10.2 Sometimes a polynomial with 4 terms can be factored like:

Factor: $\underbrace{x^3 + 2x^2}$ $+$ $\underbrace{5x + 10}$ (note: not factored; four pieces... but <u>added</u>, not multiplied)

break it into two groups

$= (x^3 + 2x^2) + (5x + 10)$

with each group use its GCF

$= x^2(x+2) + 5(x+2)$

Lucky: "x+2" in both places

$= x^2 \cdot y + 5 \cdot y$ (y a place holder for (x+2))

$= y(x^2 + 5)$

$= (x+2)(x^2+5)$ ← Two things multiplied! It's factored!

Ex Factor $2x^3 - 5x^2 - 12x + 30$

$= x^2(2x-5) - 6(2x-5)$

$= (2x-5)(x^2-6)$

Negative signs, watch out...

2nd group is $-12x + 30$

Leading coef negative ⟹ factor out a negative

be careful!

Ex Factor $4x^3 - 12x^2 + x - 3$

$= 4x^2(x-3) + x - 3$

$= 4x^2(x-3) + 1\cdot(x-3)$ ← in this situation factor out a "1"

$= (x-3)(\quad)$

$= (x-3)(4x^2+1)$

Ex Factor: $xy^2 - 10y^2 - 2x + 20$ 4 terms? try grouping

$= (xy^2 - 10y^2) + (-2x+20)$

$= y^2(\quad) - 2(\quad)$

$= y^2(x-10) - 2(x-10)$

$= (x-10)(y^2-2)$ Now it's factored.

1. Factor each polynomial. In some cases, some grouping parentheses are already in place to help you finish the factoring.

a) $x(x+2)-4(x+2)$

$= (x+2)(x-4)$

b) $3x(x+y)-(x+y)$

$= (x+y)(3x-1)$

c) $7x^2(5x+4)+5x+4$

$= (5x+4)(7x^2+1)$

d) $x^2+3x-5x-15$

$= (x+3)(x-5)$

e) x^3-x^2+2x-2

$= (x-1)(x^2+2)$

f) $xy-x+5y-5$

$= (y-1)(x+5)$

g) $3x^3-2x^2-6x+4$

$= (3x-2)(x^2-2)$

h) $x^2+2xy+3xz+6yz$

$= x(x+2y)+3z(x+2y)$

$= (x+2y)(x+3z)$

2. Decide if the statement or math work is true/correct or false/incorrect. If it is false/incorrect, explain what exactly is wrong with it.

a) $a(x-7)+b(7-x)$
$= a(x-7)+b(-1)(x-7)$
$= a(x-7)-b(x-7)$
$= (a-b)(x-7)$

Good, Correct!

b) a^2+b^2
$= a^2+ab-ab+b^2$
$= a(a+b)-b(a+b)$
$= (a+b)(a-b)$

Need to factor out $-b$ so this should be subtraction!

c) $-4x^2+12x$ can be factored as $-4x(x-3)$ or $4x(-x+3)$.

didn't... factored out neg...

both are correct.

d) Since the GCF of $9x^3+6x^2+3x$ is $3x$, it is not necessary to write a 1 when $3x$ is factored from the last term.

$3x(3x^2+2x+1)$

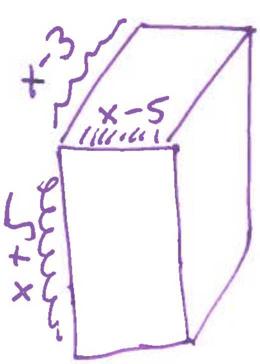
3. There was a rectangular box with all three edges of different length. After you had computed the volume of the box, you had found that the volume was

$$x^3-3x^2-25x+75$$

measured in cubic inches, where x is in inches. Find one possibility for the height, width, and depth of the box.

$= x^2(x-3)-25(x-3)$
$= (x-3)(x^2-25)$
$= (x-3)(x+5)(x-5)$

$50 = 10\cdot 5 = 5\cdot 5\cdot 2$



$V = (\ldots)(\ldots)(\ldots) = x^3-3x^2-25x+75 = (x-3)(x+5)(x-5)$

10.3 Factoring Monic Trinomials

Monic ↓ the leading coefficient is 1.

Trinomials ↓ three terms...

Ex Multiply $(x+7)(x+5)$

Start with "+7" and "+5" ⋮

$= x^2 + 5x + 7x + 35$

$= x^2 + 12x + 35$

$12x$ ← $(+7)+(+5)$; 35 ← $(+7)\cdot(+5)$

Use this to reverse engineer a factoring technique...

Ex Factor $x^2 + 9x + 18$.

$= (x \quad)(x \quad)$

$= (x+3)(x+6)$

Done!

$(\ )\cdot(\ ) = 18$

Same numbers...

$(\ ) + (\ ) = 9$

3 and 6 do it...
+3 +6

Ex Factor $x^2 + 3x - 40$

$= (x+8)(x-5)$

!!!

$(\ )(\ ) = -40$

$(\ ) + (\ ) = 3$

$+8$ and -5 ...

Ex Factor $q^2 - 10q - 144$

$= (q \quad)(q \quad)$

$(\quad)(\quad) = -144$

$(\quad) + (\quad) = -10$

Give up on guessing...

Factor pair tables

Multiply	Add
$(1)\cdot(-144)$	-143
$(2)(-72)$	-70
$(3)(-48)$	-45
$(4)(-36)$	-32
skip 5 → $(6)(-24)$	-18
skip 7 → $(8)(-18)$	-10

winning combo!

$= (q+8)(q-18)$

Check by multiplying back...

$(q+8)(q-18)$

= use FOIL

$= q^2 - 18q + 8q - 144$

$= q^2 - 10q - 144$

	q	$+8$
q	q^2	$8q$
-18	$-18q$	-144

$= q^2 - 10q + 144$

1. Factor these polynomials.

a) $x^2 + 7x + 10$

$= (x+2)(x+5)$

b) $x^2 - 7x + 12$

$(x-3)(x-4)$

c) $y^2 - 8y + 15$

$= (y-5)(y-3)$

d) $y^2 + 10y - 39$

$= (y+13)(y-3)$

e) $x^2 - 2x - 8$

$= (x-4)(x+2)$

f) $w^2 - 30w - 64$

$= (w-32)(w+2)$

g) $r^2 + 12r + 27$

$= (r+3)(r+9)$

h) $x^2 - 8xy + 15y^2$

$= (x-3y)(x-5y)$

2. You dive directly upward from a board that is 32 feet high. After t seconds, your height above the water is described by the polynomial

$$-16t^2 + 16t + 32$$

a) Evaluate this polynomial at $t = 2$. (2 seconds after launch, ... going to tell us how we are above water).

$-16(2)^2 + 16(2) + 32$
$= -16(4) + 32 + 32$
$= -64 + 64$
$= 0$ (reached the water's surface)

b) Factor the polynomial completely. The first step should be to look for a Greatest Common Factor.

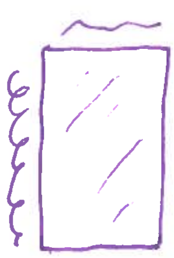
$= -16(t^2 - t - 2)$
$= -16(t+1)(t-2)$

c) Evaluate your answer from part (b) at $t = 2$. Do you get the same result as you did in part (a)? Describe what this answer means in the context of the dive.

$-16(2+1)(2-2)$
$= -16(3)(0)$
$= 0$

3. There was a rectangle with two edges of different length. After you had computed the area of the rectangle, you had found that the area was

$$2x^2 - 10x - 48$$



measured in square inches, where x is in inches. What is one possibility for the dimensions (height and width) of the rectangle. (There are several possibilities.)

$= 2(x^2 - 5x - 24)$
$= 2(\quad)(\quad)$
$= 2(x-8)(x+3)$

$(\ )\cdot(\ ) = -24$
$(\ ) + (\ ) = -5$

$A = (\text{~~~})(\text{eeee})$
$= 2x^2 - 10x - 48$
$= \underbrace{2(x-8)}_{L}\underbrace{(x+3)}_{H}$

4. Factor each polynomial completely. If it can't be done, then label the polynomial as prime. Note that these are all trinomials with a leading coefficient of 1.

a) $2x^2 - 16x + 30$

$= 2(x^2 - 8x + 15)$

$= 2(x-3)(x-5)$

b) $5a^2 - 90a + 225$

c) $4y^2 - 4y - 8$

d) $x^2 - 3x + 6$

this polynomial can't be factored... it's prime

Mult to 6

(1)(6)	7
(2)(3)	5
(-1)(-6)	-7
(-2)(-3)	-5

Never got -3...

e) $2r^3 + 6r^2 + 4r$

f) $y^8 - 7y^7 + 5y^6$

$= y^6(\quad\quad)$

$= y^6(y^2 - 7y + 5)$

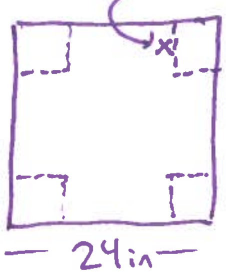
this part is prime...

g) $2w^4 - 26w^3 - 96w^2$

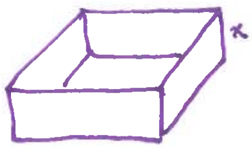
h) $x^5y^2 + 3x^4y^2 - 4x^3y^2$

5. Start with a square of cardboard that is 24 in on each side (sketch a picture of this). You will cut out a little square of the same size from each corner (add this to your picture). After folding up the tabs that this leaves, you end up with a box that is open on its top (sketch a 3D picture of this).

a) If x is the the length (in inches) of the little square that you cut out, then this box has volume $4x^3 - 96x^2 + 576x$ cubic inches. Factor this polynomial completely.



24in



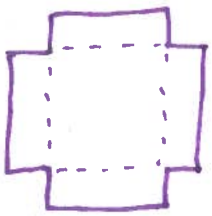
Box has volume

$4x^3 - 96x^2 + 576x$

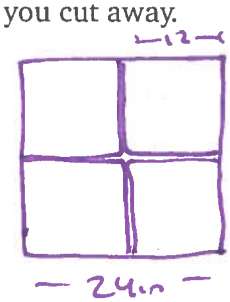
$= 4x(x^2 - 24x + 144)$

$= 4x(x - 12)(x - 12)$

$= 4x(x - 12)^2$



b) Thinking about the physical piece of cardboard and the cuts that you make, what volume would you expect to have in the box if $x = 12$? Note how much cardboard this would mean you cut away.



→ leaves no cardboard!

would have a 0-volume box

c) What does your factored polynomial evaluate to if $x = 12$?

well, $4(12)(12-12)^2$

$= 4(12)(0)^2$

$= 0$