Section 10.4 : Factoring trinomials when $a \neq 1$

To factor ax^2+bx+c . . .

1. multiply $a \cdot c$
2. find two numbers that multiply to ac and add to b.
3. split the bx term into two terms using those numbers.
4. factor by grouping.

Ex 1: $2x^2+11x+12$

use the ac method!

$a \cdot c = 2 \cdot 12 = 24$

find two #s that multiply to 24 and add to 11: 8 and 3

$$= \underbrace{2x^2+8x}+\underbrace{3x+12}$$

$$= 2x(x+4)+3(x+4)$$

$$= (2x+3)(x+4)$$

Ex 2: $2x^2+1x-15$

$a \cdot c = 2(-15) = -30$

find two #s that multiply to -30 and add to 1: -5 and 6

$$= \underbrace{2x^2-5x}+\underbrace{6x-15}$$

$$= x(2x-5)+3(2x-5)$$

$$= (x+3)(2x-5)$$

we could check these by FOILing.

1. Factor each polynomial completely. Note that these are all trinomials with a leading coefficient other than 1. Check your factorization by multiplying out the factored version.

a) $5y^2 - 16y + 3$

$a \cdot c = 3 \cdot 5 = 15$
mult to 15, add to -16: -15 and -1

$= \underbrace{5y^2 - 15y} \underbrace{- 1y + 3}$

$= 5y(y-3) - 1(y-3)$

$= (5y-1)(y-3)$

b) $3x^2 + 13x - 10$

$a \cdot c = 3(-10) = -30$
mult to -30, add to 13: 15 and -2 (not 10 and 3!)

$= \underbrace{3x^2 + 15x} \underbrace{- 2x - 10}$

$= 3x(x+5) - 2(x+5)$

$= (3x-2)(x+5)$

c) $6w^2 - 11w + 4$

$a \cdot c = 6 \cdot 4 = 24$
mult to 24, add to -11: -3 and -8

$= \underbrace{6w^2 - 3w} \underbrace{- 8w + 4}$

$= 3w(2w-1) - 4(2w-1)$

$= (3w-4)(2w-1)$

d) $14y^2 + 15y - 9$

$a \cdot c = 14(-9) = -126$
mult to -126, add to 15? -6 and 21

1, 126
2, 63
3, 42
6, 21 ✓

$= \underbrace{14y^2 - 6y} \underbrace{+ 21y - 9}$

$= 2y(7y-3) + 3(7y-3)$

$= (2y+3)(7y-3)$

e) $18r^2 + 27r + 9$

GCF = 9

$= 9(2r^2 + 3r + 1)$ $a \cdot c = 2 \cdot 1 = 2$
2 and 1

$= 9(\underbrace{2r^2 + 2r} \underbrace{+ 1r + 1})$

$= 9[2r(r+1) + 1(r+1)]$

$= 9(2r+1)(r+1)$

f) $35t^2 + 28t - 7$

GCF = 7

$= 7(5t^2 + 4t - 1)$ $a \cdot c = 5(-1) = -5$
5 and -1

$= 7(\underbrace{5t^2 + 5t} \underbrace{- 1t - 1})$

$= 7[5t(t+1) - 1(t+1)]$

$= 7(5t-1)(t+1)$

g) $4x^9 + 18x^8 + 14x^7$

GCF = $2x^7$

$= 2x^7(2x^2 + 9x + 7)$ $a \cdot c = 2 \cdot 7 = 14$
2 and 7

$= 2x^7(\underbrace{2x^2 + 2x} \underbrace{+ 7x + 7})$

$= 2x^7[2x(x+1) + 7(x+1)]$

$= 2x^7(2x+7)(x+1)$

h) $5r^2 + 17rx + 14x^2$

$a \cdot c = 5 \cdot 14 = 70$
10 and 7

$= \underbrace{5r^2 + 10rx} \underbrace{+ 7rx + 14x^2}$

$= 5r(r+2x) + 7x(r+2x)$

$= (5r+7x)(r+2x)$

2. A rectangle has area $10x^2 + 3x - 27$ square centimeters, where x is some unknown number. Its two side lengths are nice, simple, linear binomials.

   a) What are the lengths of ~~the~~ each side, expressed as an expression in x?

area = length · width

$a \cdot c = 10(-27) = -270$

1, 270
2, 135
3, 90
5, 54
6, 45
9, 30
10, 27
15, 18

$$10x^2+3x-27 =$$
$$\underbrace{10x^2-15x}+\underbrace{18x-27} =$$
$$5x(2x-3)+9(2x-3) =$$
$$(5x+9)(2x-3)$$

$\boxed{\text{The lengths are } 5x+9 \text{ and } 2x-3.}$

   b) Based on the previous answer, what must x be larger than to guarantee the rectangle has positive length and positive width?

make sure that $2x-3>0$.

$$2x - 3 + 3 > 0 + 3$$
$$2x > 3$$
$$2x \div 2 > 3 \div 2$$
$$x > 1.5$$

3. When you stand on top of a certain skyscraper and throw a javelin straight up in the air, it eventually turns and falls all the way to the street below. Since the building height is 407 feet, and you throw the javelin with an initial speed of 14 feet per second, the height of the javelin after t seconds is

$$-16t^2 + 14t + 407$$

(That -16 has to do with how strong gravity is on Earth.)

   a) Factor that polynomial. This may take time. When you look for factor pairs of AC, there are 20 of them (not counting negatives). Use a simple calculator to help speed up finding the factor pairs.

88 and -74

$$\underbrace{-16t^2+88t}\underbrace{-74t+407}$$
$$=-8t(2t-11)-37(2t-11)$$
$$=(-8t-37)(2t-11)$$
$$=-(8t+37)(2t-11)$$

   b) Based on the factorization, can you predict the time when the javelin will hit the ground?

let's save this for lesson 10.7 :)

Section 10.5: Factoring special patterns

$$(A+B)^2 = (A+B)(A+B) = A^2 + AB + AB + B^2$$
$$= A^2 + 2AB + B^2$$

Ex 1: factor $x^2 + 10x + 25$

x squared $\quad 2\cdot x\cdot 5 \quad$ 5 squared

$$= (x+5)(x+5) \text{ or } (x+5)^2$$

$$(A-B)^2 = (A-B)(A-B) = A^2 - AB - AB + B^2$$
$$= A^2 - 2AB + B^2$$

Ex 2: factor $x^4 - 14x^2 + 49$

$(x^2)^2 \quad -2\cdot x^2\cdot 7 \quad 7^2$

$$= (x^2-7)(x^2-7) \text{ or } (x^2-7)^2$$

$$(A+B)(A-B) = A^2 - \cancel{AB} + \cancel{AB} - B^2$$
$$= A^2 - B^2$$

Ex 3: Factor $w^6 - 100$

$(w^3)^2 \quad 10^2$

$$= (w^3+10)(w^3-10)$$

$A^2 + B^2$ does NOT factor.

Ex 4: $x^2 + 16$ is prime.

try $(x+4)(x+4)$?
$= x^2 + 4x + 4x + 16$ NO.

try $(x+4)(x-4)$?
$= x^2 - 4x + 4x - 16$ NO.

1. Factor each polynomial (completely). If it can't be factored, then label the polynomial as prime. Note that most of these are polynomials with some special pattern that allows you to factor them quickly. Check your factorization by multiplying out the factored version.

a) x^2-25 $=$

$(x+5)(x-5)$

b) $1-49x^2$ $=$

$(1+7x)(1-7x)$

c) $x^{10}-9$ $=$

$(x^5+3)(x^5-3)$

d) $16x^4-81$ $=$

$(4x^2+9)(4x^2-9) =$

$(4x^2+9)(2x+3)(2x-3)$

e) x^2+36

prime

f) $18y^2-2$ $=$

$2(9y^2-1) =$

$2(3y+1)(3y-1)$

g) x^2+2x+1 $=$

$(x+1)(x+1) =$

$(x+1)^2$

h) $x^2-14x+49$ $=$

$(x-7)(x-7) =$

$(x-7)^2$

i) $x^2+22x+121$ $=$

$(x+11)(x+11) =$

$(x+11)^2$

j) $x^2+10x+100$ $=$

$(x+$ wait a minute...

I was expecting $x^2+20x+100$!

$$\begin{array}{l} \underline{100} \\ 1,100 \\ 2,50 \\ 4,25 \\ 5,20 \\ 10,10 \end{array} \Bigg\}\ \text{none of these add to 10:}$$

prime

2. Find the prime factorization of the number 899 by observing that it equals $900-1$, and then observing that $900 = 30^2$.

$$899 = 900 - 1 = 30^2 - 1^2$$
$$= (30+1)(30-1)$$
$$= 31 \cdot 29$$

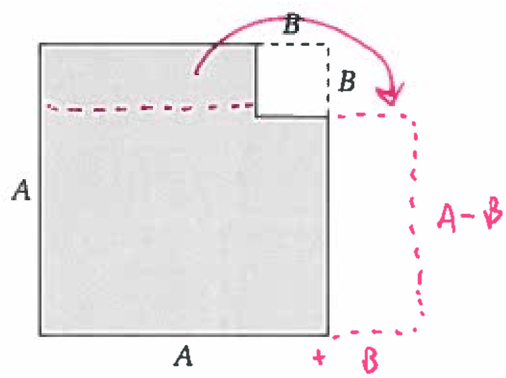
3. Without using a calculator, find $\sqrt{10609}$ by observing that 10609 equals $10000+600+9$, and that $10000 = 100^2$.

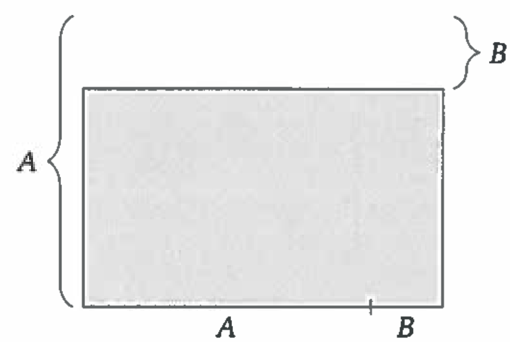
$$= 100^2 + 2 \cdot 100 \cdot 3 + 3^2$$
$$A^2 + 2AB + B^2$$

$$\sqrt{10609} = 103$$

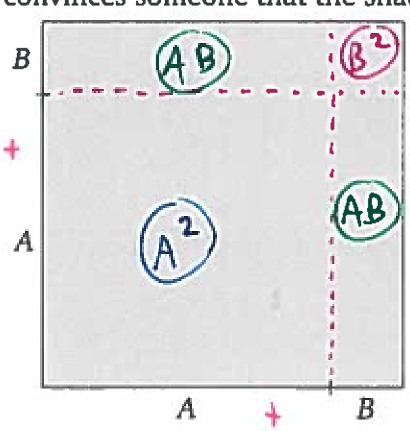
$$= (100+3)(100+3)$$
$$= 103 \cdot 103$$

4. The shaded area on the left represents $A^2 - B^2$. And the shaded area on the right represents $(A+B)(A-B)$. Recently we have learned that $A^2 - B^2 = (A+B)(A-B)$. Enhance these pictures to reveal an explanation for why this is true. You could draw some lines and shade some sections of the images in additional ways. Your goal is to draw a picture that convinces someone that the left shaded area equals the right shaded area.





5. The shaded area represents $(A+B)^2$. Recently we have learned that $A^2+2AB+B^2 = (A+B)^2$. Enhance the picture to reveal an explanation for why this is true. You could draw some lines and shade some sections of the images in additional ways. Your goal is to draw a picture that convinces someone that the shaded area is the same as $A^2+2AB+B^2$.



$$A^2 + 2AB + B^2$$