11.4

Ex You work a 30-hour week, always 5 days per week, 6-hour shifts.

You are paid \$20 per hour.
Also you get a one-time \$100 bonus.

Let $f(x)$ be how much you are paid after working x hours

So... $f(x) = 20 \cdot x + 100$.

input is a # of hours
output is a # of dollars.

change...

$f: \underset{(hr)}{\text{time}} \longmapsto \text{dollar}$

Let $g(x)$ be how much you are paid after working x weeks.

$g: \underset{(wk)}{\text{time}} \longmapsto \text{dollar}$

Note x weeks is the same as $30 \cdot x$ hours.

$g(x) = f(\underbrace{30x}_{\text{so many hours}}) = 20(30x)+100 \quad f(\ ) = 20 \cdot (\ ) + 10$

so many ~~hours~~ weeks

$g(x) = f(30x) = 600x + 100$

$g(x) = 600x + 100$

Ex Let $f(x) = x^2 + 3x - 4$.

Find/simplify $f(-x)$.

$f(\ ) = (\ )^2 + 3(\ ) - 4$

$f(-x) = (-x)^2 + 3(-x) - 4$

$\boxed{f(-x) = x^2 - 3x - 4}$

Ex Find/simplify $-f(x)$.

$f(x) = x^2 + 3x - 4$

$-f(x) = -(x^2 + 3x - 4)$

$\boxed{-f(x) = -x^2 - 3x + 4}$

Ex $h(x) = \dfrac{2x}{x-8}$

Find/simplify $h(7x)$.

$h(\ ) = \dfrac{2(\ )}{(\ ) - 8}$

$h(7x) = \dfrac{2(7x)}{(7x) - 8}$

$\boxed{h(7x) = \dfrac{14x}{7x - 8}}$

Ex $g(x) = 0.3x + 2$

Find / simplify $\frac{1}{2}g(2x) = \frac{1}{2}\cdot\left(0.3(\quad) + 2\right)$

multiplied

$= \frac{1}{2}\left(0.3(2x) + 2\right)$

$= \frac{1}{2}\cdot\left(0.6x + 2\right)$

$\boxed{\frac{1}{2}g(2x) = 0.3x + 1}$

Ex $k(x) = x^2 + 12x$

Find / simplify $k(x+3)$. $= (\quad)^2 + 12($

$= (x+3)^2 + 12(x+3)$

watch out for the common mistake here

$= x^2 + 6x + 9 + 12x + 36$

$\boxed{k(x+3) = x^2 + 18x + 45}$

Ex $f(x) = \frac{1}{x^2}$.

Find / simplify $f(2x+1)+3 = \frac{1}{(\quad)^2} + 3$

$= \frac{1}{(2x+1)^2} + 3$ leave alon

$\boxed{f(2x+1)+3 = \frac{1}{4x^2+4x+1} + 3}$ or this

11.5 Technical Definition of a Function.

A <u>function</u> is a collection of ordered pairs ((first number, second number)) where for each "first number" in the collection, there is at most one "second number".

<u>Ex</u> $\{(3,9), (4,4), (6,5), (7,5)\}$ is a function. (according to technical definition)

we <u>interpret</u> this as a process for turning numbers into other numbers like..
$f(3)=9$ and $f(4)=4$ etc.

<u>Ex</u> $\{(9,3), (4,4), (5,6), (5,7)\}$ is a collection of ordered pairs but NOT a function.

because "input" 5 is paired with more than one "output".

we'd not know what is $f(5)$.

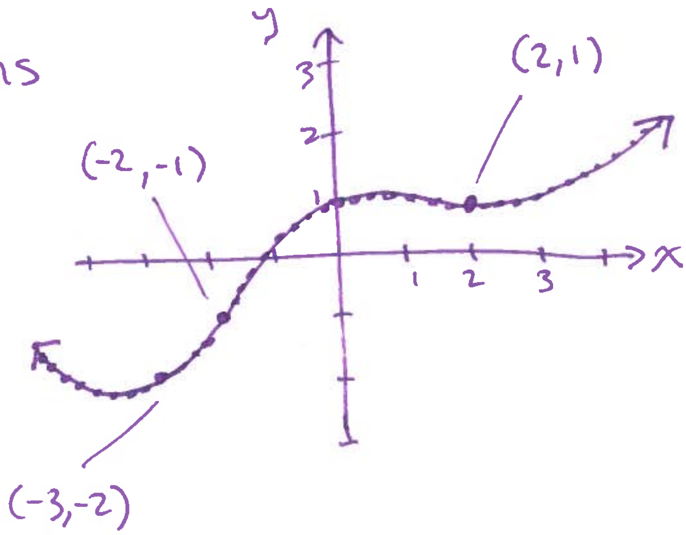
<u>Ex</u> formula $f(x) = x^2$. How is this meeting the technical def?

"same" as

$\{(0,0), (1,1), (4,16), (-3,9), (1.5, 2.25),$

.... <u>very</u> long list... $\}$ counts as a function.

<u>Ex</u> Graphs

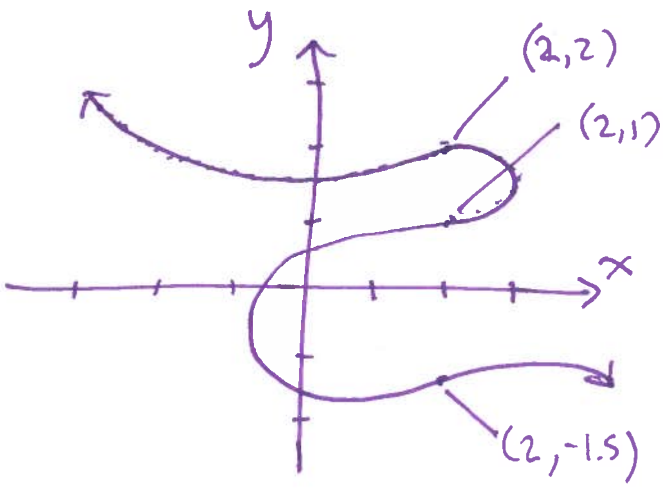


"same" as

$\{(-3,-2), (-2,-1),$
$(2,1),$ ... <u>very</u> long list $\}$

counts as a function.

<u>Ex</u>



This collection pairs up some x-values (namely with multiply y-values.

<u>NOT</u> a function.

Ex

x	y
5	15
7	22
8	22
13	30

Is $\underbrace{y}_{\text{"output"}}$ a function $\underbrace{\text{of } x}_{\text{"input"}}$

"Same" as

$\{(5,15), (7,22), (8,22), (13,30)\}$

is a function.

Ex

x	y
5	15
5	16
6	17

Not a function....

What is $f(5)$?

Can't be two things at once.

Ex Equations in x and y....

$$2x + 3y = 8$$

plug in input here...

...would be able to solve for y with basic algebra ...only one result.

This equation makes y a function of x.

$$y^2 = x^2 + 1$$

plug in inp (try 0)

$$y^2 = 1$$

1   -1

This equation does not make y a function of

Simplifying Function Notation

1. Suppose f is a function where $f(x) = 2x + 3$. Simplify the formula for...

a) $f(5x)$

$= 2(\ ) + 3$
$= 2(5x) + 3$
$= 10x + 3$

b) $f(-x)$

$= 2(\ ) + 3$
$= 2(-x) + 3$
$= -2x + 3$

c) $f(x+7)$

$= 2(\ ) + 3$
$= 2(x+7) + 3$
$= 2x + 14 + 3$
$= 2x + 17$

2. Suppose g is a function where $g(x) = x^2 - 6$. Simplify the formula for...

a) $g(5x)$

$= (\ )^2 - 6$
$= (5x)^2 - 6$
$= 25x^2 - 6$

b) $g(-x)$

$= (\ )^2 - 6$
$= (-x)^2 - 6$
$= x^2 - 6$

c) $g(x+7)$

$= (\ )^2 - 6$
$= (x+7)^2 - 6$
$= x^2 + 14x + 49 - 6$
$= x^2 + 14x + 43$

3. Suppose h is a function where $h(x) = x^2 + 2x - 7$. Simplify the formula for...

a) $h(5x)$

$= (\ )^2 + 2(\ ) - 7$
$= (5x)^2 + 2(5x) - 7$
$= 25x^2 + 10x - 7$

b) $h(-x)$

$= (\ )^2 + 2(\ ) - 7$
$= (-x)^2 + 2(-x) - 7$
$= x^2 - 2x - 7$

c) $h(x+7)$

$= (\ )^2 + 2(\ ) - 7$
$= (x+7)^2 + 2(x+7) - 7$
$= x^2 + 14x + 49 + 2x + 14 -$
$= x^2 + 16x + 56$

4. Suppose k is a function where $k(x) = \dfrac{2x+3}{x+9}$. Simplify the formula for...

a) $k(5x)$

$= \dfrac{2(\ )+3}{(\ )+9}$

$= \dfrac{2(5x)+3}{5x+9}$

$= \dfrac{10x+3}{5x+9}$

b) $k(-x)$

$= \dfrac{2(\ )+3}{(\ )+9}$

$= \dfrac{2(-x)+3}{(-x)+9}$

$= \dfrac{-2x+3}{-x+9}$

c) $k(x+7)$

$= \dfrac{2(\ \ )+3}{(\ \ )+9}$

$= \dfrac{2(x+7)+3}{(x+7)+9}$

$= \dfrac{2x+14+3}{x+16}$

$= \dfrac{2x+17}{x+16}$

5. Let f be the function with formula $f(x) = x^3 - 6x^2 + 11x - 6$.

a) Use graphing technology to plot a graph of f.

b) Simplify the formula for $f(2x)$.

$8x^3 - 24x^2 + 22x - 6$

c) Use graphing technology to plot a graph of $y = f(2x)$, using your formula from the previous part.

d) Do you notice anything about how the two graphs relate to each other?

Second is compressed toward y-axis by a factor of 2

6. Let g be the function with formula $g(x) = \dfrac{1}{x^2+1}$.

a) Use graphing technology to plot a graph of g.

b) Simplify the formula for $g(x+2)$.

$\dfrac{1}{x^2+4x+5}$

c) Use graphing technology to plot a graph of $y = g(x+2)$, using your formula from the previous part.

d) Do you notice anything about how the two graphs relate to each other?

Second is shifted ~~right~~ left by 2 units.

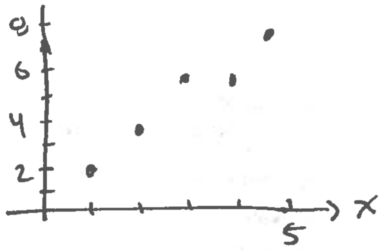
Technical Definition of a Function

1. A function named C is given by $\{(1,2),(2,4),(3,6),(4,6),(5,8)\}$.

   a) What are the domain and range of C? Give your answers using set notation (with the curly braces).

   $\{1,2,3,4,5\}$ $\{2,4,6,8\}$

   b) Graph C.

   c) What is $C(2)$? What is $C(6)$?

   $C(2) = 4$

   6 is not in C's doma
   C(6) is undefined.

   d) Solve the equation $C(x) = 8$.

   output

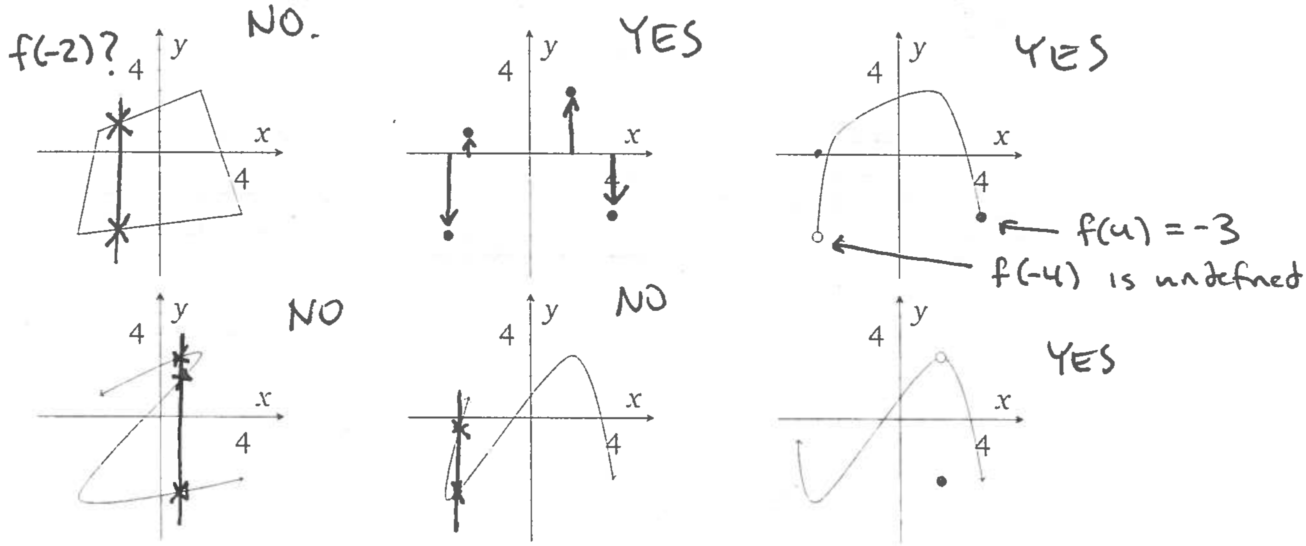
   $x = 5$

   e) Solve the equation $C(x) = 0$.

   This equation has no solutions.

   output

2. Here are some graphs that give relations between x and y. Your job is to determine if the relation can be used to define y as a function of x. For each graph, after you make your decision, say a little bit about why. If you decide that the relation *can* be used to define a function, then you can just say that. But if it *cannot*, then give a reason why. For example, you might say something like "There is more that one possibility for what $f(2)$ would be."



3. Which of these tables describe y as a function of x? For each table, after you make your decision, say a little bit about why. If you decide that the relation *can* be used to define a function, then you can just say that. But if it *cannot*, then give a reason why. For example, you might say something like "There is more that one possibility for what $f(2)$ would be."

x	y
1	-12
2	10
3	8
4	5
5	13
6	11

Yes.

x	y
10	3
15	4
20	3
25	4
30	3
35	4

Yes

x	y
-12	9
10	8
8	3
5	-2
13	1
11	1

Yes

x	y
2	-2
4	17
5	1
8	8
4	13
5	10

x-value 4 got paired up with more than one thing: No.

x	y
3	-1
8	10
7	14
7	14
12	13
16	-9

Yes

x	y
red	Mercury
orange	Venus
yellow	Earth
green	Mars
blue	Jupiter
purple	Saturn

Yes.

4. For each of these relations between x and y, decide if y is a function of x.

a) $y = 3x - 2$ YES

one input

one output

b) $2y = 3x - 2$ YES

input...

one output

c) $y^2 = 3x - 2$

input... 2

$y^2 = 4$

$y = 2$ or $y = -2$

$(2, 2)$

$(2, -2)$

No.

d) $x^2 + y^2 = 4$ NO

0

$y^2 = 4$

$2 \quad -2$

e) $|x| - y^2 = x^2$ NO

$\frac{1}{2}$

$\frac{1}{2} - y^2 = \frac{1}{4}$

$-y^2 = -\frac{1}{4}$

$y^2 = \frac{1}{4} \implies y = \frac{1}{2}$ or $y = \frac{-1}{2}$

f) $y = \pm\sqrt{x}$ NO

makes two y-values

5. Think about one specific person, living or dead. Let a represent the age of that person, and h represent their height.

a) Is h a function of a? If not, explain why not.

age as input

height as output.

f: age $\longmapsto$ height

b) Is a a function of h? If not, explain why not.

height as input

age as output...

NO.

f: height $\longrightarrow$ age

$f(5'9'')$ multiple outputs

6. Let w represent the weight of a (1 foot) × (1 foot) × (1 foot) package you want to send through the postal service, and C represent the cost to ship it.

a) Is C a function of w? If not, explain why not.

YES

b) Is w a function of C? If not, explain why not.

NO.