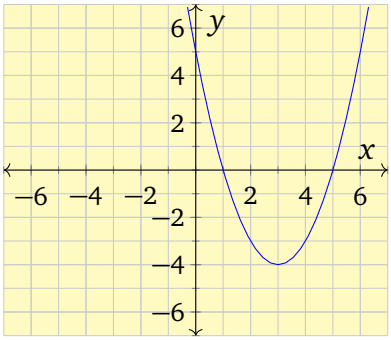
Quadratic Graphs and Vertex Form

1. Here is the graph of a quadratic function.



a) Does it open upward or downward?

b) Where is its vertex?

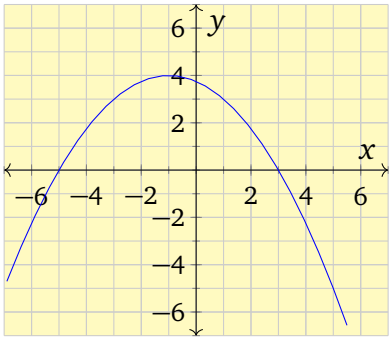
c) What is its axis of symmetry?

d) What is the function's domain and range?

e) What is its vertical intercept?

f) Does the function have horizontal intercepts? If so, what are they?

2. Here is the graph of a quadratic function.



a) Does it open upward or downward?

b) Where is its vertex?

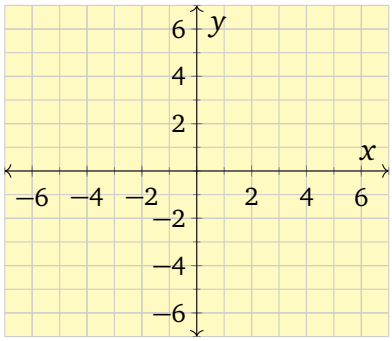
c) What is its axis of symmetry?

d) What is the function's domain and range?

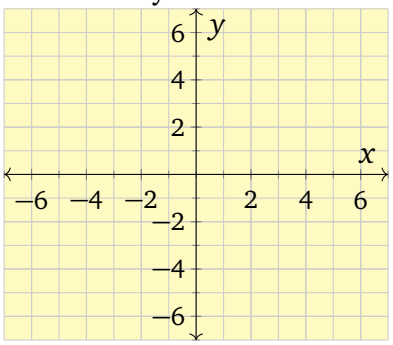
e) What is its vertical intercept?

f) Does the function have horizontal intercepts? If so, what are they?

3. Sketch the graph of a quadratic function that does not have any horizontal intercepts.



4. Sketch the graph of a quadratic function that opens downward and has exactly one horizontal intercept.



5. Let f be a function defined by $f(x) = 4x^2 + 9x + 55$. In the following, it is OK to use a calculator to do arithmetic, but please do not use a calculator to do any graphing.

   a) Where is the vertex of the graph of f?

   b) In the graph of f, where is the y-intercept?

   c) In the graph of f, are there any x-intercepts, and if so, where are they?

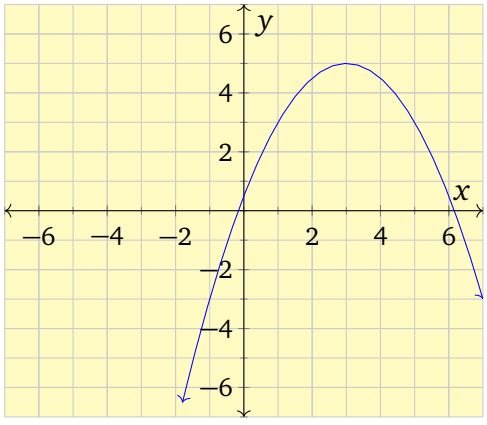
   d) Use your answers so far to sketch a graph of f. The "special points" may not be at nice whole numbers, so you should label them.

6. An object was launched from the top of a hill (at 30 feet above sea level) with an upward vertical velocity of 100 feet per second. The height of the object can be modeled by the function $h(t) = -16t^2 + 100t + 30$, where t represents the number of seconds after the launch. Assume the object landed on the ground at sea level. How high did the object get before it started to fall back down? How many seconds did it take to get that high? (It is OK to use a calculator to do arithmetic.)

7. Where is the vertex of $y = 3(x - 9.1)^2 - 3.6$?

8. Write the vertex form for the quadratic function f, whose vertex is $(1, 9)$ and has leading coefficient $a = -8$.

9. A graph of a function f is given. Use the graph to write a formula for f in vertex form. You will need to identify the vertex and also one more point on the graph to find the leading coefficient.



10. Currently, an artisan can sell 60 handmade dining tables every year at the price of \$900 per table. Each time she raises the price by \$20.00, she sells 1 fewer table per year. Let f be a function where the input is how many times she raises the price by \$20, and the output is how much revenue she takes in. Then

$$f(x) = (\text{price}) \cdot (\text{number sold})$$
$$f(x) = (900 + 20x)(6 - x)$$
$$f(x) = 5400 - 780x - 20x^2$$

How many times should she raise the price by \$20 to maximize revenue?

11. You will build a rectangular sheep enclosure next to a river. There is no need to build a fence along the river, so you only need to build on three sides. You have a total of 440 feet of fence to use. Find the dimensions of the pen such that you can enclose the maximum possible area. One approach is to let x represent the length of fencing that runs perpendicular to the river, and write a formula for a function of x that outputs the area of the enclosure. Then find its vertex and interpret it.