Exploring Rational Functions

1. Let $f(x) = \dfrac{x+1}{x^4+1}$.

   a) Make a table of values for f. Decimals are OK. Use $x = -5$ to $x = 5$.

   b) What does your table suggest will happen when x gets very large?

   c) Use graphing technology to see the graph of f. If appropriate, adjust the viewing window.

2. Let $g(x) = \dfrac{x+1}{2x^2-3x-14}$.

   a) Make a table of values for g. Decimals are OK. Use $x = -5$ to $x = 5$.

   b) Use graphing technology to see the graph of g. If appropriate, adjust the viewing window.

   c) What does your graph suggest will happen when x gets very large?

   d) What is the domain of g? Use the formula to figure this out. Look back on the graph to see how it agrees with your answer.

3. Let $h(x) = \dfrac{x^3+1}{x-1}$.

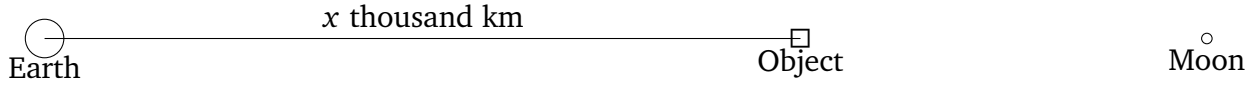
   a) Make a table of values for h. Decimals are OK. Use $x = -5$ to $x = 5$.

   b) Use graphing technology to see the graph of h. If appropriate, adjust the viewing window.

   c) What does your graph suggest will happen when x gets very large?

   d) What is the domain of h? Use the formula to figure this out. Look back on the graph to see how it agrees with your answer.

4. Suppose an object is in between the Earth and the Moon, x thousand kilometers from the center of the Earth.



An object like experiences some gravitational acceleration from these two things pulling on it. This acceleration (measured in meters per second, per second) is a function of x, and has formula

$$a(x) = \frac{-393.67x^2 + 306425x - 58895000}{x^2(384.4 - x)^2}$$

a) Standing here on the surface of the Earth, we are 6.371 thousand kilometers from the center of the Earth. Find $a(6.371)$ (using a calculator) and write a sentence that uses that number in context.

b) Use a graphing calculator to find where the object would need to be for it to feel no net gravitational pull. (Where it should be for the pull of the Earth and the pull of the Moon to cancel each other out.)

c) From the center of the earth to the surface of the moon is about 382.66 thousand kilometers. What is the acceleration from gravity on the moon's surface?

d) Compared to being on the Moon, how many times heavier are you on Earth?

e) What is the domain of the rational function given by the formula above?

f) What is the domain of a? Hint: this is different from the last question, because in context there are x-values that would not make any sense. Use Google to look of the radius of the Earth, the radius of the Moon, and the distance between them.

5. Engineers are building a curved section of freeway onramp that needs to be banked. They have discovered that if the speed of cars is about 40 mph, then there is a relation between the radius R of the curve and the slope m of the banking. $m(R) = \frac{1600-2R}{15R}$. Plot this rational function using graphing technology. What radius would yield a banking of 10%? (That is, a slope of 0.1?)