MTH 95

Final Exam Review

Complete the following exercises to help you review for the final exam. This review sheet is intended as a problem set for you to practice. The solution key is posted along with this document in our final module.

Part 1: No Calculator

Factoring and Solving Quadratic Equations by Factoring

1) Factor the following polynomials completely.

a) x^2-25 difference of squares
$=(x+5)(x-5)$

b) $x^2-12x+35$ monic quadratic trinomial
$=(x-5)(x-7)$

c) $2y^3-6y^2-56y$ GCD first
$=2y(y^2-3y-28)$
$=2y(y+4)(y-7)$

d) $p^2+12p+36$ perfect square trinomial
$=(p+6)^2$

2) Solve the following quadratic quadratic equations by factoring.

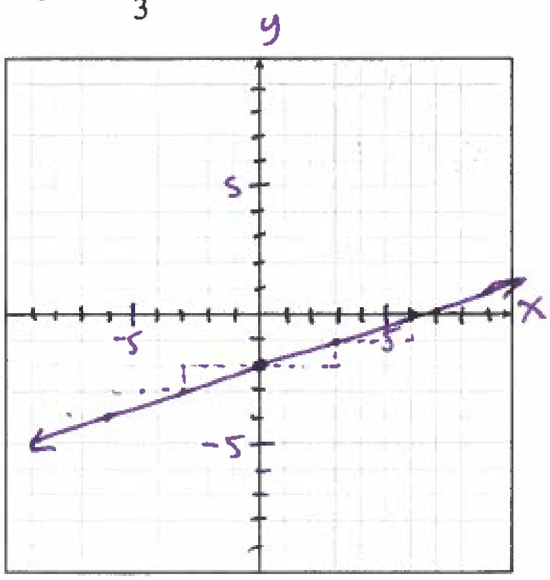
a) $2x^2-7x+6=0$ $A\cdot C=12$
$(-3)(-4)=12$ ✓
$(-3)+(-4)=-7$
$2x^2-3x-4x+6=0$
$x(2x-3)-2(2x-3)=0$
$(2x-3)(x-2)=0$
$2x-3=0$ or $x-2=0$
$2x=3$
$x=\frac{3}{2}$ or $x=2$

b) $2(x+3)=x(x+1)$
$2x+6=x^2+x$
$0=x^2-x-6$
$0=(x+2)(x-3)$
$x=-2$ or $x=3$

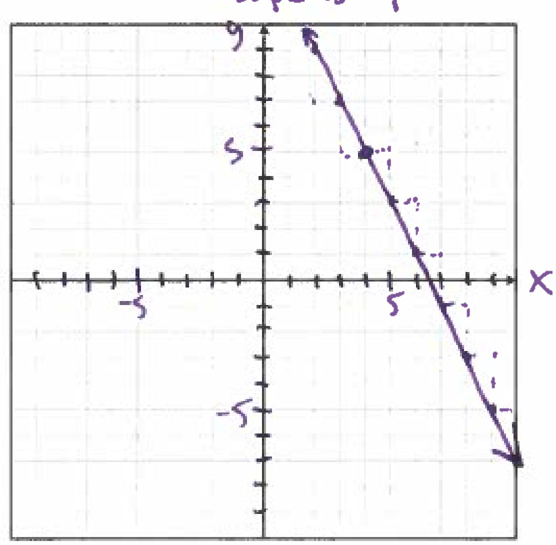
Sketching Graphs of Linear Functions by Hand

3) Sketch the graph of $y=f(x)$ for each of the following linear functions WITHOUT a calculator. Show at least two points.

a) $f(x)=\frac{1}{3}x-2$



b) $f(x)=-2(x-4)+5$
(4,5) is on this
slope is $\frac{-2}{1}$



Evaluating Functions and Determining Their Domain and Range

The domain of a function is the set of all possible inputs, which are typically x-values. The range of a function is the set off all possible outputs, which are typically y-values.

When determining the domain of a function based on the formula:

- Exclude any numbers that cause **division by zero.**
- Exclude any numbers that cause **the square root of a negative number** to occur.

4) Algebraically determine the domain of the following functions. State each domain using set-builder notation AND interval notation. Then find $f(0)$ and $f(-2)$ for each function.

Function	Domain (SB Notation)	Domain (Interval Notation)	$f(0)$	$f(-2)$
$f(x)=-5x+7$	$\{x \mid x$ is a real #$\}$	$(-\infty,\infty)$	7	17
$f(x)=\sqrt{-2x+3}$	Need $-2x+3\geq 0$ $-2x\geq -3$ $x\leq 1.5$ $\{x \mid x\leq 1.5\}$	$(-\infty, 1.5]$	$\sqrt{3}$	$\sqrt{7}$
$f(x)=-3(x-1)^2-4$	$\{x \mid x$ is a real #$\}$	$(-\infty,\infty)$	-7	-31
$f(x)=\lvert 3x-6\rvert$	$\{x \mid x$ is a real #$\}$	$(-\infty,\infty)$	6	12
$f(x)=\dfrac{4x+1}{3x-6}$	Need $3x-6\neq 0$ $3x\neq 6$ $x\neq 2$ $\{x \mid x\neq 2\}$	$(-\infty,2)\cup(2,\infty)$	$-\frac{1}{6}$	$\frac{7}{12}$
$f(x)=\dfrac{x-6}{x^2+x-6}$	Need $x^2+x-6\neq 0$ $(x+3)(x-2)\neq 0$ $x\neq -3$ and $x\neq 2$ $\{x \mid x\neq -3, x\neq 2\}$	$(-\infty,-3)\cup(-3,2)\cup(2,\infty)$	1	2

5) State the domain and range of each function below (Figure 1 and Figure 2) using interval notation.

a) Domain: $(-\infty,\infty)$

Range: $[-4,\infty)$

b) Domain: $[-6,-4)\cup(-1,1]$

Range: $(-3,-1]\cup(2,6]$

FIGURE 1

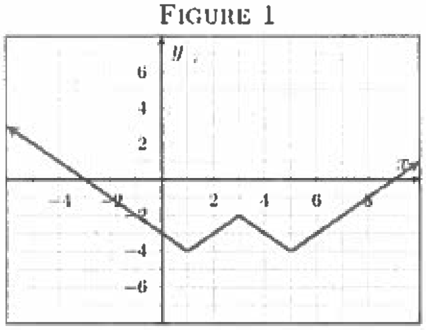
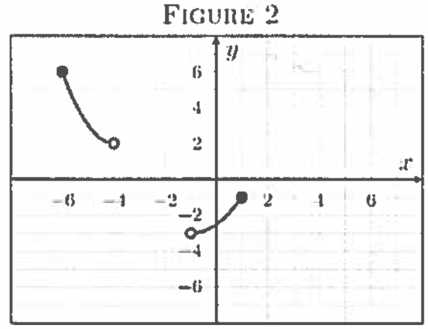


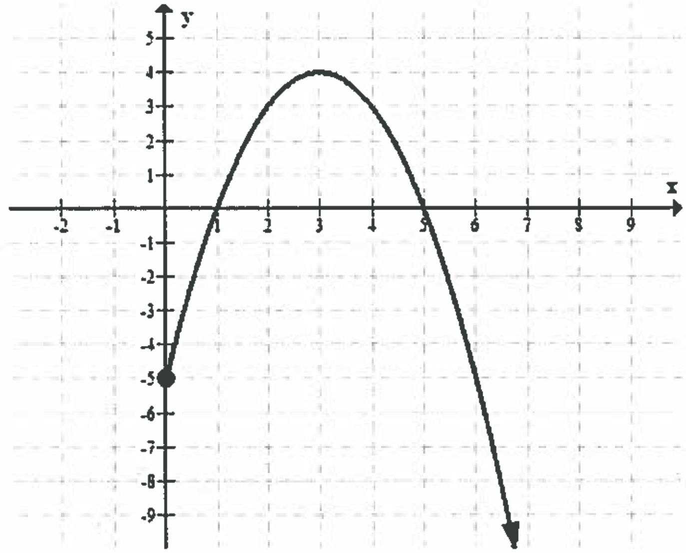
FIGURE 2



Evaluating, Solving, and Determining Domain and Range Using the Graph of a Function

6) For the following graph of f in Figure 3, determine

Figure 3



a) $f(2) = 3$

b) $f(0) = -5$

c) Any x-value(s) for which $f(x) = 0$

$x = 1$ or $x = 5$

d) Any x-value(s) for which $f(x) = -5$

$x = 0$ or $x = 6$

e) Any x-value(s) for which $f(x) = 4$

$x = 3$

f) Any x-value(s) for which $f(x) > 3$

$(2, 4)$

g) Any x-value(s) for which $f(x) \leq 0$

$[0, 1] \cup [5, \infty)$

h) The domain of f in interval notation.

$[0, \infty)$

I) The range of f in interval notation.

$(-\infty, 4]$

Simplifying Expressions with Function Notation

7) Let $p(x) = 4x^2 - 9x + 7$. Find:

a) $p(x-3)$
$$= 4(x-3)^2 - 9(x-3) + 7$$
$$= 4(x^2 - 6x + 9) - 9x + 27 + 7$$
$$= 4x^2 - 24x + 36 - 9x + 34$$
$$= 4x^2 - 33x + 70$$

b) $p(-5x)$
$$= 4(-5x)^2 - 9(-5x) + 7$$
$$= 4(25x^2) + 45x + 7$$
$$= 100x^2 + 45x + 7$$

c) $p(x) + 4$
$$= 4x^2 - 9x + 7 + 4$$
$$= 4x^2 - 9x + 11$$

d) $8p(x)$
$$= 8(4x^2 - 9x + 7)$$
$$= 32x^2 - 72x + 56$$

Simplifying Rational Expressions

There are four important facts about rational expressions and equations:

- Only **factors** cancel.
- Restrictions occur when the original expression or function was undefined, but the simplified expression or function is not. These need to be stated in order for equivalence to hold.
- A rational **expression** is equivalent to another rational expression when BOTH the **numerator and denominator** are multiplied by the SAME expression.
- An **equation** with rational expressions is equivalent to another equation when BOTH **sides of the equation** are multiplied by the SAME expression.

8) Perform the indicated operation and simplify the expression as much as possible. Include any necessary restrictions.

a) $\frac{4}{t^2+9t+8}\cdot\frac{t+8}{2t-6} = \frac{\cancel{4}\,2}{\cancel{(t+8)}(t+1)}\cdot\frac{\cancel{(t+8)}}{\cancel{2}(t-3)}$

$= \frac{2}{(t+1)(t-3)},\ t \neq -8$

Factor and flip divisor

b) $\frac{2x+4}{x^2-5x+6}\div\frac{x^2+4x+4}{4x-8} = \frac{2\cancel{(x+2)}}{\cancel{(x-2)}(x-3)}\cdot\frac{4\cancel{(x-2)}}{(x+2)^{\cancel{2}}}$

$= \frac{8}{(x-3)(x+2)},\ x \neq 2$

Need common denoms.
Start by clarifying factors

c) $\frac{y+2}{y-3}-\frac{y+1}{y+4} = \frac{(y+2)}{(y-3)} - \frac{(y+1)}{(y+4)}$

$= \frac{(y+2)(y+4)}{(y-3)(y+4)} - \frac{(y+1)}{(y+4)}\frac{(y-3)}{(y-3)}$

$= \frac{y^2+6y+8}{(y-3)(y+4)} - \frac{y^2-2y-3}{(y-3)(y+4)}$

$= \frac{8y+11}{(y-3)(y+4)}$

d) $\frac{3}{x+9}+\frac{15}{x^2+13x+36} = \frac{3}{(x+9)} + \frac{15}{(x+9)(x+4)}$

$= \frac{3}{(x+9)}\cdot\frac{(x+4)}{(x+4)} + \frac{15}{(x+9)(x+4)}$

$= \frac{3x+12}{(x+9)(x+4)} + \frac{15}{(x+9)(x+4)}$

$= \frac{3x+27}{(x+9)(x+4)}$

$= \frac{3\cancel{(x+9)}}{\cancel{(x+9)}(x+4)}$

$= \frac{3}{x+4},\ x \neq -9$

LCM of internal denoms is $t(t+4)(t-4)$

9) Simplify the complex fractions below as much as possible. Include any necessary restrictions.

LCM of internal denoms is $(x-3)$

a) $\dfrac{\dfrac{3}{\cancel{(x-3)}}}{\dfrac{x}{(x-3)}+2} \cdot \dfrac{\cancel{(x-3)}}{(x-3)}$

$= \dfrac{3}{\frac{x}{\cancel{(x-3)}}\cancel{(x-3)}+2(x-3)}$

$= \dfrac{3}{x+2x-6}$

$= \dfrac{3}{3x-6}$

$= \dfrac{3}{3(x-2)}$

$= \dfrac{1}{x-2}, \quad x \neq 3$

b) $\dfrac{\dfrac{t}{t^2-16}}{\dfrac{4t-16}{t^2+4t}} = \dfrac{\dfrac{t}{\cancel{(t+4)(t-4)}}}{\dfrac{4(t-4)}{\cancel{t(t+4)}}} \cdot \dfrac{t\cancel{(t+4)(t-4)}}{\cancel{t(t+4)}(t-4)}$

$= \dfrac{t^2}{4(t-4)^2}, \quad t \neq 0, -4$

Completing the Square

10) Write the quadratic function $q(x)=x^2+12x+5$ in vertex form by completing the square. Identify the vertex.

$\left(\frac{12}{2}\right)^2 = 6^2$
$= 36$

$= x^2+12x+36-36+5$
$= (x+6)^2-31$

The vertex is at $(-6,-31)$.

11) Solve the quadratic equation $x^2-6x+1=-5$ using completing the square. Clearly state all solutions in a solution set.

$\left(\frac{-6}{2}\right)^2 = (-3)^2$
$= 9$

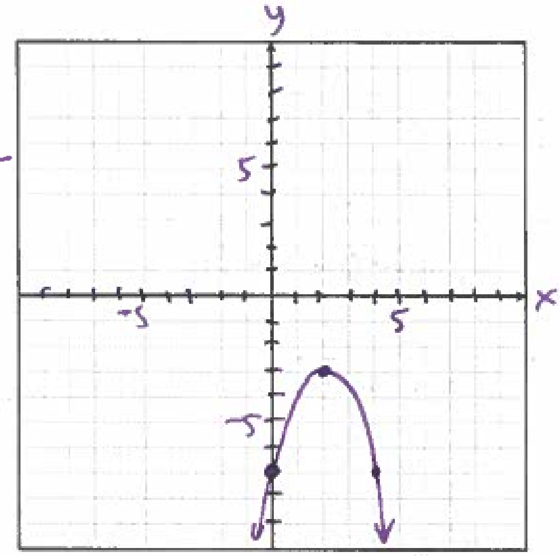
$x^2-6x = -6$
$x^2-6x+9 = -6+9$
$(x-3)^2 = 3$
$x-3 = \pm\sqrt{3}$
$x = 3 \pm \sqrt{3}$

The solution set is $\{3-\sqrt{3},\ 3+\sqrt{3}\}$

12) Graph $f(x) = -x^2 + 4x - 7$ by algebraically determining its key features. Then state the domain and range of the function.

$h = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2$

$k = -2^2 + 4(2) - 7 = -4 + 8 - 7 = -3$



Vertex: $(2,-3)$

x-intercepts: NONE

y-intercept: $(0,-7)$

Domain: $(-\infty,\infty)$

Range: $(-\infty,-3]$

$-x^2+4x-7=0$

$x^2-4x+7=0$

$x^2-4x = -7$

$x^2-4x+4=-3$

$(x-2)^2=-3$

Not possible! There are no x-intercepts.

Solving Equations and Inequalities Algebraically

13) Algebraically solve the following equations. Check solutions and state the solutions in a solution set.

a) $|2x-9|-4=7$

$|2x-9| = 11$

$2x-9=11$ or $2x-9=-11$

$2x=20$ or $2x=-2$

$x=10$ or $x=-1$

b) $2x^2 = -11x-5$

$2x^2+11x+5=0$

$A \cdot C = 10$

$(10)(1)=10$

$(10)+(1)=11$

$2x^2+10x+x+5=0$

$2x(x+5)+1(x+5)=0$

$(x+5)(2x+1)=0$

$x+5=0$ or $2x+1=0$

$x=-5$ or $2x=-1$

$x=-\frac{1}{2}$

c) $\frac{x}{x^2-10x+25} = \frac{-4}{x^2-x-20}$

$\frac{x}{(x-5)^2} = \frac{-4}{(x-5)(x+4)}$

LCM of denominators is $(x-5)^2(x-4)$

$\frac{x}{\cancel{(x-5)^2}}\cancel{(x-5)^2}(x+4) = \frac{-4}{\cancel{(x-5)(x+4)}}(x-5)^{\cancel{2}}\cancel{(x+4)}$

$x(x+4) = -4(x-5)$

$x^2+4x=-4x+20$

$x^2+8x+20=0$

$(x+10)(x-2)=0$

$x=-10$ or $x=2$

d) $\sqrt{2x+18}+3=x$

$\sqrt{2x+18} = x-3$

$2x+18 = (x-3)^2$

$2x+18 = x^2-6x+9$

$0 = x^2-8x-9$

$0 = (x-9)(x+1)$

$x=9$ or $x=-1$

✓ doesn't check out!

Solution set is $\{9\}$.

e) $2(x+3)=3(x-7)$

$2x+6 = 3x-21$

$27 = x$

$x = 27$

f) $x(x+4)=-4(x-5)$

$x^2+4x = -4x+20$

$x^2+8x-20 = 0$

$(x+10)(x-2) = 0$

$x=-10$ or $x=2$

g) $\frac{1}{x+2}+\frac{1}{x-2}=\frac{4}{x^2-4}$

$(x+2)(x-2)\left(\frac{1}{(x+2)}+\frac{1}{(x-2)}\right) = (x+2)(x-2)\cdot\frac{4}{(x+2)(x-2)}$

$x-2 + x+2 = 4$

$2x = 4$

$x = 2$

But $x=2$ doesn't check!
So there are no solutions!

h) $|-3+3x|=|-2x-6|$

$-3+3x = -2x-6$ or $-3+3x = -(-2x-6)$

$5x = -3$ $\qquad -3+3x = 2x+6$

$x = -3/5$ or $x = 9$

14) Solve $R=\frac{gs}{g+s}$ *for* s.

$R\cdot(g+s) = g\cdot s$

$R\cdot g + R\cdot s = g\cdot s$

$R\cdot s - g\cdot s = -R\cdot g$

$s(R-g) = -Rg$

$s = \frac{-Rg}{R-g}$

15) Algebraically solve the following compound inequalities and state the solution set in interval notation.

a) $-8<-4x+12\le 16$

$-8<-4x+12$ and $-4x+12\le 16$

$-20<-4x$ $\quad$ $-4x\le 4$

$5>x$ $\quad$ $x\ge -1$

$x<5$

$[1,5)$

b) $2x+5<-17$ *or* $-4x+10\le 34$

$2x<-22$ $\quad$ $-4x\le 24$

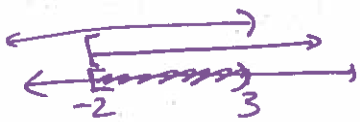
$x<-11$ or $x\ge -6$

$(-\infty,-11)\cup[-6,\infty)$

c) $2-4t\le 10$ *and* $4+2t\le 10$

$-4t\le 8$ $\quad$ $2t\le 6$

$t\ge -2$ and $t\le 3$



$[-2,3)$

d) $1-3x\ge -5$ *or* $x-5\le -8$

$-3x\ge -6$ $\quad$ $x\le -3$

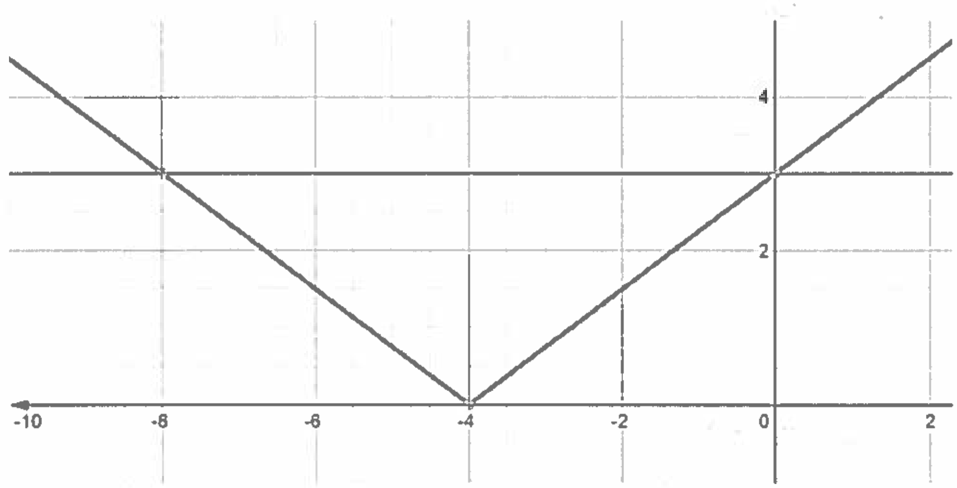
$x\le 2$ or $x\le -3$

$(-\infty,2]$

Solving Equations and Inequalities Graphically

16) The equations $y=\left|\frac{3}{4}x+3\right|$ and $y=3$ are plotted in Figure 4. Solve the equation and inequalities graphically. State the solution set using interval notation where applicable.

Figure 4



a) $\left|\frac{3}{4}x+3\right|=3$

$x=-8$ or $x=0$

b) $\left|\frac{3}{4}x+3\right|>3$

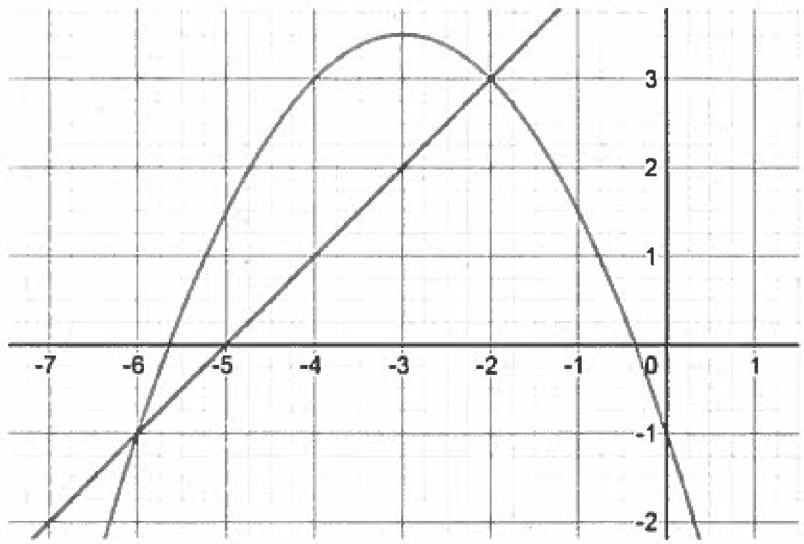
$(-\infty,-8)\cup(0,\infty)$

c) $\left|\frac{3}{4}x+3\right|\le 3$

$[-8,0]$

17) The equations $y=-0.5x^2-3x-1$ and $y=x+5$ are plotted in Figure 5. Solve the equation and inequalities graphically.

Figure 5



a) $-0.5x^2-3x-1=x+5$

$x=-6$ or $x=-2$

b) $-0.5x^2-3x-1\geq x+5$

$[-6,-2]$

c) $-0.5x^2-3x-1<x+5$

$(-\infty,-6)\cup(2,\infty)$

Part 2: Calculator Permitted

<u>Applications</u>

18) Because of startup expenses, suppose that it costs \$100 to make the first specialty cookie and \$1 for each cookie after that. The average cost, in dollars per cookie, to make *x* cookies is given by $C(x)=\dfrac{x+99}{x}$.

a) Find *C*(1) and interpret the result. Use a whole sentence and include units.

$C(1)=\dfrac{100}{1}=100$

The average cost per cookie to make 1 cookie is \$100 per cookie.

b) Find *C*(200) and interpret the result. Use a whole sentence and include units.

$C(200)=\dfrac{299}{200}=1.995$

The average cost per cookie to make 200 cookies is \$1.995 per cookie.

c) Solve *C*(*x*) = 1.19 algebraically, and interpret the result. Use a whole sentence and include units.

$\dfrac{x+99}{x}=1.19$

$x+99=1.19x$

$99=0.19x$

$\dfrac{99}{0.19}=x$

$x\approx 521.05\ldots$

In order to get the cost per cookie down to \$1.19, you would have to make 521 or 522 cookies.

19) Georgia bicycles 12 mph with no wind. Against the wind, she bikes 8 mi in the same time that it takes to bike 14 mi with the wind. What is the speed of the wind? State your final solution using a complete sentence.

Let x be this, in mph.

	distance (mi)	speed	time
with wind	14	$12+x$	$\frac{14}{12+x}$
against wind	8	$12-x$	$\frac{8}{12-x}$

Supposed to be equal

$$\frac{14}{12+x} = \frac{8}{12-x}$$

$$14(12-x) = 8(12+x)$$

$$168 - 14x = 96 + 8x$$

$$-22x = -72$$

$$x = \frac{72}{22} = \frac{36}{11} \approx 3.27$$

The wind speed is about 3.27 mph.

20) Suppose one painter can paint an entire house in twelve hours, and a second painter takes eight hours. How long would it take the two painters working together to paint the house? State your final solution using a complete sentence.

Let this be x, in hrs.

	Amount (houses)	time (hr)	rate
one painter	1	12	$\frac{1}{12}$
another	1	8	$\frac{1}{8}$
together	1	x	$\frac{1}{x}$

should add up to this

$$\frac{1}{12} + \frac{1}{8} = \frac{1}{x}$$

LCM of denominators is $24x$

$$24x\left(\frac{1}{12} + \frac{1}{8}\right) = 24x \cdot \frac{1}{x}$$

$$2x + 3x = 24$$

$$\cancel{20x = 96}$$

$$5x = 24$$

$$x = \frac{24}{5} = 4.8$$

Together it would take them 4.8 hours to paint the house.

21) You drive 90 miles along the Pacific Coast Highway and then take a 5-mile run along a hiking trail. Your driving rate is nine times that of your running rate. If the total time for driving and running is 3 hours, find the average rate of running and the average rate of driving. Write your final answer using a complete sentence including units.

Let x be your running rate (mph).

	distance (mi)	rate	time
drive	90	$9x$	$\frac{90}{9x}$
hike/run	5	x	$\frac{5}{x}$

in total, was 3 hours

$$\frac{90}{9x} + \frac{5}{x} = 3$$
$$\frac{10}{x} + \frac{5}{x} = 3$$
$$\frac{15}{x} = 3$$
$$15 = 3x$$
$$5 = x$$

You run at 5 mph, and drive at 45 mph.

22) Working together, Erica and Shawna can answer a day's worth of technical support questions in 4 hr. Working alone, Erica takes 6 hr longer than Shawna. How long would it take Shawna to answer the questions alone?

Let this be x

	amount (day's worth of questions)	time	rate
Erica	1	$x+6$	$\frac{1}{x+6}$
Shawna	1	x	$\frac{1}{x}$
together	1	4	$\frac{1}{4}$

Should add up to

$$\frac{1}{x+6} + \frac{1}{x} = \frac{1}{4}$$
$$4x(x+6)\left(\frac{1}{x+6} + \frac{1}{x}\right) = 4x(x+6)\frac{1}{4}$$
$$4x + 4(x+6) = x(x+6)$$
$$4x + 4x + 24 = x^2 + 6x$$
$$0 = x^2 - 2x - 24$$
$$0 = (x-6)(x+4)$$

$x = 6$ or $x = -4$

In context only $x=6$ makes sense. It takes Shawna 6 hours to answer the questions if she works alone.