12.1 Intro to Rational Functions

being careful, thoughtful

calculating something

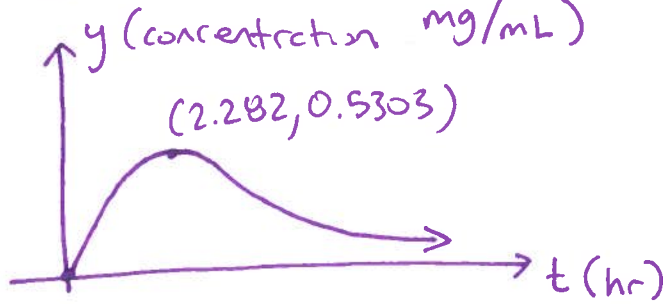
dividing $A \div B$, or $\frac{A}{B}$

a ratio

A function f where $f(x) = \frac{\text{polynomial}}{\text{polynomial}}$

<u>Ex</u> $C(t) = \frac{3t}{t^2+8}$ numerator & denom. both polynomials

Ingest a medication. Look at concentration of drug in bloodstream as time passes.



← $y = C(t)$ looks like this

What is C's <u>domain</u>? $[0, \infty)$

What is C's <u>range</u>? $[0,05303]$

What is $C(0)$? $C(0) = 0$ $C(0) = \frac{3 \cdot 0}{0^2+8} = \frac{0}{8} = 0$

0 mg/mL

How long until blood concentration peaks?

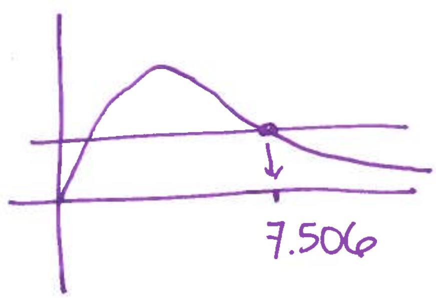
2.282 hours...

If it's appropriate to take another dose once concentration is only 0.35 mg/mL, how long until 2nd pill?

$$C(\overset{?}{t}) = 0.35$$

$$\underbrace{\frac{3t}{t^2+8}}_{\text{graph this}} = \underbrace{0.35}_{\text{graph this too}}$$

$$y = \frac{3t}{t^2+8} \qquad y = .35$$



So... wait 7.5 hours for 2nd dose.

Ex $f(x) = \dfrac{1}{x-2}$ Make a plot... by had

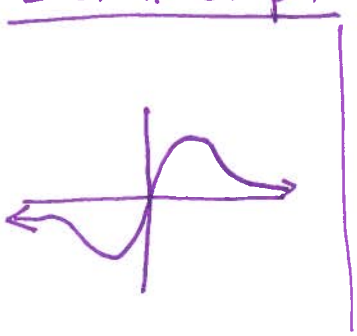
Make a table

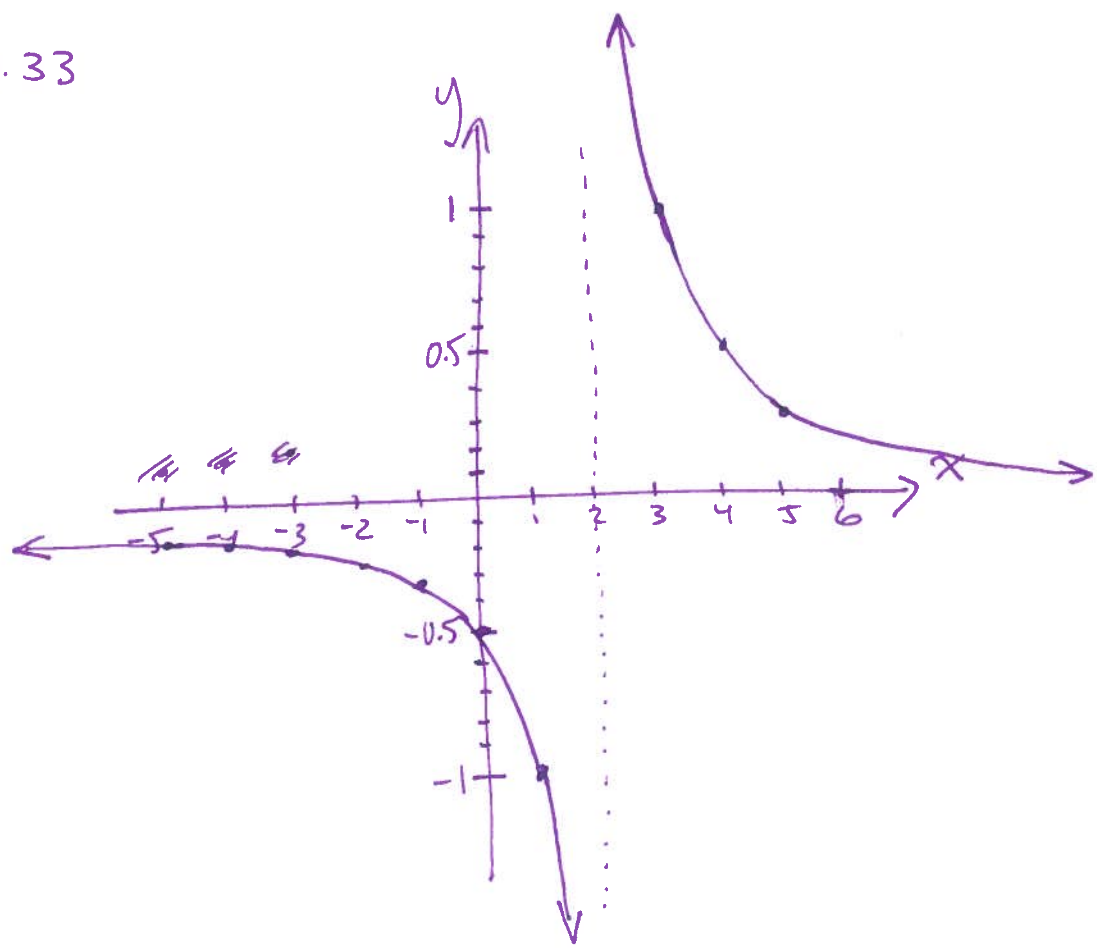
x	$f(x)$
-5	$-1/7 \approx 0.14$
-4	$-1/6 \approx -0.17$
-3	$-1/5 \approx -0.2$
-2	$-1/4 = -0.25$
-1	$-1/3 \approx -0.33$
0	$-1/2 = -0.5$
1	$-1/1 = -1$
2	undefined
3	1
4	$1/2$
5	$1/3 \approx 0.33$

$$f(-5) = \frac{1}{-5-2} = \frac{1}{-7} = -\frac{1}{7}$$

$\leftarrow$ $f(2) = \dfrac{1}{2-2} = \dfrac{1}{0}$ is undefined

Last Graph

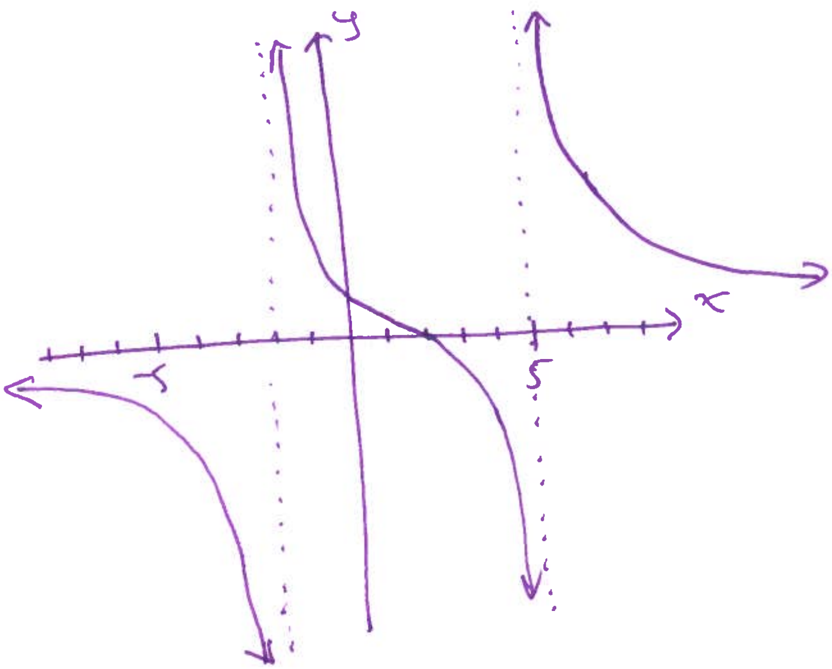




$\underline{Ex}$ $g(x) = \dfrac{3x-6}{x^2-3x-10}$

rational function ✓

Goal: graph this.



Some help from GeoGebra.

what is g's domain?

worry about $x^2-3x-10=0$

$(x-5)(x+2)=0$

$x-5=0$ or $x+2=0$

$x=5$ $\quad$ $x=-2$

consequences for graph.

12.2 Simplifying, Multiplying, Dividing Rational Expressions

Ex Consider $\frac{x+3}{x+4}$

Do not cancel. These "x"s are terms (pieces added to other pieces)

Only cancel factors. (Pieces being multipled with other pieces)

Ex
$$\frac{\overbrace{(x+3)(x-2)}^{\text{factors}}}{\underbrace{(x+4)(x+3)}_{\text{factors}}}$$

In WeBWorK
(x-2)/(x+4), x != -3

$$\frac{\cancel{(x+3)}(x-2)}{(x+4)\cancel{(x+3)}} = \frac{x-2}{x+4}, \quad x \neq -3$$

canceled (x+3)

$x+3=0$

$x=-3$

optionally, can also exclude -4, but not required.

Ex Simplify $\dfrac{3x-12}{x^2+x-20}$ Need factors...

$$= \frac{3(x-4)}{(x+5)(x-4)}$$

$$= \frac{3}{x+5}, \quad \underbrace{x \neq 4}_{\text{"domain restriction"}}$$

Ex Simplify $\dfrac{-y-2y^2}{2y^3-y^2-y}$

$$= \frac{-y(1+2y)}{y(2y^2-y-1)}$$

Try to factor $2y^2-y-1$

$A \cdot C = -2$

$(-2)(1) = -2$

$(-2)+(1) = -1$

$\underbrace{2y^2-2y}+y-1$

$2y(y-1)+1(y-1)$

$(y-1)(2y+1)$

$$= \frac{\cancel{-y}\,\cancel{(1+2y)}\cdot 1}{\cancel{y}(y-1)\cancel{(2y+1)}}$$

$$= \frac{-1}{y-1}, \quad y \neq 0, -\tfrac{1}{2}$$

$y=0$ causes trouble

$1+2y=0$ causes trouble

$2y=-1$

$y=-\tfrac{1}{2}$

Ex $\quad \dfrac{x+5}{2x-1} \cdot \dfrac{x-3}{x+5} = \dfrac{\cancel{(x+5)}(x-3)}{(2x-1)\cancel{(x+5)}}$

legit to cancel...

$$= \frac{x-3}{2x-1}, \quad x \neq -5$$

Ex $\quad \dfrac{x^2-4x}{x^2-4} \cdot \dfrac{x^2-4x+4}{x^2+x-20}$

$$= \frac{x\cancel{(x-4)}}{(x+2)\cancel{(x-2)}} \cdot \frac{(x-2)^{\cancel{2}}}{(x+5)\cancel{(x-4)}}$$

factored; look for cancellation

$$= \frac{x(x-2)}{(x+2)(x+5)}, \quad x \neq 4, 2$$

Ex

$$8 \div 4 = 2$$

$$8 \cdot \frac{1}{4} = 2$$

Ex $\quad \dfrac{x+2}{x+5} \div \dfrac{x-3}{x-3}$ "flip"

leave alone

change

$$\frac{x+2}{x+5} \cdot \frac{x-3}{x+2}$$

$$= \frac{x-3}{x+5}, \quad x \neq -2, 3$$

canceled $\dfrac{(x+2)}{(x+2)}$

3 was a bad input on original expression.

Exploring Rational Functions

1. Let $f(x) = \dfrac{x+1}{x^4+1}$.

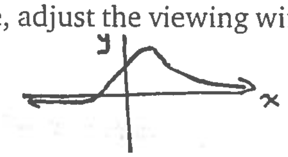
a) Make a table of values for f. Decimals are OK. Use $x = -5$ to $x = 5$.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$	-0.006	-0.012	-0.024	-0.059	0	1	1	0.176	0.049	0.019	0.010

b) What does your table suggest will happen when x gets very large?

It appears f(x) gets close to 0 when x is large.

c) Use graphing technology to see the graph of f. If appropriate, adjust the viewing window.

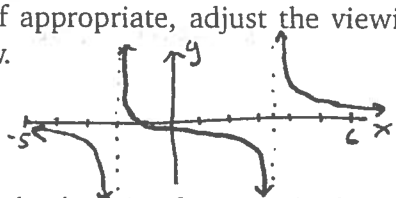


2. Let $g(x) = \dfrac{x+1}{2x^2-3x-14}$.

a) Make a table of values for g. Decimals are OK. Use $x = -5$ to $x = 5$.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$g(x)$	-0.078	-0.1	-0.154	undef.	0	-0.071	-0.133	-0.25	-0.8	0.833	0.286

b) Use graphing technology to see the graph of g. If appropriate, adjust the viewing window.



c) What does your graph suggest will happen when x gets very large?

As x gets large, g(x) gets close to 0.

d) What is the domain of g? Use the formula to figure this out. Look back on the graph to see how it agrees with your answer.

$2x^2-3x-14=0$

$(x+2)(2x-7)=0$

$x=-2$ or $x=\frac{7}{2}$

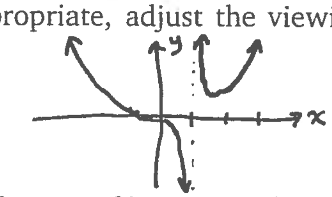
The domain is $(-\infty,-2)\cup(-2,\frac{7}{2})\cup(\frac{7}{2},\infty)$

3. Let $h(x) = \dfrac{x^3+1}{x-1}$.

a) Make a table of values for h. Decimals are OK. Use $x = -5$ to $x = 5$.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$h(x)$	20.7	12.6	6.5	2.33	0	-1	undef	9	14	21.7	31.5

b) Use graphing technology to see the graph of h. If appropriate, adjust the viewing window.



c) What does your graph suggest will happen when x gets very large?

As x gets large, h(x) will become large too.

d) What is the domain of h? Use the formula to figure this out. Look back on the graph to see how it agrees with your answer.

$x-1=0$

$x=1$

The domain is $(-\infty,1)\cup(1,\infty)$

4. Suppose an object is in between the Earth and the Moon, x thousand kilometers from the center of the Earth.

An object like experiences some gravitational acceleration from these two things pulling on it. This acceleration (measured in meters per second, per second) is a function of x, and has formula

$$a(x) = \frac{-393.67x^2 + 306425x - 58895000}{x^2(384.4 - x)^2}$$

a) Standing here on the surface of the Earth, we are 6.371 thousand kilometers from the center of the Earth. Find $a(6.371)$ (using a calculator) and write a sentence that uses that number in context.

$f(6.371) \approx -9.82$. This says that on Earth's surface, gravity's acceleration is about 9.82 meters per second per second.

b) Use a graphing calculator to find where the object would need to be for it to feel no net gravitational pull. (Where it should be for the pull of the Earth and the pull of the Moon to cancel each other out.)

Need to solve $a(x) = 0$. Graph shows $x \approx 346$. So about 346 thousand km from Earth's surface toward the moon, gravity's acceleration is 0 m/s².

c) From the center of the earth to the surface of the moon is about 382.66 thousand kilometers. What is the acceleration from gravity on the moon's surface?

$a(382.66) \approx 1.617$. So on the moon's surface, gravity's acceleration is about 1.617 meters per second per second.

d) Compared to being on the Moon, how many times heavier are you on Earth?

$\frac{9.82}{1.617} \approx 6.07$, so you are over 6 times heavier on Earth than on the moon.

e) What is the domain of the rational function given by the formula above?

Need $x \neq 0$, $x \neq 384.4$. So the domain is

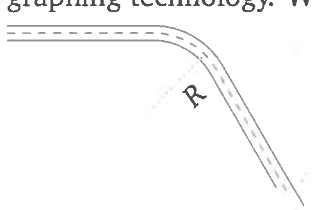
$(-\infty, 0) \cup (0, 384.4) \cup (384.4, \infty)$

f) What is the domain of a? Hint: this is different from the last question, because in context there are x-values that would not make any sense. Use Google to look of the radius of the Earth, the radius of the Moon, and the distance between them.

In context, need to be between Earth's surface and Moon's surface, so domain is $[6.371, 382.66]$

5. Engineers are building a curved section of freeway onramp that needs to be banked. They have discovered that if the speed of cars is about 40 mph, then there is a relation between the radius R of the curve and the slope m of the banking. $m(R) = \frac{1600-2R}{15R}$. Plot this rational function using graphing technology. What radius would yield a banking of 10%? (That is, a slope of 0.1?)

in feet



Solve $0.10 = \frac{1600 - 2R}{15R}$

Graphically: $R \approx 457$.

So the radius should be about 457 feet.

Multiplying, Dividing, and Simplifying Rational Functions

1. Simplify each of the rational expressions below. If there is only one variable, it is important to note any restrictions on the simplified version of the expression.

a) $\frac{18t^3}{6t}$

$= \frac{2\cdot 3\cdot 3\cdot t\cdot t\cdot t}{2\cdot 3\cdot t}$

$= 3t^2, \; t\neq 0$

b) $(x+2)\frac{x-6}{x+2}$

$= \frac{x+2}{1}\cdot\frac{x-6}{x+2}$

$= x-6, \; x\neq -2$

c) $\frac{(3x+2)(x-7)}{x(x-7)}$

$= \frac{3x+2}{x}, \; x\neq 7$

d) $\frac{x^2+5x+4}{x^2+2x-8}$

$= \frac{(x+4)(x+1)}{(x+4)(x-2)}$

$= \frac{x+1}{x-2}, \; x\neq 2$

e) $\frac{2}{x-3}\cdot\frac{x}{x+1}$

$= \frac{2x}{(x-3)(x+1)}$

$= \frac{2x}{x^2-2x-3}$

both OK

WBWK wants factored answers

f) $\frac{x+1}{y-2}\div\frac{2x+2}{y-2}$

$= \frac{x+1}{y-2}\cdot\frac{y-2}{2x+2}$

$= \frac{\cancel{(x+1)}\cancel{(y-2)}\cdot 1}{\cancel{(y-2)}\cdot 2\cancel{(x+1)}}$

$= \frac{1}{2}$ (restrictions not necessary...)

g) $\frac{x^2+x}{2x+6}\div\frac{x}{x+3}$

$= \frac{x(x+1)}{2(x+3)}\cdot\frac{x+3}{x}$

$= \frac{x+1}{2}, \; x\neq 0, -3$

h) $\frac{4z^2}{r^3}\cdot\frac{5r}{2z}\cdot\frac{r}{z}$ $= \frac{20r^2z^2}{2r^3z^2}$

$= \frac{10}{r}$

i) $\frac{(x-5)(x+3)}{3x-1}\cdot\frac{x(3x-1)}{x-5}$

$= (x+3)\cdot x, \; x\neq 5, \frac{1}{3}$

j) $\frac{x^2+4x+4}{(x-3)(x^2-1)}\cdot\frac{x^2-2x-3}{x^2-4}$

$= \frac{(x+2)^2(x-3)(x+1)}{(x-3)(x+1)(x-1)(x+2)(x-2)}$

$= \frac{(x+2)}{(x-1)(x-2)}, \; x\neq -2, 3, -1$

k) $\frac{x^2+x-12}{2x^2-9x-5}\div\frac{x^2+7x+12}{2x^2-7x-4}$

$= \frac{(x+4)(x-3)}{(2x+1)(x-5)}\cdot\frac{(2x+1)(x-4)}{(x+3)(x+4)}$

$= \frac{(x-3)(x-4)}{(x-5)(x+3)}, \; x\neq -4, -\frac{1}{2}, 4$

from cancellations

bad in original, but $(x-4)$ was flipped.

l) $\frac{x+1}{y-2}\div\frac{2x+2}{y-2}\div\frac{x-3}{x}$

$= \frac{\cancel{x+1}}{\cancel{y-2}}\cdot\frac{\cancel{y-2}}{2\cancel{(x+1)}}\cdot\frac{x}{x-3}$

$= \frac{x}{2(x-3)}$

2. The area of a rectangle is $5x^2+12x+4$ and its width W is $x+2$. Find (and simplify) the length L of the rectangle. (In general, $A = LW$. So that means $L = \frac{A}{W}$.)

$$L = \frac{5x^2+12x+4}{x+2}$$
$$= \frac{(x+2)(5x+2)}{x+2}$$
$$= 5x+2, \quad x \neq -2$$

3. There are $x+4$ bakers who *each* bake x^2-9 cupcakes.

a) How many total cupcakes did they make?

$(x+4)(x^2-9)$

b) There will be a party with all the bakes plus an additional x^2-16 guests. How many total ~~guests~~ people is that?

$$x+4+x^2-16$$
$$= x^2+x-12$$

c) If the cupcakes are supposed to be divided evenly among all of the guests, how many cupcakes (in terms of x) does each person get?

$$\frac{\#\text{ cupcakes}}{\#\text{ people}} = \frac{(x+4)(x^2-9)}{x^2+x-12}$$
$$= \frac{(x+4)(x+3)(x-3)}{(x+4)(x-3)}$$
$$= x+3, \quad x \neq -4, 3$$

4. Pretend that you are the teacher of a math lesson, and a student writes:

$$\frac{x+4}{x+2} = \frac{\cancel{x}+4}{\cancel{x}+2} = \frac{4}{2} = 2$$

Discuss within your group how you should respond. What should you write on this student's paper to do the best job of preventing them from making the same mistake in the future?

This is not valid algebra cancellation. The "x"s are <u>terms</u>: pieces being <u>added</u> to other pieces. Only <u>factors</u> may be canceled when they match, and when one is a factor of the numerator and the other is a factor of the denominator