)	15. (4, 1)	16. (2, 4)
17. (–2, 5)	18. (7, 11)	19. no solution	20. lines coincide

STEM AND LEAF PLOTS #38

A **stem and leaf plot** is a way to display data that shows each individual value from a set of numbers and how they are distributed. The vertical "stem" part of the graph represents all of the digits in a number except the last one and the horizontal "leaves" represent the last digit of the number. A stem-and-leaf plot includes a key to make the arrangement of the plot clear.

Put the values in order, from lowest to highest. The stem represents the leading digits. Place the "leaf" part in numerical order, lowest to highest.

Example 1

Make a stem and leaf plot of this set of data: 83, 90, 82, 74, 91, 91, 99, 105, 100, 64, and 67.

6	4 7
7	4
8	2 3
9	0 1 1 9
10	0 5

Key:
8 | 2 means 82

Example 2

Make a stem and leaf plot of this set of data: 5.3, 5.8, 4.2, 6, 6.1, 6, 0.8, 1.2, and 1.4.

0	8
1	2 4
4	2
5	3 8
6	0 0 1

Key:
4 | 2 means 4.2

Problems

Make a stem and leaf plot of each set of data.

1. 16, 22, 33, 41, 42, 41, 41, 40, 27, and 18
2. 121, 90, 83, 97, 96, 84, 110, 118, 128, and 120
3. 41, 38, 47, 48, 22, 28, 25, 40, and 30
4. 7, 10, 11, 12, 11, 11, 13, 8, 22, and 23
5. 1.6, 2.4, 1.8, 2.3, 1.9, 1.6, 2, 3.1, 3.8, 4.1, and 4
6. 10.8, 9.7, 6.1, 8.1, 8.3, 8.5, 8.5, 9.2, 9.1, and 9.1
7. 76, 83, 54, 43, 44, 44, 103, 99, and 94
8. 8, 12, 14, 11, 11, 15, 9, 21, 25, and 27

Answers

1.

1	6 8
2	2 7
3	3
4	0 1 1 1 2

Key:
2 | 2 means 22

2.

8	3 4
9	0 6 7
10	
11	0 8
12	0 1 8

Key:
9 | 6 means 96

3.

2	2 5 8
3	0 8
4	0 1 7 8

Key:
2 | 2 means 22

4.

0	7 8
1	0 1 1 1 2 3
2	2 3

Key:
0 | 7 means 7

5.

1	6 6 8 9
2	0 3 4
3	1 8
4	1

Key:
2 | 3 means 2.3

6.

6	1
7	
8	1 3 5 5
9	1 1 2 7
10	8

Key:
8 | 1 means 8.1

7.

4	3 4 4
5	4
6	
7	6
8	3
9	4 9
10	3

Key:
5 | 4 means 54

8.

0	8 9
1	1 1 2 4 5
2	1 5 7

Key:
0 | 9 means 9

SUBSTITUTION AND EVALUATION #39

SUBSTITUTION is replacing one symbol with an equivalent symbol (a number, a variable, or an expression). One application of the substitution property is replacing a variable name with a number in an expression or equation. A **VARIABLE** is a letter used to represent one or more numbers (or other algebraic expression). The numbers are the values of the variable. A variable expression has numbers and variables with arithmetic operations performed on it.

In general, if $a = b$, then a may replace b and b may replace a.

Examples

Evaluate each variable expression for $x = 3$.

a. $5x \Rightarrow 5(3) \Rightarrow 15$

b. $x + 10 \Rightarrow (3) + 10 \Rightarrow 13$

c. $\frac{18}{x} \Rightarrow \frac{18}{3} \Rightarrow 6$

d. $\frac{x}{3} \Rightarrow \frac{3}{3} \Rightarrow 1$

e. $3x - 5 \Rightarrow 3(3) - 5 \Rightarrow 9 - 5 \Rightarrow 4$

f. $5x + 3x \Rightarrow 5(3) + 3(3) \Rightarrow 15 + 9 \Rightarrow 24$

Problems

Evaluate each of the variable expressions below for the values $x = -4$ and $y = 3$. Be sure to follow the order of operations as you simplify each expression.

1. $x + 4$
2. $x - 1$
3. $x + y + 3$
4. $y - 3 + x$
5. $x^2 - 5$
6. $-x^2 + 5$
7. $x^2 + 4x - 3$
8. $-3x^2 + 2x$
9. $x + 3 + 2y$
10. $y^2 + 3x - 2$
11. $x^2 + y^2 + 3^2$
12. $2^2 + y^2 - 2x^2$

Evaluate the expressions below using the values of the variables in each problem. These problems ask you to evaluate each expression twice, once with each of the values.

13. $3x^2 - 2x + 5$ for $x = -3$ and $x = 4$
14. $2x^2 - 3x + 6$ for $x = -2$ and $x = 5$
15. $-3x^2 + 7$ for $x = -3$ and $x = 2$
16. $-2x^2 + 5$ for $x = -4$ and $x = 5$

Evaluate the variable expressions for $x = -3$ and $y = 4$.

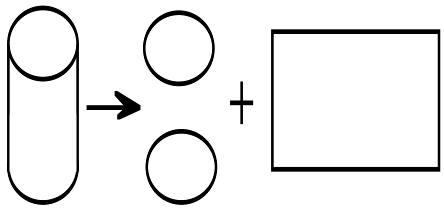
17. $x(x + 3x)$
18. $2(x + 2x)$
19. $2(x + y) + 4\left(\frac{y+2}{x}\right)$
20. $3\left(y^2 + 2\left(\frac{x+7}{2}\right)\right)$
21. $2y\left(x + x^2 - 2y\right)$
22. $(3x + y)(2x + 4y)$

Answers

1. 0	2. –5	3. 2	4. –4	5. 11
6. –11	7. –3	8. –56	9. 5	10. –5
11. 34	12. –19	13. 38; 45	14. 20; 41	15. –20; –5
16. –27; –45	17. 36	18. –18	19. 20	20. 36
21. –16	22. –50			

SURFACE AREA OF A CYLINDER #40

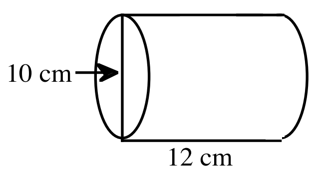
The surface area of a cylinder is the sum of the two base areas and the lateral surface area. The lateral (side) surface is a rectangle when the cylinder's bases are removed, the lateral side is cut vertically, and the curved surface is laid flat. The formula for the surface area is:



$$\mathbf{SA} = \boldsymbol{2r^2\pi + 2\pi rh}$$

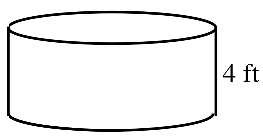
where r = radius and h = height of the cylinder.

Example 1



$$\begin{aligned} \text{SA} &= 2r^2\pi + 2\pi rh \\ &= 2\,(5)^2\pi + 2\pi \cdot 5 \cdot 12 \\ &= 50\pi + 120\pi \\ &= 170\pi \approx 534 \text{ cm}^2 \end{aligned}$$

Example 2



If the volume of the tank above is 2826 ft^3, what is the surface area?

$$\begin{aligned} V &= r^2\pi h \\ 2826 &= r^2\pi(4) \\ \tfrac{2826}{4\pi} &= r^2 \\ 225 &= r^2 \\ 15 &= r \end{aligned}$$

$$\begin{aligned} SA &= 2r^2\pi + 2\pi rh \\ &= 2(15)^2\pi + 2\pi(15)(4) \\ &= 450\pi + 120\pi \\ &= 570\pi \approx 1789.8 \text{ ft}^2 \end{aligned}$$

Problems

Find the surface area of each cylinder.

1. $r = 7$ cm, $h = 7$ cm
2. $r = 8$ cm, $h = 11$ cm
3. $r = 9$ cm, $h = 13$ cm
4. $r = 10$ cm, $h = 15$ cm
5. $r = 15$ cm, $h = 9$ cm
6. $r = 16$ cm, $h = 8$ cm
7. $r = 17$ cm, $h = 6$ cm
8. $r = 18$ cm, $h = 4$ cm
9. $r = 32$ cm, $h = 2$ cm
10. $r = 10$ cm, $h = 1$ cm

Find the height of each cylinder.

11. $r = 10$ cm, SA = 690.8 cm^2
12. $r = 5$ cm, SA = 408.2 cm^2

Answers

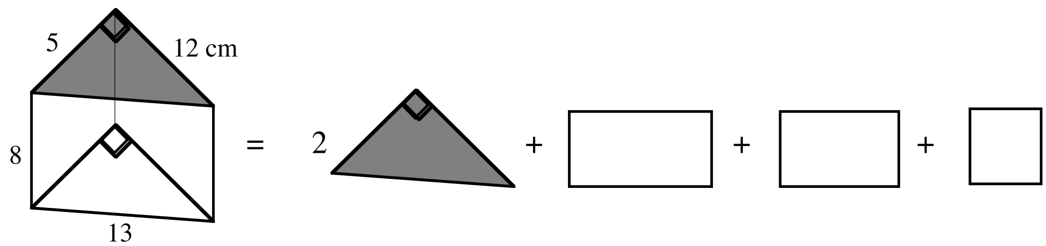
1. 615.44 cm^2
2. 954.56 cm^2
3. 1243.44 cm^2
4. 1570.80 cm^2
5. 2260.8 cm^2
6. 2411.52 cm^2
7. 2455.48 cm^2
8. 2486.88 cm^2
9. 6832.64 cm^2
10. 690.8 cm^2
11. 0.99 cm
12. 7.99 cm

SURFACE AREA OF A PRISM #41

The **SURFACE AREA OF A PRISM** (SA) is the sum of the areas of all of the faces, including the bases. Surface area is expressed in square units.

Example

Find the surface area of the triangular prism below.



Subproblem 1: Find the area of each face.

$$SA = 2(\tfrac{1}{2}\cdot 5\cdot 12) + 8\cdot 13 + 8\cdot 12 + 8\cdot 5$$

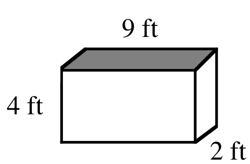
Subproblem 2: Find the sum of the areas.

$$SA = 60 + 104 + 96 + 40$$
$$= 300 \text{ cm}^2$$

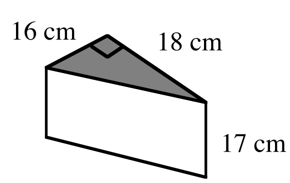
Problems

Calculate the surface area of each prism. One base of some figures is shaded.

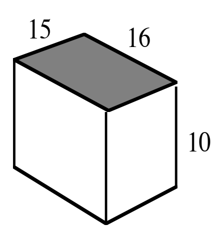
1. Rectangular prism



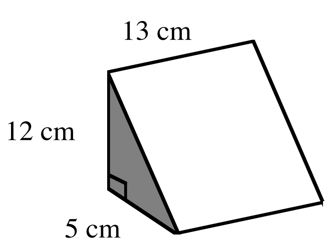
2. Right triangular prism



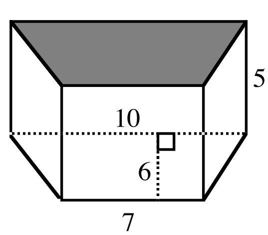
3. Rectangular prism



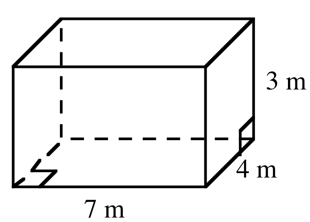
4. Right triangular prism



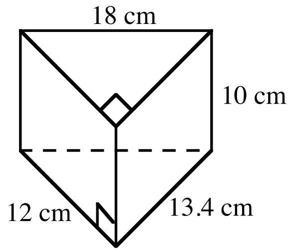
5. Trapezoidal prism



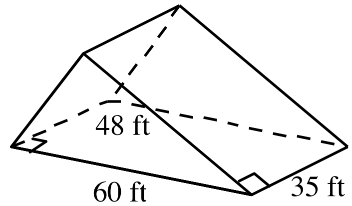
6. Rectangular prism



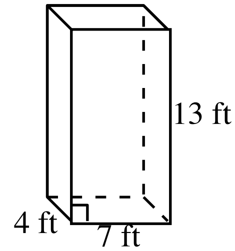
7. Right triangular prism



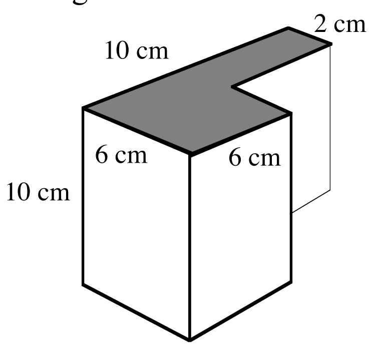
8. Right triangular prism



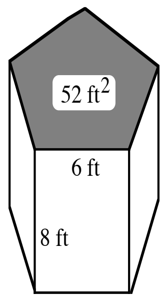
9. Rectangular prism



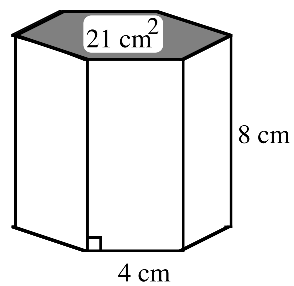
10. All angles on the prism bases and faces are right angles.



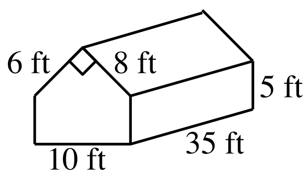
11. Regular pentagonal prism



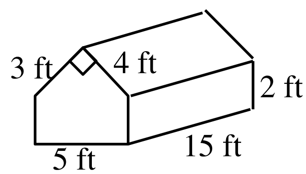
12. Regular hexagonal prism



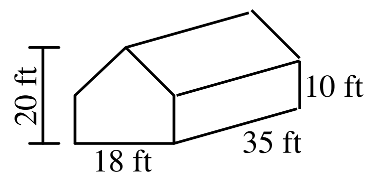
13.



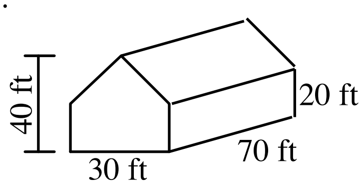
14.



15.



16.

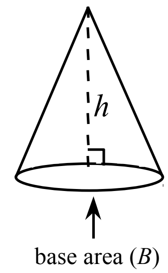


Answers

1. 124 feet^2
2. 1274 cm^2
3. 1100 units^2
4. 450 cm^2
5. 249 units^2
6. 122 m^2
7. 594.8 cm^2
8. 6768 feet^2
9. 342 feet^2
10. 408 cm^2
11. 344 feet^2
12. 234 cm^2
13. 1338 feet^2
14. 272 feet^2
15. 2342.5 feet^2
16. 10,225 feet^2

VOLUME OF A CONE #42

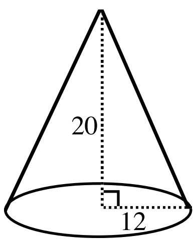
Every cone has a **VOLUME** that is one-third the volume of the cylinder with the same base and height. To find the volume of a cone, use the same formula as the volume of a cylinder and divide by three. The formula for the volume of a cone of base area B and height h is:



$$\textbf{Volume} = \tfrac{1}{3}\textbf{(Area of Base)(height) or } V = \frac{B \cdot h}{3} = \frac{r^2 \pi h}{3} = \frac{1}{3} r^2 \pi h$$

Example 1

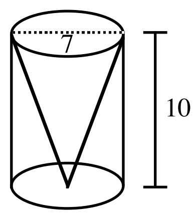
Find the volume of the cone below.



$$\begin{aligned}\text{Volume} &= \tfrac{1}{3}(12)^2\pi(20)\\ &= \tfrac{2880\pi}{3}\\ &\approx 3014.4 \text{ units}^3\end{aligned}$$

Example 2

Find the volume of the cone below.



$$\begin{aligned}\text{radius} &= 3.5\\ \text{Volume} &= \tfrac{1}{3}(3.5)^2\pi(10)\\ &= \tfrac{122.5\pi}{3}\\ &\approx 128.22 \text{ units}^3\end{aligned}$$

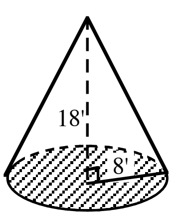
Example 3

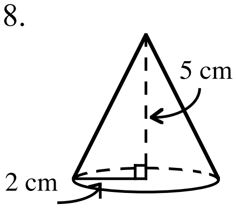
If the volume of a cone is 1441.26 cm^3 and its radius is 9 cm, find its height.

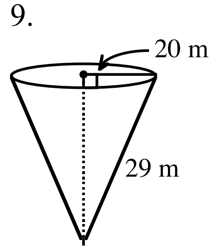
$$\begin{aligned}\text{Volume} &= \tfrac{1}{3} r^2 \pi h\\ 1441.26 &= 13(9)^2\pi \cdot h\\ 4323.78 &= (81)\pi \cdot h\\ \tfrac{4323.78}{81\pi} &= h\\ 17 \text{ cm} &= h\end{aligned}$$

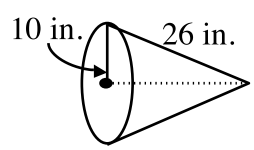
Problems

Find the volume of each cone. Use $\pi \approx 3.14$. Round to the nearest cubic unit. Hint: First solve a subproblem in problems 9 and 10.

1. base area = 50 cm^2
   h = 3 cm
2. base area = 17 cm^2
   h = 10 cm
3. r = 5 cm
   h = 12 cm
4. r = 7.5 in.
   h = 3.6 in.
5. diameter = 28 cm
   h = 5 cm
6. d = 29 cm
   h = 41 cm
7. 

8. 

9. 

10. 

Find the missing part of each cone. Use $\pi \approx 3.14$.

11. If the volume is 355 ft^3 and the height is 12 ft, find the radius of the base.

12. If the volume is 4000 ft^3 and the height is 25.4 ft, find the radius of the base.

13. If the volume is 864 ft^3 and the height is 9 ft, find the diameter of the base.

14. If the volume is 864 ft^3 and the height is 30 ft, find the diameter of the base.

15. If the volume is 14,736.02 $inches^3$ and the radius is 19 inches, find the height.

16. If the volume is 22,372.5 $inches^3$ and the radius is 25 inches, find the height.

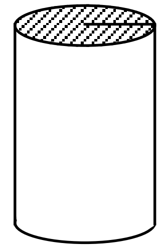
17. If the circumference of the base is 25 cm and the height is 8 cm, find the volume. Round to the nearest cubic cm.

18. If the circumference of the base is 14 cm and the height is 11 cm, find the volume. Round to the nearest cubic cm.

Answers

1. 50 cm^3	2. 57 cm^3	3. 314 cm^3	4. 212 $in.^3$	5. 1026 cm^3
6. 9023 cm^3	7. 1206 ft^3	8. 21 cm^3	9. 8796 m^3	10. 2513 $in.^3$
11. 5.3 ft	12. 12.3 ft	13. 19.2 ft	14. 10.4 ft	15. 39 in.
16. 34.2 in.	17. 133 cm^3	18. 57 cm^3		

VOLUME OF A CYLINDER #43



The volume of a cylinder is the area of its base multiplied by its height:

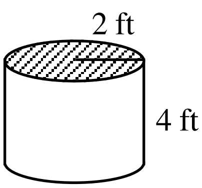
Volume = (Area of Base)(height) or $V = B \cdot h$.

Since the base of a cylinder is a circle of area $A = r^2\pi$, we can write:

$$V = r^2\pi h.$$

Example 1

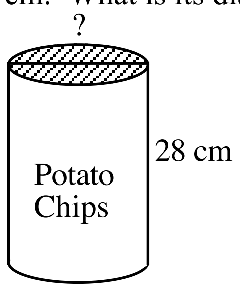
Find the volume of the cylinder.



$$\begin{aligned} \text{Volume} &= r^2\pi h \\ &= (2)^2\pi\,(4) \\ &= 16\pi \\ &= 50.27 \text{ ft}^3 \end{aligned}$$

Example 2

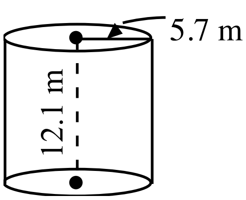
The can has volume 3165 cm^3 and height 28 cm. What is its diameter?

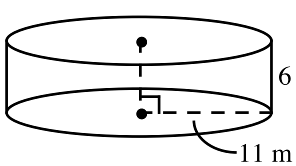


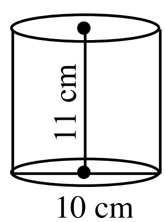
$$\begin{aligned} \text{Volume} &= r^2\pi h \\ 3165 &= r^2\pi\,(28) \\ \tfrac{3165}{28\pi} &= r^2 \\ 36 &= r^2 \\ \text{radius} &= 6 \\ \text{diameter} &= 2(6) = 12 \text{ cm} \end{aligned}$$

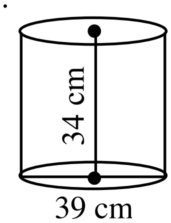
Problems

Find the volume of each cylinder. Use 3.14 for π. Round your answer to the nearest cubic unit.

1. base area = 50 cm^2
   h = 7 cm
2. base area = 17 cm^2
   h = 10 cm
3. r = 5 cm
   h = 12 cm
4. r = 7.5 in.
   h = 3.6 in.
5. diameter = 28 cm
   h = 5 cm
6. d = 29 cm
   h = 41 cm
7. 

8. 

9. 

10. 

Find the missing part of each cylinder.

11. If the volume is 355 ft^3 and the height is 12 ft, find the radius, to the nearest foot.

12. If the volume is 4000 ft^3 and the height is 25.4 ft, find the radius, to the nearest foot.

13. If the volume is 864 ft^3 and the height is 9 ft, find the diameter, to the nearest foot.

14. If the volume is 864 ft^3 and the height is 30 ft, find the diameter, to the nearest foot.

15. If the volume is 14,736.02 $inches^3$ and the radius is 19 inches, find the height, to the nearest inch.

16. If the volume is 22,372.5 $inches^3$ and the radius is 25 inches, find the height, to the nearest inch.

17. If the circumference is 25 cm and the height is 8 cm, find the volume. Round to the nearest cubic cm.

18. If the circumference is 14 cm and the height is 11 cm, find the volume. Round to the nearest cubic cm.

Answers

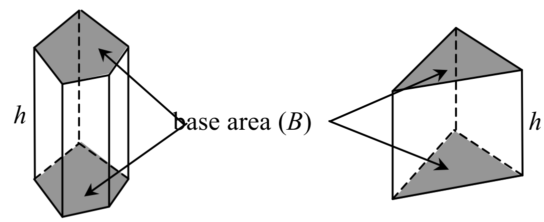
1.	350 cm^3	2.	170 cm^3	3.	942 cm^3	4.	636 $in.^3$	5.	3077 cm^3
6.	27, 068 cm^3	7.	1234 m^3	8.	2280 m^3	9.	864 cm^3	10.	40, 595 cm^3
11.	3 ft	12.	7 ft	13.	11 ft	14.	6 ft	15.	13 in.
16.	11 in.	17.	398 cm^3	18.	172 cm^3				

VOLUME OF A PRISM **#44**

Volume is a three-dimensional concept. It measures the amount of interior space of a three-dimensional figure based on a cubic unit, that is, the number of 1 by 1 by 1 cubes that will fit inside a figure.

The volume of any prism is the area of either base (B) times the height (h) of the prism.

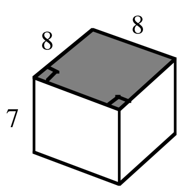
V = (Area of base) · (height) or **V = *Bh***



Example 1

Find the volume of each figure below. Show the steps you use in your subproblems.

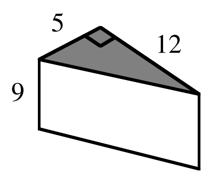
a.



This is a square prism.
The base is a square with area (B) $8 \cdot 8 = 64$ units^2.

Volume $= B(h)$
$= 64(7)$
$= 448$ units^3

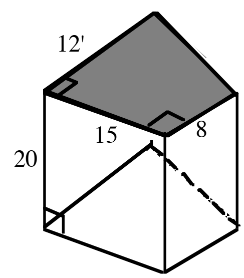
b.



This is a triangular prism.
The base is a right triangle with area (B).
$\frac{1}{2}(5)(12) = 30$ units^2.

Volume $= B(h)$
$= 30(9)$
$= 270$ units^3

c.



This is a trapezoidal prism with area (B).
$\frac{1}{2}(12 + 8)(15) = 150$ ft^2.

Volume $= B(h)$
$= 150(20)$
$= 3000$ ft^3

Example 2

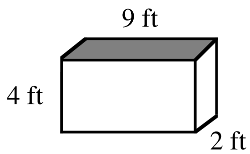
Find the height of the prism with a volume of 3240 cm^3 and base area of 40.5 cm^2.

$$\begin{aligned} \text{Volume} &= B(h) \\ 3240 &= 40.5(h) \\ \frac{3240}{40.5} &= h \\ 80 \text{ cm} &= h \end{aligned}$$

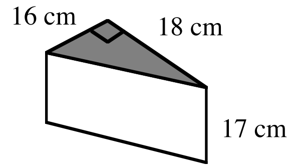
Problems

Calculate the volume of each prism. One base in some figures is shaded.

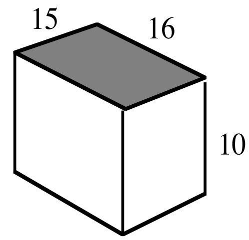
1. Rectangular prism



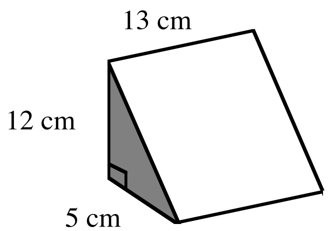
2. Right triangular prism



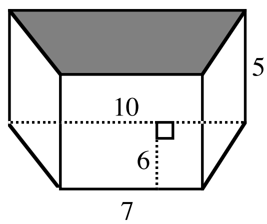
3. Rectangular prism



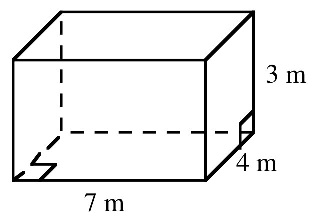
4. Right triangular prism



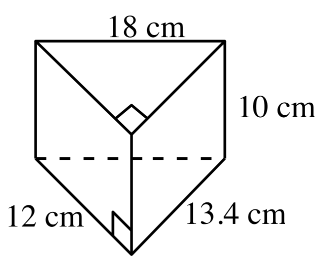
5. Trapezoidal prism



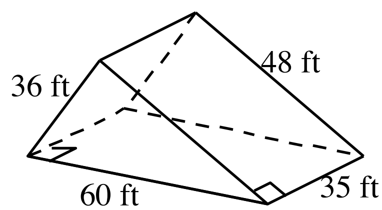
6. Rectangular prism



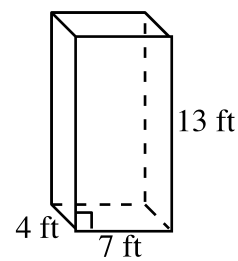
7. Right triangular prism



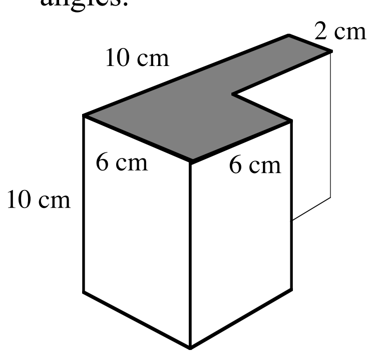
8. Right triangular prism



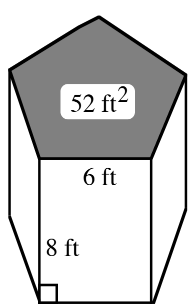
9. Rectangular prism



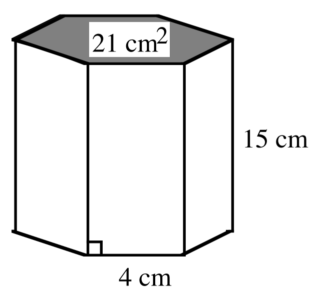
10. All angles on the prism bases and faces are right angles.



11. Regular pentagonal prism

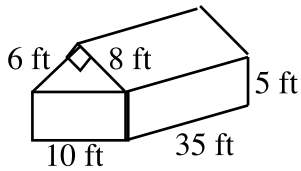


12. Regular hexagonal prism

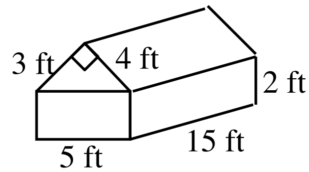


In problems 13 through 16, all quadrilaterals are rectangles.

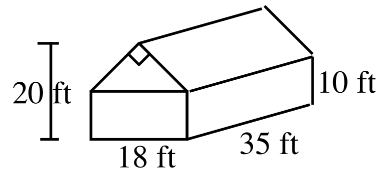
13.



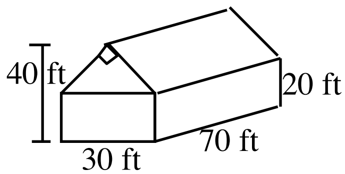
14.



15.



16.



17. Find the volume of a prism with base area 44 cm^2 and height 1.5 cm.

18. Find the volume of a prism with base area 75 cm^2 and height 26.2 cm.

19. Find the height of a prism with base area 32 cm^2 and volume 179 cm^3.

20. Find the area of the base of a prism with volume 760.48 cm^3 and height 9.8 cm.

Answers

1.	72 ft^3	2.	2448 cm^3	3.	2400 $units^3$	4.	390 cm^3	5.	255 $units^3$
6.	84 m^3	7.	804 cm^3	8.	51,840 ft^3	9.	364 ft^3	10.	440 cm^3
11.	416 ft^3	12.	315 cm^3	13.	2590 ft^3	14.	240 ft^3	15.	9450 ft^3
16.	66,150 ft^3	17.	66 cm^3	18.	1965 cm^3	19.	5.6 cm	20.	77.6 cm^2

WRITING AND GRAPHING LINEAR EQUATIONS #45

SLOPE (rate of change) is a number that indicates the steepness (or flatness) of a line, that is, its rate of change, as well as its direction (up or down) left to right.

SLOPE (rate of change) is determined by the ratio: $\dfrac{\text{vertical change}}{\text{horizontal change}} = \dfrac{\text{change in } y}{\text{change in } x}$

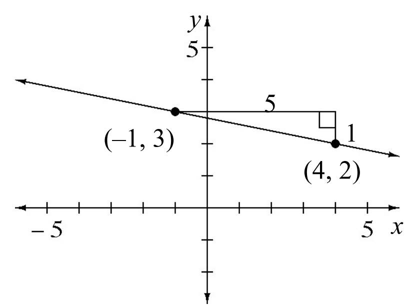
between <u>any</u> two points on a line. Some books and teachers refer to this ratio as the rise (y) over the run (x).

For lines that go **up** (from left to right), the sign of the slope is **positive**. For lines that go **down** (left to right), the sign of the slope is **negative**.

Any linear equation written as $\boldsymbol{y = mx + b}$, where m and b are any real numbers, is said to be in **SLOPE-INTERCEPT FORM**. **m** is the **SLOPE** of the line. **b** is the ***Y*-INTERCEPT**, that is, the point (0, b) where the line intersects (crosses) the y-axis.

Example 1

Write the slope of the line containing the points (–1, 3) and (4, 2).



First graph the two points and draw the line through them.

Look for and draw a slope triangle using the two given points.

Write the ratio $\dfrac{\text{vertical change in } y}{\text{horizontal change in } x}$ using the legs of the right triangle: $\frac{1}{5}$.

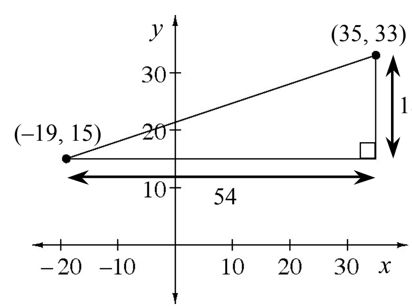
Assign a positive or negative value to the slope depending on whether the line goes up (+) or down (–) from left to right. The slope is $-\frac{1}{5}$.

Example 2

Write the slope of the line containing the points (–19, 15) and (35, 33).

Since the points are inconvenient to graph, use a "Generic Slope Triangle," visualizing where the points lie with respect to each other and the axes.

Make a sketch of the points.



Draw a slope triangle and determine the length of each leg. Write the ratio of y to x: $\frac{18}{54} = \frac{1}{3}$.
The slope is $\frac{1}{3}$.

Example 3

Given a table, determine the rate of change (slope) and the equation of the line.

+2 +2 +2

x	–2	0	2	4
y	1	4	7	9

+3 +3 +3

rate of change = $\frac{3}{2}$

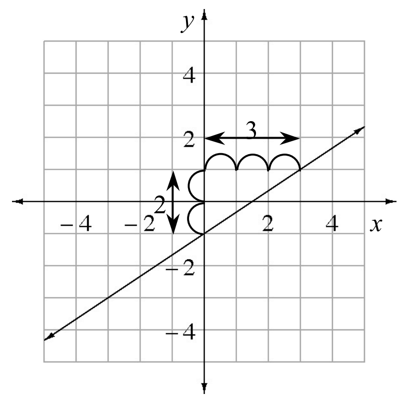
y - intercept = (0, 4)

so the equation of the line is $y = \frac{3}{2}x + 4$.

Example 4

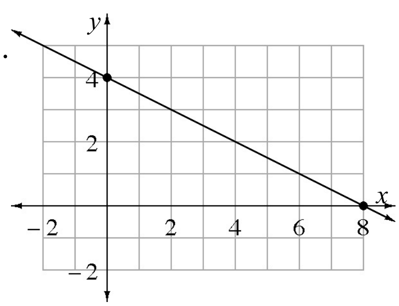
Graph the linear equation $y = \frac{2}{3}x - 1$

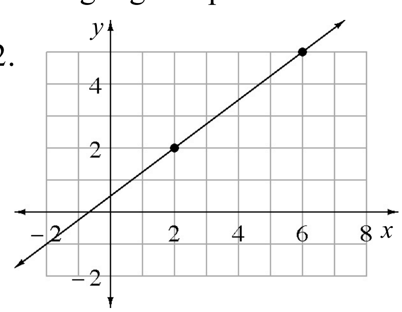
Using $y = mx + b$, the slope in $y = \frac{2}{3}x - 1$ is $\frac{2}{3}$ and the y-intercept is the point (0, –1). To graph, begin at the y-intercept (0, –1). Remember that slope is $\frac{\text{vertical change}}{\text{horizontal change}}$ so go up 2 units (since 2 is positive) from (0, –1) and then move right 3 units. This gives a second point on the graph. To create the graph, draw a straight line through the two points.

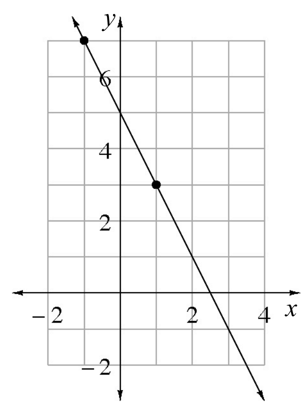


Problems

Determine the slope of each line using the highlighted points.

1.


2.


3.


Write the slope of the line containing each pair of points. Sketch a slope triangle to visualize the vertical and horizontal change.

4. (2, 3) and (5, 7)
5. (2, 5) and (9, 4)
6. (1, –3) and (7, –4)
7. (–2, 1) and (3, –3)
8. (–2, 5) and (4, 5)
9. (5, 8) and (3, 5)

Use a Generic Slope Triangle to write the slope of the line containing each pair of points:

10. (50, 40) and (30, 75)   11. (10, 39) and (44, 80)   12. (5, –13) and (–51, 10)

Identify the slope and y-intercept in each equation.

13. $y = \frac{1}{2}x - 2$ 14. $y = -3x + 5$ 15. $y = 4x$

16. $y = -\frac{2}{3}x + 1$ 17. $y = x - 7$ 18. $y = 5$

Draw a graph to find the equation of the line with:

19. slope = $\frac{1}{2}$ and passing through (2, 3).   20. slope = $\frac{2}{3}$ and passing through (3, –2).

21. slope = $-\frac{1}{3}$ and passing through (3, –1).   22. slope = –4 and passing through (–3, 8).

For each table, determine the rate of change and the equation. Be sure to record whether the rate is positive or negative for both x and y.

23.

x	–2	–1	0	1	2
y	–5	–2	1	4	7

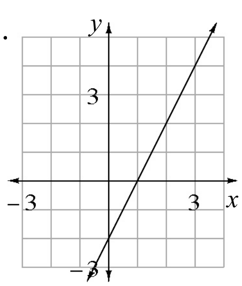
24.

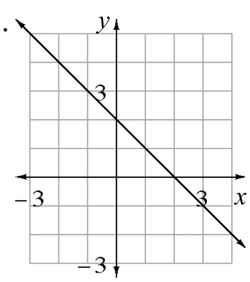
x	–2	0	2	4	6
y	7	3	–1	–5	–9

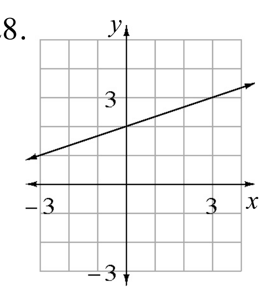
25.

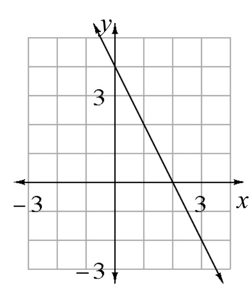
x	–6	–3	0	3	6
y	–3	–1	1	3	5

Using the slope and y-intercept, determine the equation of the line.

26.


27.


28.


29.


Graph the following linear equations on graph paper.

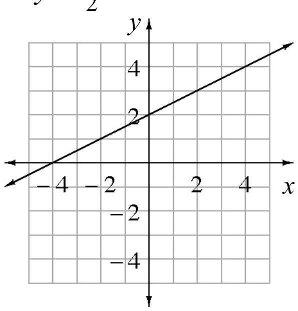
30. $y = \frac{1}{2}x + 2$ 31. $y = -\frac{3}{5}x + 1$ 32. $y = -4x$

33. $y = 2x + \frac{1}{2}$ 34. $3x + 2y = 12$

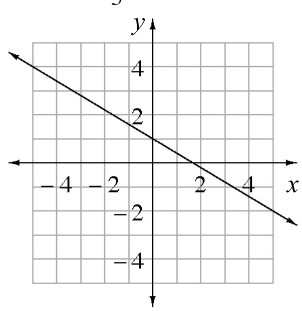
Answers

1. $-\frac{1}{2}$	2. $\frac{3}{4}$	3. -2	4. $\frac{4}{3}$
5. $-\frac{1}{7}$	6. $\frac{1}{6}$	7. $-\frac{4}{5}$	8. 0
9. $\frac{3}{2}$	10. $-\frac{35}{20}=-\frac{7}{4}$	11. $\frac{41}{34}$	12. $-\frac{33}{71}$
13. $\frac{1}{2}$; (0, –2)	14. -3; (0, –5)	15. 4; (0, 0)	16. $-\frac{2}{3}$; (0, 1)
17. 1; (0, –7)	18. 0; (0, 5)	19. $y=\frac{1}{2}x+2$	20. $y=\frac{2}{3}x-4$
21. $y=-\frac{1}{3}$	22. $y=-4x-4$	23. 3; $y=3x+1$	24. $-\frac{4}{2}=-2$; $y=-2x+3$
25. $\frac{2}{3}$; $y=\frac{2}{3}x+1$	26. $y=2x-2$	27. $y=-x+2$	28. $y=\frac{1}{3}x+2$
29. $y=-2x+4$			

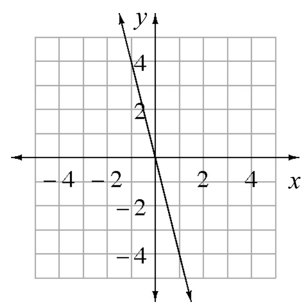
30. $y=\frac{1}{2}x+2$



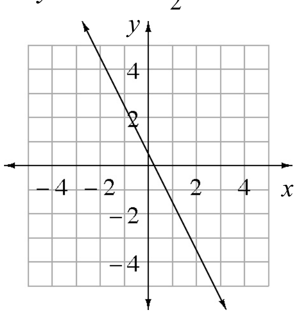
31. $y=-\frac{3}{5}x+1$



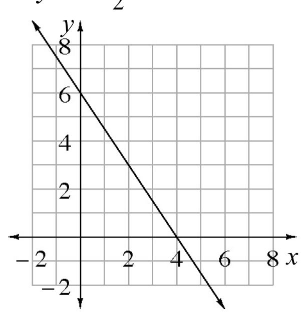
32. $y=-4x$



33. $y=-2x+\frac{1}{2}$



34. $y=-\frac{3}{2}x+6$



WRITING EQUATIONS USING THE 5-D PROCESS #46

You have used the 5-D Process to solve problems. However, solving complicated problems with the 5-D Process can be time consuming and it may be difficult to find the correct solution if it is not an integer. The patterns developed in the 5-D Process can be generalized by using a variable to write an equation. Once you have an equation for the problem, it is often more efficient to solve the equation than to continue to use the 5-D Process. Most of the problems here will not be complex so that you can practice writing equations using the 5-D Process.

Example 1

A box of fruit has 4 times as many oranges as grapefruit. Together there are 60 pieces of fruit. How many pieces of each type of fruit are there?

Describe: Number of oranges is 4 times the number of grapefruit.
Number of oranges plus number of grapefruit equals 60.

	Define		Do	Decide
	# of Grapefruit	# of Oranges	Total Pieces of Fruit	60?
Trial 1:	10	40	50	too low
Trial 2:	15	60	75	too high

After several trials to establish a pattern in the problem, you can generalize it using a variable. Since we could try any number of grapefruit, use x to represent it. The pattern for the number of oranges is four times the number of grapefruit, or $4x$. The total pieces of fruit is the sum of column one and column two, so our table becomes:

Define		Do	Decide
# of Grapefruit	# of Oranges	Total Pieces of Fruit	60?
x	$4x$	$x + 4x$	$= 60$

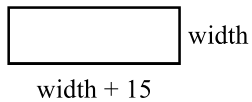
Since we want the total to agree with the check, our equation is $x + 4x = 60$. Simplifying this yields $5x = 60$, so $x = 12$ (grapefruit) and then $4x = 48$ (oranges).

Declare: There are 12 grapefruit and 48 oranges.

Example 2

The perimeter of a rectangle is 90 feet. If the length of the rectangle is 15 feet more than the width, what are the dimensions (length and width) of the rectangle?

Describe/Draw:



	Define		Do	Decide
	Width	Length	Perimeter	90?
Trial 1:	10	25	$(10 + 25) \cdot 2 = 70$	too low
Trial 2:	20	35	110	too high

Again, since we could guess any width, we labeled this column x. The pattern for the second column is that it is 15 more than the first: $x + 15$. Perimeter is found by multiplying the sum of the width and length by 2. Our table now becomes:

Define		Do	Decide
Width	Length	Perimeter	90?
x	$x + 15$	$(x + x + 15) \cdot 2$	$= 90$

Solving the equation:

$$(x + x + 15) \cdot 2 = 90$$
$$2x + 2x + 30 = 90$$
$$4x + 30 = 90$$
$$4x = 60$$

So $x = 15$ (width) and $x + 15 = 30$ (length).

Declare: The width is 15 feet and the length is 30 feet.

Example 3

Jorge has some dimes and quarters. He has 8 more dimes than quarters and the collection of coins is worth \$7.45. How many dimes and quarters does Jorge have?

Describe: The number of quarters plus 8 equals the number of dimes.
The total value of the coins is \$7.45.

	Define				**Do**	**Decide**
	Quarters	Dimes	Value of Quarters	Value of Dimes	Total Value	\$7.45?
Trial 1:	10	18	2.50	1.80	4.30	too low
Trial 2:	20	28	5.00	2.80	7.80	too high
	x	$x + 8$	$0.25x$	$0.10(x + 8)$	$0.25x + 0.10(x + 8)$	

Since you need to know both the number of coins and their value, the equation is more complicated. The number of quarters becomes x, but then in the table the Value of Quarters column is $0.25x$. Thus the number of dimes is $x + 8$, but the value of dimes is $0.10(x + 8)$. Finally, to find the numbers, the equation becomes $0.25x + 0.10(x + 8) = 7.45$.

Solving the equation:

$$0.25x + 0.10x + 0.8 = 7.45$$
$$0.35x + 0.8 = 7.45$$
$$0.35x = 6.65 \qquad \text{So } x = 19$$

Declare: There are 19 quarters worth \$4.75 and 27 dimes worth \$2.70 for a total value of \$7.45.

Problems

Start the problems using the 5-D Process. Then write an equation. Solve the equation.

1. A wooden board 100 centimeters long is cut into two pieces. One piece is 16 centimeters longer than the other. What are the lengths of the two pieces?

2. Thu is six years older than her brother Tuan. The sum of their ages is 48. What are their ages?

3. Tomas is thinking of a number. If he doubles his number and subtracts 27, the result is 407. Of what number is Tomas thinking?

4. Two consecutive numbers have a sum of 327. What are the two numbers?

5. Two consecutive even numbers have a sum of 186. What are the numbers?

6. Joanne's age is two times Devin's age and Devin is eight years older than Christena. If the sum of their ages is 76, what is Christena's age? Joanne's age? Devin's age?

7. Farmer Fran has 47 barnyard animals, consisting of only chickens and sheep. If these animals have 138 legs, how many of each type of animal are there?

8. A wooden board 228 centimeters long is cut into three parts. The two longer parts are the same length and are 18 centimeters longer than the shortest part. How long are the three parts?

9. Juan has 16 coins, all nickels and dimes. This collection of coins is worth $1.15. How many nickels and dimes are there?

10. Tickets to the school play are $7.00 for adults and $5.50 for students. If the total value of all the tickets sold is $3825 and 100 more students bought tickets than adults, how many adults and students bought tickets?

11. A wooden board 180 centimeters long is cut into six pieces: four short ones of equal length and two that are each 18 centimeters longer than the shorter ones. What are the lengths of the boards?

12. Conrad has a collection of three types of coins: nickels, dimes, and quarters. There are five more nickels than quarters but four times as many dimes as quarters. If the entire collection is worth $5.85, how many nickels, dimes, and quarters are there?

Answers (Equations may vary.)

1. $x + (x + 16) = 100$
   The lengths of the boards are 42 cm and 58 cm.

2. $x + (x + 6) = 48$
   Thu is 27 years old and her brother is 21 years old.

3. $2x - 27 = 407$
   Tomas is thinking of the number 217.

4. $x + (x + 1) = 327$
   The two consecutive numbers are 163 and 164.

5. $x + (x + 2) = 186$
   The two consecutive numbers are 92 and 94.

6. $x + (x + 8) + 2(x + 8) = 76$
   Christena is 13, Devin is 21, and Joanne is 42 years old.

7. $2x + 4(47 - x) = 138$
   Farmer Fran has 22 sheep and 25 chickens.

8. $x + (x + 18) + (x + 18) = 228$
   The lengths of the boards are 64, 82, and 82 cm.

9. $0.05x + 0.10(16 - x) = 1.15$
   Juan has 9 nickels and 7 dimes.

10. $\$7x + \$5.50(x + 100) = \$3825$
    There were 262 adult and 362 student tickets purchased for the play.

11. $4x + 2(x + 18) = 180$
    The lengths of the boards are 24 and 42 cm.

12. $0.25x + 0.05(x + 5) + 0.10(4x) = 5.85$
    Conrad has 8 quarters, 13 nickels, and 32 dimes.