10.4 Factoring non-monic trinomials

$\underbrace{\text{non-monic}}_{\text{leading coefficient is not 1}}$ $\underbrace{\text{trinomials}}_{\text{3 terms}}$

Ex Some are easy... Factor $2x^2+14x+24$

<look for GCF first>

$$= 2(\underbrace{x^2}_{\text{monic}}+7x+12)$$

$$= 2(x+4)(x+3)$$

Ex Factor $2x^2+9x+9$

A=2, B, C=9

← no GCF here...

← 3 terms, not 4
can't use grouping

← trinomial... not monic.

The "AC" method.

$A \cdot C = 18$ → Factor pairs of 18

	Add
1 (18)	19
(2)(9)	11
(3)(6) ←	9 ← is **B**

Now use 3 and 6...

$$2x^2+9x+9$$
$$= 2x^2+3x+6x+9$$
$$= x(2x+3)+3(2x+3) \quad \leftarrow \text{"}2x+3\text{" twice}$$
$$= (2x+3)(\quad\quad)$$
$$= (2x+3)(x+3) \quad \text{Now it's factored.}$$

Ex Factor $8x^2 - 2x - 21$

$$= \underbrace{8x^2 + 12x}\ \underbrace{-14x - 21}$$

$$= 4x(\quad) - 7(\quad)$$

$$= 4x(2x+3) - 7(2x+3)$$

$$= (2x+3)(\quad)$$

$$= (2x+3)(4x-7)$$

$A \cdot C = 8(-21) = -168$

Do factor pairs on $\longrightarrow$

	sum
$1(-168)$	-167
$2(-84)$	-82
$3(-56)$	-53
$4(-42)$	-38
$6(-28)$	-22
$7(-24)$	-17
$8(-21)$	-13
$12(-14)$	-2

winners!

looking for $B = -2$

Why do we factor?

Ex Currently, we sell digital microscopes for \$41 online. Currently, we have 998 sales each month. (How much revenue? $\$41 \cdot 998 = \$40{,}918$)

We consider adding x dollars to our price. (maybe even x is negative...)

Economics: If we do this, ~~profit~~ revenue will be

$$-31x^2 - 273x + 40918$$

$$= \underbrace{-31x^2 + 998x}\ \underbrace{-1271x + 40918}$$

$$= -x(31x - 998) - 41(31x - 998)$$

$$= (31x - 998)(-x - 41)$$

$$= -(31x - 998)(x + 41)$$

Let's factor using AC method...

$A \cdot C = -1{,}268{,}458$

$1(-1268458)$ $\quad -1268457$

$\vdots \qquad \vdots$

$998(-1271)$ $\quad -273$

revenue $= -(31x-998)\underbrace{(x+41)}$

this makes some sense in $ context

it would be the new price.

revenue $=$ (# sales)$\cdot$(price)

revenue $= \underbrace{-(31x-998)}_{\text{new \# of sales!}} \underbrace{(x+41)}_{\text{new price}}$

More simply: new # sales $= -31x + 998$

$y = mx + b$

slope

rate of change

rate of change in $\frac{\text{\# sales}}{\text{dollar added}}$

This has been foreshadowing how factoring becomes unexpectedly useful.

1. Factor each polynomial completely. Note that these are all trinomials with a leading coefficient other than 1. Check your factorization by multiplying out the factored version.

a) $5y^2 - 16y + 3$

$= 5y^2 - 15y - y + 3$

$= 5y(y-3) - 1(y-3)$

$= (y-3)(5y-1)$

15 sum
1(15) 16
3(5)
⋮
(-1)(-15) -16 !!!

b) $3x^2 + 13x - 10$

$= 3x^2 + 15x - 2x - 10$

$= 3x(x+5) - 2(x+5)$

$= (x+5)(3x-2)$

c) $6w^2 - 11w + 4$

$= 6w^2 - 8w - 3w + 4$

$= 2w(3w-4) - 1(3w-4)$

$= (3w-4)(2w-1)$

d) $14y^2 + 15y - 9$

$= 14y^2 - 6y + 21y - 9$

$= 2y(7y-3) + 3(7y-3)$

$= (7y-3)(2y+3)$

$AC = -126$

1(-126) -125
2(-63) -61
3(-42) -39
6(-21) -15
close!
(-6)(21) 15
winners!

e) $18r^2 + 27r + 9$

$= 9(2r^2 + 3r + 1)$

$= 9(2r^2 + r + 2r + 1)$ $A \cdot C = 2$ $1 \cdot (2)$ 3✓

$= 9(r(2r+1) + 1(2r+1))$

$= 9(2r+1)(r+1)$

f) $35t^2 + 28t - 7$

$= 7(5t^2 + 4t - 1)$

$= 7(5t^2 + 5t - t - 1)$

$= 7((5t)(t+1) - 1(t+1))$

$= 7(t+1)(5t-1)$

g) $4x^9 + 18x^8 + 14x^7$

$= 2x^7(2x^2 + 9x^1 + 7)$

$= 2x^7(2x^2 + 2x + 7x + 7)$

$= 2x^7(2x(x+1) + 7(x+1))$

$= 2x^7(2x+7)(x+1)$

h) $5r^2 + 17rx + 14x^2$ $AC = 70 \ldots$

$= (?r + ?x)(?r + ?x)$

$= 5r^2 + 10rx + 7rx + 14x^2$

$= \cancel{5r(2r+5x}$

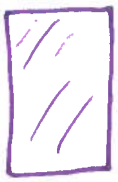
$= 5r(r+2x) + 7x(r+2x)$

$= (r+2x)(5r+7x)$

2. A rectangle has area $10x^2 + 3x - 27$ square centimeters, where x is some unknown number. Its two side lengths are nice, simple, linear binomials.

   a) What are lengths of the each side, expressed as an expression in x?

5x+9



2x−3

Factor: $AC = -270$

$(1)(-270)$ ⋮ $(18)(-15)$

-269 ⋮ 3

$$= 10x^2 + 18x - 15x - 27$$
$$= 2x(5x+9) - 3(5x+9)$$
$$= (5x+9)(2x-3)$$

heyht width

   b) Based on the previous answer, what must x be larger than to guarantee the rectangle has positive length and positive width?

Need $2x - 3 > 0$

$2x > 3$

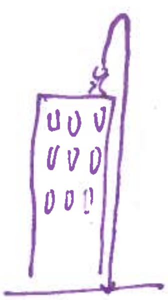
$x > 1.5$

it must be loger than 1.5

3. When you stand on top of a certain skyscraper and throw a javelin straight up in the air, it eventually turns and falls all the way to the street below. Since the building height is 407 feet, and you throw the javelin with an initial speed of 14 feet per second, the height of the javelin after t seconds is

$$-16t^2 + 14t + 407$$

(That -16 has to do with how strong gravity is on Earth.)



   a) Factor that polynomial. This may take time. When you look for factor pairs of AC, there are 20 of them (not counting negatives). Use a simple calculator to help speed up finding the factor pairs.

$$-16t^2 + 88t - 74t + 407$$
$$= -8t(2t - 11) - 37(2t - 11)$$
$$= (2t-11)(-8t-37)$$

   b) Based on the factorization, can you predict the time when the javelin will hit the ground?

heyht = 0

$$-(2t-11)(8t+37) = 0$$

$2t - 11 = 0$

$2t = 11$

$t = 5.5$ → It takes 5.5 seconds

10.5 Factoring Special Patterns

Ex Factor $x^2 - 36$.

$= (x+6)(x-6)$

$\leftarrow$ Pattern is $A^2 - B^2$

$x^2 - 6^2$

$x^2 + 0x - 36$

Ex Factor $x^2 + 36$.

This pattern $(A^2 + B^2)$ is automatically prime (doesn't factor).

Ex Factor $25p^2 - 81$.

$= (5p+9)(5p-9)$

$\leftarrow$ See "$A^2 - B^2$"

$(5p)^2 - 9^2$

Ex Factor $28x^2 - 700$

$= 7(4x^2 - 100)$

$= 7 \cdot 4 \cdot (x^2 - 25)$

$= 28(x+5)(x-5)$

Ex Factor $x^2 + 6x + 9$

perfect square? yes... x

match!

double it...

perfect square? yes... 3

multiply: $3x$

$$= (x+3)^2$$

$$A^2 + 2AB + B^2$$

$A = x$ $B = 3$

$$= (A+B)^2$$

Ex Factor $x^2 + 20x + 36$ $\underset{\text{monic trinomial method}}{\overset{\text{use}}{=}} (x+2)(x+18)$

NO!!

perf square x

double?

perf. square 6

$6x$

Ex Factor $y^2 - 20y + 100$

y

double

10

$10y$

$$= (y-10)^2$$

Ex Factor $4x^2 + 28x + 49$

$2x$

double

7

$14x$

$$= (2x+7)^2$$

1. Factor each polynomial (completely). If it can't be factored, then label the polynomial as prime. Note that most of these are polynomials with some special pattern that allows you to factor them quickly. Check your factorization by multiplying out the factored version.

$A^2 - B^2$

$1^2 - (7x)^2$

a) $x^2 - 25$

$= (x+5)(x-5)$

b) $1 - 49x^2$

$= (1+7x)(1-7x)$

c) $x^{10} - 9$

$(x^5)^2 \quad 3^2$

$(A+B)(A-B)$

$= (x^5+3)(x^5-3)$

d) $16x^4 - 81$

$A^2 - B^2$

$(4x^2)^2 - 9^2$

$= (4x^2+9)(4x^2-9)$

$= (4x^2+9)(2x+3)(2x-3)$

e) $x^2 + 36$

prime!

f) $18y^2 - 2$

$= 2(9y^2-1)$

$= 2(3y+1)(3y-1)$

g) $x^2 + 2x + 1$

$= (x+1)^2$

h) $x^2 - 14x + 49$

$(x-7)^2$

i) $x^2 + 22x + 121$

$= (x+11)^2$

j) $x^2 + 10x + 100$

x NO double... 10

$10x$

prime!

can't use a special pattern...

$(\ )(\ ) = 100$

but $(\ ) + (\ ) = 10$

won't find them.

2. Find the prime factorization of the number 899 by observing that it equals $900-1$, and then observing that $900 = 30^2$.

$899 = 30^2 - 1^2$

$899 = (30+1)(30-1)$

$899 = 31 \cdot 29$

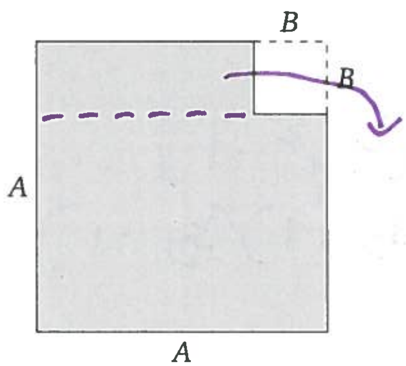
3. Without using a calculator, find $\sqrt{10609}$ by observing that 10609 equals $10000+600+9$, and that $10000 = 100^2$.

$\sqrt{10609} = \sqrt{10000 + 600 + 9} = \sqrt{(100+3)^2} = 103$

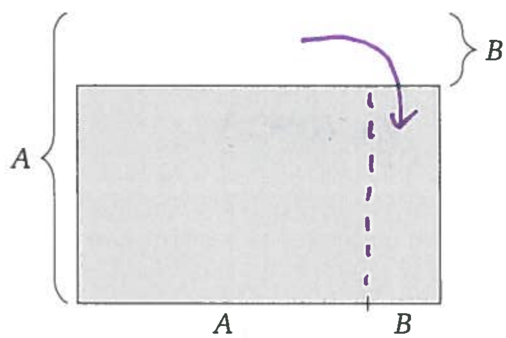
100 300 3

4. The shaded area on the left represents $A^2 - B^2$. And the shaded area on the right represents $(A+B)(A-B)$. Recently we have learned that $A^2-B^2 = (A+B)(A-B)$. Enhance these pictures to reveal an explanation for why this is true. You could draw some lines and shade some sections of the images in additional ways. Your goal is to draw a picture that convinces someone that the left shaded area equals the right shaded area.

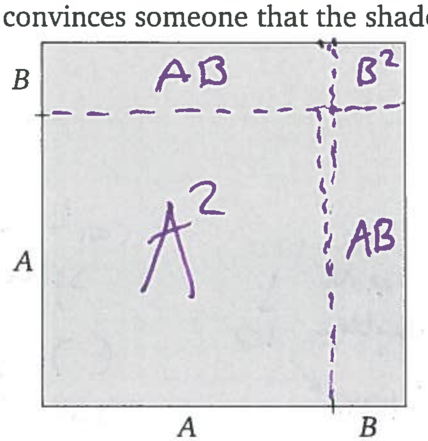
A^2-B^2



$(A+B)(A-B)$



5. The shaded area represents $(A+B)^2$. Recently we have learned that $A^2+2AB+B^2 = (A+B)^2$. Enhance the picture to reveal an explanation for why this is true. You could draw some lines and shade some sections of the images in additional ways. Your goal is to draw a picture that convinces someone that the shaded area is the same as $A^2+2AB+B^2$.



$(A+B)^2$

$= A^2 + 2AB + B^2$