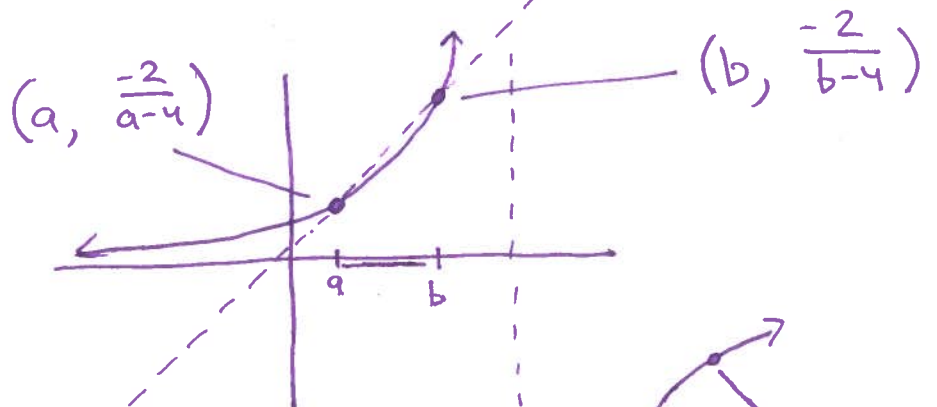


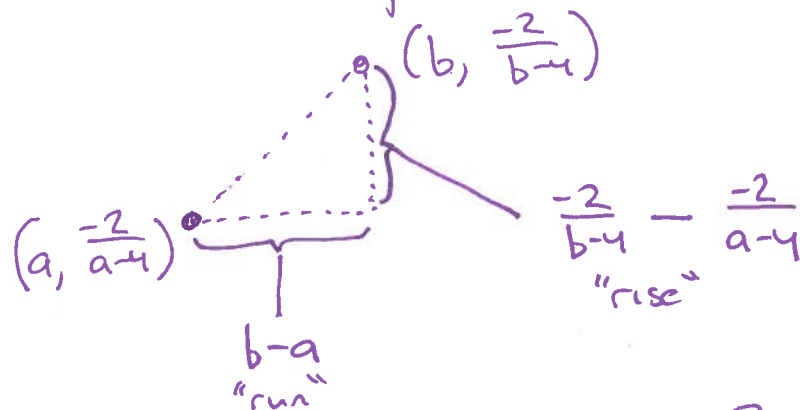
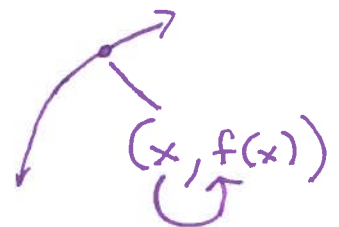
Section 12.4

Complex Rational Expressions (Complex Fractions)

$$y = \frac{-2}{x-4}$$



what is
this line's slope?



So slope = $\frac{\frac{-2}{b-4} - \frac{-2}{a-4}}{b-a}$] 3-leveled or 4-leveled
rational expression
"complex
rational
expression"

aka a "complex fraction"

Can always be
simplified to 1-leveled
or 2-leveled rational expression

Ex

Simplify

$$\frac{\frac{x-8}{x+1}}{x+2}$$

1) Note the main fraction bar.

$$= \frac{\frac{x-8}{x+1}}{x+2} \cdot \frac{x+1}{x+1}$$

aligned with the main fraction bar.

2) Identify all the internal denominator factors. Here: $(x+1)$

$$= \frac{\frac{(x-8)}{(x+1)}}{(x+2)} \cdot \frac{(x+1)}{(x+1)}$$

3) Use these...
More details soon

Use more parentheses to make factors clear

$$= \frac{(x-8)}{(x+2)(x+1)}$$

Ex

Simplify

$$\frac{\frac{x+3}{x+5}}{\frac{x+2}{x+1}}$$

1) Note main fraction bar

2) Internal denominators $(x+5)$, $(x+1)$

$$= \frac{\frac{(x+3)}{(x+5)}}{\frac{(x+2)}{(x+1)}} \cdot \frac{(x+5)(x+1)}{(x+5)(x+1)}$$

Build LCM
3) $(x+5)(x+1)$

$$= \frac{(x+3)(x+1)}{(x+2)(x+5)}, x \neq -1$$

from canceling $\frac{x+1}{x+1}$ and we don't still see that -1 is bad in final answer.

Ex Simplify $\frac{\frac{x+2}{x^2-9}}{\frac{x+1}{x^2-7x+12}}$

$$= \frac{\frac{(x+2)}{(\cancel{x+3})(\cancel{x-3})}}{\frac{(x+1)}{(\cancel{x-3})(\cancel{x-4})}} \cdot \frac{(\cancel{x+3})(\cancel{x-3})(x-4)}{(x+3)(\cancel{x-3})(\cancel{x-4})}$$

$$= \frac{(x+2)(x-4)}{(x+1)(x+3)}, \quad x \neq 3, 4$$

1) Main ber...

2) List internal denominators...

$$(x+3), (x-3), (x-4)$$



3)

LCM

$$(x+3)(x-3)(x-4)$$

Ex Simplify $\frac{2 + \frac{3x}{x-8}}{\frac{4}{x-8}}$

$$= \frac{2 + \frac{(3x)}{(x-8)}}{\frac{4}{(\cancel{x-8})}} \cdot \frac{(x-8)}{(x-8)}$$

$$= \frac{2(x-8) + \frac{3x}{(\cancel{x-8})(\cancel{x-8})}}{4}$$

$$= \frac{2x - 16 + 3x}{4}$$

$$= \frac{5x - 16}{4}, \quad x \neq 8$$

1) Main ber...

2) internal denoms:

$$(x-8)$$

3) LCM is $(x-8)$

Ex Simplify

$$\frac{\frac{x}{x+1}}{\frac{x+2}{x+1} + \frac{x+3}{x+5}}$$

$$= \frac{\frac{x}{\cancel{(x+1)}}}{\frac{(x+2)}{(x+1)} + \frac{(x+3)}{(x+5)}} \cdot \frac{\cancel{(x+1)}(x+5)}{(x+1)(x+5)}$$

$$= \frac{x(x+5)}{\frac{(x+2)}{\cancel{(x+1)}} \cancel{(x+1)}(x+5) + \frac{(x+3)}{\cancel{(x+1)}} (x+1) \cancel{(x+5)}}$$

$$= \frac{x(x+5)}{(x+2)(x+5) + (x+3)(x+1)} = \frac{x(x+5)}{x^2 + 2x + 5x + 10 + x^2 + x + 3x + 3}$$

$$= \frac{x(x+5)}{2x^2 + 11x + 13}$$

← can this factor?
 $(?) + (??) = 11$
 $(?) \cdot (??) = 26$ Not possible.

$$= \frac{x(x+5)}{2x^2 + 11x + 13}, x \neq -1, -5$$

Complex Rational Expressions

1. Simplify each of these complex rational expressions. If there is only one variable, state any restricted domain.

a) $\frac{\frac{1}{2} - 3}{\frac{1}{2} + 3}$

$$= \dots$$

$$= -\frac{5}{7}$$

b) $\frac{\frac{7}{y}}{y^2 + 2y}$

$$= \dots$$

$$= \frac{7}{y^2(y+2)}$$

c) $\frac{\frac{3}{z^2 - 4}}{\frac{z}{z^2 - 4}}$

$$= \dots$$

$$= \frac{3}{z}, z \neq 2, -2$$

d) $\frac{5 + \frac{1}{x-1}}{\frac{1}{x-1}}$

$$= \dots$$

$$= 5x - 4, x \neq 1$$

e) $\frac{\frac{1}{p^2q} + \frac{1}{pq^2}}{\frac{p+q}{pq}}$

internal
denoms
 $p \cdot p \cdot q, p \cdot q \cdot q, p \cdot q$
LCM copy
 $p \cdot p \cdot q \cdot q$

$$= \frac{\frac{1}{p^2q} + \frac{1}{pq^2}}{\frac{p+q}{pq}} \cdot \frac{p^2 \cdot q^2}{p^2 \cdot q^2}$$

$$= \frac{(q+p) \cdot 1}{(p+q) p \cdot q} = \frac{1}{pq}$$

f) $\frac{\frac{2}{x-1}}{\frac{1}{x-1} + \frac{1}{x-2}} \cdot \frac{(x-1)(x-2)}{(x-1)(x-2)}$

$$= \frac{2(x-2)}{\left(\frac{1}{x-1}\right)(x-1)(x-2) + \left(\frac{1}{x-2}\right)(x-1)(x-2)}$$

$$= \frac{2(x-2)}{x-2 + x-1}$$

$$= \frac{2(x-2)}{2x-3}, x \neq 1, 2$$

$$g) \frac{\frac{x}{x^2-4} - \frac{1}{x^2-4}}{\frac{1}{x+4}}$$

$$= \frac{(x-1)(x+4)}{(x-2)(x+2)}, x \neq -4$$

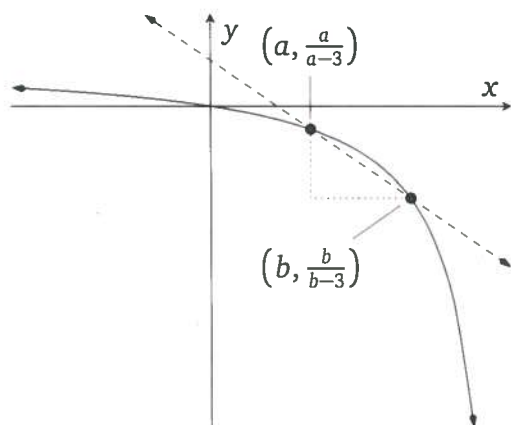
$$h) \frac{\frac{a^2+4a+3}{a+5} + \frac{2}{a+4}}{\frac{a+5}{a+4} + \frac{2}{a+2}} = \frac{(a+3)(a+1)}{\frac{a+5}{a+4} + \frac{2}{a+2}} \cdot \frac{(a+4)(a+2)}{(a+4)(a+2)}$$

$$= \frac{(a+3)(a+1)(a+4)(a+2)}{(a+5)(a+2) + 2(a+4)}$$

$$= \frac{(a+3)(a+1)(a+4)(a+2)}{a^2+7a+10+2a+8}$$

$$= \frac{(a+3)(a+1)(a+4)(a+2)}{a^2+9a+18}$$

2. In this picture, we see the graph of $y = \frac{x}{x-3}$. Two points are singled out. The straight line connecting them is overlaid, including a slope triangle.



$$= \frac{(a+3)(a+1)(a+4)(a+2)}{(a+6)(a+3)}$$

$$= \frac{(a+1)(a+4)(a+2)}{a+6}, a \neq -4, -2, 3$$

- a) What is the “run” on the slope triangle? (What is the length of the horizontal leg of the triangle?) The answer is a pretty simple math expression using a and b .

$$b-a$$

- b) What is the “rise” on the slope triangle? (What is the length of the vertical leg of the triangle?) The answer is a math expression that you should not try to simplify.

$$\frac{b}{b-3} - \frac{a}{a-3}$$

- c) What is the slope of the line? The answer should start out as a complex rational expression, and then you should simplify it.

$$\frac{\frac{b}{b-3} - \frac{a}{a-3}}{b-a} \cdot \frac{(b-3)(a-3)}{(b-3)(a-3)} = \frac{b(a-3) - a(b-3)}{(b-a)(b-3)(a-3)}$$

$$= \frac{ab-3b-ab+3a}{(b-a)(b-3)(a-3)} = \frac{-3b+3a}{(b-a)(b-3)(a-3)} = \frac{-3(b-a)}{(b-a)(b-3)(a-3)}$$

$$= \frac{-3}{(b-3)(a-3)}$$

12.5 Solving Rational Equations

Ex $\frac{x}{2} = \frac{5x}{4}$

(Solve for x.)

List denoms: 2, 4

LCM: 4.

We can multiply by 4
on each side...

"clearing denominators".

$$\frac{x}{2} \cdot 4 = \frac{5x}{4} \cdot 4$$

$$\frac{x \cdot 4}{2 \cdot 1}$$

$$x \cdot 2 = 5x$$

$$2x = 5x$$

$$\xrightarrow{-5x}$$

$$-3x = 0$$

$$\frac{-3x}{-3} = \frac{0}{-3}$$

$$x = 0$$

Your job is to
report what x could be...

denominators
gone!

Ex Solve $\frac{x}{x+1} = \frac{3}{2}$

this is a rational equation

$$\frac{x}{(x+1)} = \frac{3}{2}$$

two denoms:

$$(x+1), 2$$

LCM: $(x+1) \cdot 2$

$$\frac{x}{\cancel{(x+1)}} \cdot \cancel{(x+1)} \cdot 2 = \frac{3}{2} (x+1) \cdot \cancel{2}$$

$$x \cdot 2 = 3(x+1)$$

$$2x = 3x + 3$$

$\leftarrow -3x$

$$-x = 3$$

Report:

$$x = -3$$

Check: $\frac{-3}{-3+1} \stackrel{?}{=} \frac{3}{2}$
 $\frac{-3}{-2} \stackrel{\checkmark}{=} \frac{3}{2}$

Ex Solve $\frac{x+1}{x-3} = \frac{x+2}{x-7}$

$$\frac{(x+1)}{(x-3)} = \frac{(x+2)}{(x-7)}$$

LCM: $(x-3)(x-7)$

$$\frac{(x+1)}{\cancel{(x-3)}} \cdot \cancel{(x-3)} (x-7) = \frac{(x+2)}{\cancel{(x-7)}} (x-3) \cdot \cancel{(x-7)}$$

$$(x+1)(x-7) = (x+2)(x-3)$$

No denominators!

$$x^2 - 7x + x - 7 = x^2 - 3x + 2x - 6$$

$$x^2 - 6x - 7 = x^2 - x - 6$$

$$\begin{array}{c} \leftarrow -x^2 \quad \leftarrow +x \\ \leftarrow +7 \end{array}$$

$$-5x = 1$$

$$\frac{-5x}{-5} = \frac{1}{-5}$$

$$x = -\frac{1}{5}$$

To be continued