Rational Functions and Equations

Solving Rational Equations

Rational Equation

A **rational equation** is an equation that contains one or more rational expressions.

Examples:

$$\frac{1}{3} + \frac{2}{7} = \frac{1}{x} \quad , \quad \frac{4}{(x+3)} = \frac{1}{x^2 + 6x + 9} \quad , \quad x + \frac{3}{x} = 4$$

To Solve a Rational Equation

Solving Equations Containing Rational Expressions

1. Determine the LCD of all of the rational expressions in the equation, regardless of which side of the equal sign the expression occurs.

2. Make note of any domain restrictions.

3. Multiply **both** sides of the equation by the LCD.

4. Solve the resulting equation.

5. Check the proposed solution(s). This is critical, as the process for solving rational equations sometimes yields false solutions.

6. State the solution set.

Solve each equation.

$$\frac{12}{t} = -3$$

$$\frac{x-1}{2x+3} = \frac{2}{5}$$

The solution set is_____

The solution set is_____

Solve each equation.

$$\frac{1}{2x} + \frac{1}{3x} = 2$$

$$\frac{r+6}{r^2 - 36} = 0$$

The solution set is_____

The solution set is_____

Solve the equation.

$$y - \frac{14}{y} = 5$$

The solution set is_____

Solve the equation.

$$\frac{1}{x+5} = \frac{2}{2x+1}$$

The solution set is_____

Solve the equation.

$$\frac{x^2 - 1}{x + 2} = \frac{3}{x + 2}$$

The solution set is_____

Solve the equation.

$$\frac{y+3}{y+2} - \frac{y}{y^2-4} = \frac{y}{y-2}$$

The solution set is_____

Use technology to solve the rational equation graphically. Approximate your answer(s) to the nearest hundredth when appropriate.

$$\frac{5}{x^2+7} = \frac{x+2}{x-3}$$

The solution set is_____

Solve the formula for the specified variable.

$$s = \frac{(v_1 + v_2)t}{2} \quad ; \quad t$$

Solve the formula for the specified variable.

$$I = \frac{2V}{R + 2r} \quad ; \quad r$$

Solve the formula for the specified variable.

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad ; \quad p$$

Solve the formula for the specified variable.

$$\frac{E}{e} = \frac{R+r}{R} \quad ; R$$

Work Problems

If   a = the time needed for A to complete the work alone

b = the time needed for B to complete the work alone, and

t = the time needed for A and B to complete the work together,

Then

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{t}$$

Tara can finish a puzzle in 15 hours. It takes Carly 10 hours to complete the same puzzle. Working together, how long will it take them to complete the puzzle?

Together, it takes Ben and Jerry 12 hours to lay carpet in a small house. Ben, who is more experienced, can do the job 7 hours faster than Jerry can working alone. How long would it take Ben to carpet the house alone?

Two pipes are being used to fill a tank. Pipe A can fill the tank twice as fast as Pipe B. When both pipes are turned on, it takes 3 hours to fill the tank. If only Pipe A is turned on, how long would it take to fill the tank? If only Pipe B is turned on, how long will it take to fill the tank?

Motion Problems

Distance = (rate)(time)

$$d = rt$$

$$r = \frac{d}{t}$$

$$t = \frac{d}{r}$$

Eva walks 2 mph slower than John. In the time it takes Eva to walk 5 miles, John walks 8 miles. Find the speed of each person.

George's boat cruised 45 mi upstream and 45 mi back in a total of 8 hr. The speed of the river is 3 mph. Find the speed of the boat in still water?

David's boat travels 15 km/h in still water. He motors 140 km downstream in the same time it takes to travel 35 km upstream. What is the speed of the river?