Rational Functions and Equations

Multiplication and Division of Rational Expressions

To simplify a rational expression or rational function, completely factor the numerator and denominator and then apply the property below:

If A, B, and C are polynomials and C≠0, then

$$\frac{AC}{BC} = \frac{A}{B}$$

Example:

$$R(x) = \frac{x-4}{x^2-x-12}$$

$$= \frac{x-4}{(x+3)(x-4)}$$

$$= \frac{1}{x+3}, \quad x \neq 4$$

We are required to write x≠4 in the final simplification because this was a domain restriction of the original function and therefore needs to be a domain restriction in the final simplification. Otherwise, the functions would not be equivalent.*

Simplify each function, and if applicable, write the restricted domain.

$$f(t) = \frac{t^2 - 3t - 4}{t^2 + 9t + 8}$$

$$h(x) = \frac{3x^2 + 15x}{x^4 - 25x^2}$$

Simplify each expression, and if applicable, write the restricted domain on the simplified expression.

$$\frac{7a^3}{21a}$$

$$\frac{6a+30}{a+5}$$

$$\frac{-6x+12}{5x-10}$$

$$\frac{7-x}{3x-21}$$

Simplify each expression, and if applicable, write the restricted domain on the simplified expression.

$$\frac{2y^2+7y-15}{4y^2-4y-3}$$

$$\frac{-7x+8-x^2}{x^6+2x^5-3x^4}$$

Simplify each expression.

$$\frac{x^2y - xy^2}{x^2 - y^2}$$

$$\frac{4r - 8t}{r^2 - 4rt + 4t^2}$$

Multiplying and Dividing Rational Expressions

To **multiply** two rational expressions, multiply numerators and multiply denominators:

$$\frac{A}{B}\cdot\frac{C}{D}=\frac{AC}{BD}\ ,\ \ where\ \ \ B\neq 0\ ,\ D\neq 0.$$

To **divide** two rational expressions, multiply the numerator by the reciprocal of the divisor. For any rational expressions A/B and C/D with B, C, D ≠ 0,

$$\frac{A}{B}\div\frac{C}{D}=\frac{A}{B}\times\frac{D}{C}.$$

Review of Multiplying and Dividing Fractions

Multiply. $\frac{5}{6} \cdot \frac{3}{10}$ $4 \cdot \frac{7}{4} \cdot \frac{1}{2}$

Divide. $\frac{10}{9} \div \frac{5}{3}$ $-\frac{7}{11} \div 14$

Perform the indicated operation, simplify, and if applicable, write the restricted domain.

$$\frac{x}{x-5}\cdot x^2$$

$$y^{10}\cdot\frac{y+1}{y^5}$$

$$20r\div\frac{4}{r^4}$$

$$\frac{(t+2)^2}{6}\div(t+2)$$

Perform the indicated operation, simplify, and if applicable, write the restricted domain.

$$\frac{x^2+8x+16}{x^2-12x+36}\cdot\frac{x^2-4x-12}{x^2+6x+8}$$

Perform the indicated operation, simplify, and if applicable, write the restricted domain.

$$\frac{y^2+3y}{y^2-9} \div \frac{y^2+5y-14}{y^2+4y-21}$$

Perform the indicated operation, simplify, and if applicable, write the restricted domain.

$$\frac{18-6x}{25x^2-4}\cdot\frac{15x^4-6x^3}{-2x^2+5x+3}$$

Perform the indicated operation, simplify, and if applicable, write the restricted domain.

$$\frac{4a^2-1}{a^2-4} \div \frac{2a-1}{2-a}$$

Simplify each expression.

$$\frac{25a}{9b^8} \cdot \frac{3b^5}{5a^2}$$

$$\frac{\frac{x}{5y}}{\frac{x}{10y^2}}$$

$$\frac{y^2}{y^2r^2+4ry} \cdot \frac{y^2r^2+yr-12}{y}$$

$$\frac{20t^3y^2}{t-5y} \div \frac{4ty^2}{5t^2y-25ty^2}$$