Rational Functions and Equations

Addition and Subtraction of Rational Expressions

Addition and Subtraction with Like Denominators

To add or subtract when the denominators are the same, add or subtract the numerators and keep the denominator.

$$\frac{A}{C}+\frac{B}{C}=\frac{A+B}{C} \quad \textit{and} \quad \frac{A}{C}-\frac{B}{C}=\frac{A-B}{C}$$

Add or subtract the rational expressions to a single rational expression and then simplify. If applicable, state the restricted domain.

$$\frac{y+2}{y-2}+\frac{y-6}{y-2}$$

$$\frac{1}{t^2-7t+12}-\frac{t-2}{t^2-7t+12}$$

Least Common Multiple (LCM)

The least common multiple (LCM) of two polynomials can be found as follows.

1: Factor each polynomial completely.

2: List each factor the greatest number of times that it occurs in either factorization.

3: Find the product of this list of factors The result is the LCM.

Least Common Denominator (LCD)

The LCD is the LCM of the denominators. Find the least common denominator of the following rational expressions.

$\frac{1}{12}, \frac{5}{16}$

$LCD:$ _____

$\frac{7}{12ab}, \frac{5}{6a^2}$

$LCD:$ _____

$\frac{3x}{x^2-4}, \frac{x+1}{x^2+2x}$

$LCD:$ _____

$\frac{1}{4x^2+20x+25}, \frac{1}{2x^2+5x}$

$LCD:$ _____

Addition and Subtraction when Denominators are Different

To add or subtract rational expressions

1) If the denominators are not the same, determine the least common denominator (LCD) by finding the least common multiple of the denominators.
2) Write each rational expression with the LCD. To do so, multiply each expression (if necessary) by the unit fraction that builds the fraction's denominator up to the LCD.
3) Add or subtract the resulting rational expressions by adding or subtracting the numerators and leaving the result over the LCD. Do not expand the denominator.
4) Simplify the result, if possible, leaving the expression in factored form.

Add or subtract the rational expression to a single rational expression and then simplify. If applicable, state the restricted domain.

$$\frac{x+3}{x-5}+\frac{x-2}{x+4}$$

Add or subtract the rational expression to a single rational expression and then simplify. If applicable, state the restricted domain.

$$\frac{2}{x^2-9}+\frac{1}{3x+9}$$

Add or subtract the rational expression to a single rational expression and then simplify. If applicable, state the restricted domain.

$$\frac{6}{x^2-2x-8}-\frac{1}{x^2+3x+2}$$

Add or subtract the rational expression to a single rational expression and then simplify. If applicable, state the restricted domain.

$$\frac{x}{x+2} - \frac{2}{x-2} + 3$$

Add or subtract the rational expression to a single rational expression and then simplify. If applicable, state the restricted domain.

$$\frac{9x^2}{3x-y}-\frac{y^2}{3x-y}$$

$$\frac{y}{3ty}+\frac{t}{2y^2}$$

Add or subtract the rational expression to a single rational expression and then simplify. If applicable, state the restricted domain.

$$\frac{2}{rt-3}-\frac{2rt+3}{r^2t^2-9}$$