Multiply and Divide Integers

To understand negative numbers, think this way: 3 means you won $3, and -3 means you lost $3.

The word "integer" means numbers like -3, -2, -1, 0, 1, 2, 3, ... It doesn't include decimals or fractions.

"Integer operations" refer to addition, subtraction, multiplication, division and other operations involving positive and negative integers. It's not that we will not deal with decimals in this course. Once we learn integer operations well, we can easily apply the same knowledge in decimal operations.

Before talking about integer operations, first we talk about the use of multiplication sign and division sign in this course.

The Multiplication Sign

We all have fond memories of the multiplication sign from grade school: $2 \times 3 = 6$. However, to differentiate the multiplication sign with the regularly-used variable *x*, we will stop using the multiplication sign, and use a dot instead: $2 \cdot 3 = 6$.

For the sake of simplicity, we can even skip the dot, as long as there is no confusion. For example, $2x$ means "2 times *x*", and $2(-3)$ means "2 times negative 3." However, we may not skip the dot in $2 \cdot 3 = 6$. Otherwise, we would be saying "23 equals 6." We could write $2(3) = 6$, though. When there is no sign between 2 and 3 in $2(3) = 6$, it implies multiplication.

The Division Sign

Similarly, we rarely use the division sign any more. We use the fraction line instead. Instead of writing $6 \div 2 = 3$, we write $\frac{6}{2} = 3$.

However, sometimes we still use the division sign, like in reducing fractions:

$$\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

Number Line

Number line is easy: The right side is the positive direction, and the left side is the negative direction. That's it!

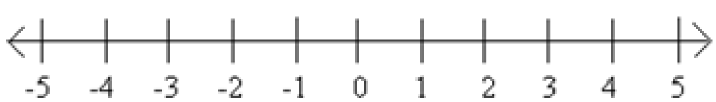

Figure 1: a number line

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Here are some examples:

$$2\cdot 3=6$$
$$-2\cdot 3=-6$$
$$2\cdot(-3)=-6$$
$$-2\cdot(-3)=6$$

$$\frac{6}{2}=3$$
$$\frac{-6}{2}=-3$$
$$\frac{6}{-2}=-3$$
$$\frac{-6}{-2}=3$$

One way to understand $-2\cdot 3=-6$ is: If I lose \$2 three times in a row, I would lose \$6. It's more difficult to understand $-2\cdot(-3)=6$.

Most people remember from middle school: negative times negative is positive. But why?

Here is one way to understand $-2\cdot(-3)=6$:

- -2 means you walk backward at the pace of 2 feet per step.
- -3 means you face backward and walk 3 steps.
- Since you face backward and walk backward, you end up walking forward by 6 feet.

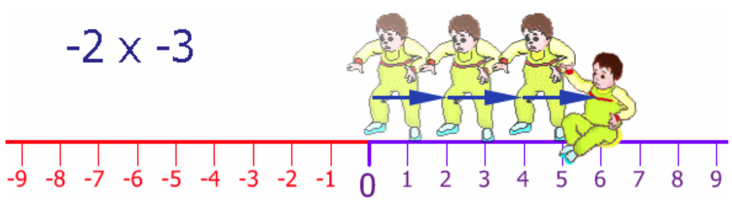

Figure 2: -2 times -3 equals +6, picture is from www.mathisfun.com

In this picture, remember the right side is positive direction in a number line.

Later, you will learn that division can be changed into multiplication. If a rule works for multiplication, most of the time it also works for division. This is why $\frac{-6}{2}=-3$ and $\frac{-6}{-2}=3$.

[**Example 1**] Calculate $(-2)(-1)(-3)(-2)$

[**Solution**] We could do this step by step: $(-2)(-1)(-3)(-2)=2(-3)(-2)=(-6)(-2)=12$.

However, an easier way is to first find out whether the final product is positive or negative. In this problem, 4 negative numbers multiply each other. Each pair of negative signs cancel each other, so the final product is positive. We can get rid of those negative signs, and the problem becomes: $(-2)(-1)(-3)(-2) = 2 \cdot 1 \cdot 3 \cdot 2$. Now we can do the rest in one step:

$$(-2)(-1)(-3)(-2) = 2 \cdot 1 \cdot 3 \cdot 2 = 12$$

[**Example 2**] Calculate $(-2)(-1)(-3) \cdot 2$

[**Solution**] This time, there are 3 negative signs. Two of them cancel each other, and one negative sign stays. So the final product is negative. We have:

$$(-2)(-1)(-3) \cdot 2 = -2 \cdot 1 \cdot 3 \cdot 2 = -12$$

Division Involving 0

Look at the difference: $\frac{0}{2} = 0$, and $\frac{2}{0} = \textit{undefined}$.

Don't try to memorize them. Otherwise you will forget soon. Try to understand why.

There are two ways to explain this, and I will show both ways.

Method 1: One way to interpret $\frac{6}{2} = 3$ is: If 2 people share 6 apples evenly, each person gets 3 apples.

Similarly, we can interpret $\frac{0}{2} = 0$ as: If 2 people share 0 apples evenly, each person gets 0 apple.

Try the same interpretation for $\frac{2}{0}$: If 0 people share 2 apples evenly, each person gets how many apples? This interpretation doesn't make sense, because we cannot say "each person" when there is "0 people." This is why $\frac{2}{0}$ is undefined.

Method 2: Multiplication and division are inverse operations. We can always turn division into multiplication, and vice versa. For example:

For $\frac{6}{2} = 3$, we have $2 \cdot 3 = 6$.

For $\frac{6}{1} = 6$, we have $1 \cdot 6 = 6$.

For $\frac{0}{2} = 0$, we have $2 \cdot 0 = 0$.

For $\frac{2}{0} = \text{a number}$, we have $0 \cdot (\text{a number}) = 2$. We cannot find such a number, because 0 times any number is always 0! This is why $\frac{2}{0}$ is undefined.

In this lesson, we learned how to multiply and divide integers. We will learn how to add and subtract integers next.