Section 12.5

Strategy: clear denominators

Ex Solve $\dfrac{x+1}{x-3} = \dfrac{x+2}{x-7}$

Report what x-values make the equation true.

Denoms: $(x-3)$ $(x-7)$

copy

LCD: $(x-3)(x-7)$

$$\cancel{(x-3)}(x-7)\cdot\frac{(x+1)}{\cancel{(x-3)}} = (x-3)\cancel{(x-7)}\frac{(x+2)}{\cancel{(x-7)}}$$

$$(x-7)(x+1) = (x-3)(x+2)$$

No denoms!

$$x^2\underbrace{-7x+x}-7 = x^2\underbrace{+2x-3x}-6$$

$$x^2-6x-7 = x^2-x-6$$

$$-x^2$$

$$-6x-7 = -x-6$$

$$+7 \qquad +x$$

$$-5x = 1$$

$$x = -\frac{1}{5}$$

This is the only possible solution.

Check

$$\frac{-\frac{1}{5}+1}{-\frac{1}{5}-3} \stackrel{?}{=} \frac{-\frac{1}{5}+2}{-\frac{1}{5}-7}$$

$$\frac{-\frac{1}{5}+1}{-\frac{1}{5}-3}\cdot\frac{5}{5} \qquad \frac{-\frac{1}{5}+2}{-\frac{1}{5}-7}\cdot\frac{5}{5}$$

$$\frac{-1+5}{-1-15} \qquad \frac{-1+10}{-1-35}$$

$$\frac{4}{-16} \qquad \frac{9}{-36}$$

$$-\frac{1}{4} \quad ✓ \quad -\frac{1}{4}$$

Ex Solve $\frac{x+1}{x-5} = \frac{2x-4}{x-5}$

Denoms: $(x-5)$ $(x-5)$

LCM: $(x-5)$

$$\cancel{(x-5)}\frac{(x+1)}{\cancel{(x-5)}} = \cancel{(x-5)}\frac{(2x-4)}{\cancel{(x-5)}}$$

$$x+1 = 2x-4$$

No denominators!

$-2x$ $\quad$ -1

$$-x = -5$$

$$x = 5$$

Check

$$\frac{5+1}{5-5} \stackrel{?}{=} \frac{2(5)-4}{5-5}$$

$\frac{6}{0}$ not equal to each other; $\frac{6}{0}$

neither is a number...

If there is a solution, it must be 5.

There is no solution.

Ex Solve $\frac{2x+1}{x-3} = \frac{x+41}{x+1}$

Denoms: $(x-3)$ $(x+1)$

LCD: $(x-3)(x+1)$

$$\cancel{(x-3)}(x+1)\frac{(2x+1)}{\cancel{(x-3)}} = (x-3)\cancel{(x+1)}\frac{(x+41)}{\cancel{(x+1)}}$$

$$(x+1)(2x+1) = (x-3)(x+41)$$

No denominators!

$$2x^2 + x + 2x + 1 = x^2 + 41x - 3x - 123$$

$-x^2$

$$x^2 + 3x + 1 = 38x - 123$$

$-38x$ $\quad$ $+123$

$$x^2 - 35x + 124 = 0$$

$$(x-31)(x-4) = 0$$

$x = 31$ or $x = 4$

Check!

31

$$\frac{2(31)+1}{31-3} \stackrel{?}{=} \frac{31+41}{31+1}$$

$$\frac{63}{28} \stackrel{?}{=} \frac{72}{32}$$

$$\frac{9}{4} \stackrel{\checkmark}{=} \frac{9}{4}$$

Check 4 as well.. it works

The solution set is $\{4, 31\}$.

So far, examples: $\dfrac{A}{B} = \dfrac{C}{D}$

a <u>proportional</u> equation.

Solving Rational Equations I

1. Solve each of the equations for x, using algebra (no calculator).

a) $\frac{5}{x} = 7$

Denoms: x

LCD: (x)

$\cancel{(x)}\frac{(5)}{\cancel{(x)}} = (x)(7)$

$5 = x \cdot 7$

$\frac{5}{7} = x$

Check!

b) $\frac{5}{x+2} = 7$

Denoms: $x+2$

LCD: $(x+2)$

$\cancel{(x+2)}\frac{(5)}{\cancel{(x+2)}} = (x+2)(7)$

$5 = 7x+14$

$-9 = 7x$

$-\frac{9}{7} = x$

Check!

c) $\frac{6}{2x+1} = x$

Denoms: $2x+1$

LCD: $(2x+1)$

$\cancel{(2x+1)}\cdot\frac{(6)}{\cancel{(2x+1)}} = (2x+1)(x)$

$6 = 2x^2+x$

$0 = 2x^2+x-6$

Use AC Method

$0 = (x+2)(2x-3)$

$x=-2$ or $x=\frac{3}{2}$

Check both!

d) $\frac{4}{x-3} = x$

Denoms: $x-3$

LCD : $(x-3)$

$\cancel{(x-3)}\frac{(4)}{\cancel{(x-3)}} = (x-3)(x)$

$4 = x^2-3x$

$0 = x^2-3x-4$

$0 = (x-4)(x+1)$

$x=4$ or $x=-1$

Check both!

e) $\frac{6}{x+2} = x+1$

$(x+2)\frac{6}{(x+2)} = (x+1)(x+2)$

$6 = x^2+3x+2$

$0 = x^2+3x-4$

$0 = (x+4)(x-1)$

$x=-4$ or $x=1$

f) $\frac{12}{x+2} = x-9$

$(x+2)\cdot\frac{12}{(x+2)} = (x-9)(x+2)$

$12 = x^2-7x-18$

$0 = x^2-7x-30$

$0 = (x-10)(x+3)$

$x=10$ or $x=-3$

g) $\frac{3}{x+2} = \frac{5}{x-8}$

$(x+2)(x-8)\frac{3}{(x+2)} = \frac{5}{(x-8)}(x+2)(x-8)$

$3(x-8) = 5(x+2)$

$3x-24 = 5x+10$

$-2x = 34$

$x = -17$

h) $\frac{x+9}{x-2} = \frac{5}{7}$

$7\cdot(x-2)\frac{(x+9)}{(x-2)} = \frac{5}{7}\cdot 7(x-2)$

$7(x+9) = 5(x-2)$

$7x+63 = 5x-10$

$2x = -73$

$x = \frac{-73}{2}$

i) $\dfrac{x+1}{x-3} = \dfrac{x+4}{x-2}$

... The solution set is

$\{5\}$.

j) $\dfrac{x+1}{x+2} = \dfrac{4x+18}{2x+31}$

... The solution set is $\{1, \frac{5}{2}\}$

2. Do you understand that in each equation from the previous exercises, each side of the equation was one of the following?

   - A rational expression
   - A polynomial

   (Yes or No)

   Yes!

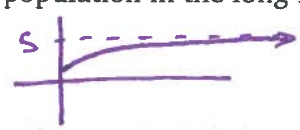
3. The size of an insect population (in thousands per acre) is modeled by $P(x) = \dfrac{5x+2}{x+1}$ where x is the time in months since April 1.

   a) Evaluate $P(3)$ and interpret the result with a sentence in context.

   $P(3) = \dfrac{5(3)+2}{3+1} = \dfrac{17}{4} = 4.25$

   After 3 months, we have 4.25 thousand insects per acre.

   b) Graph P using your graphing calculator and zoom out a lot. What happens to the insect population in the long run? Give a sentence with context.

   The insect population is getting closer and closer to 5 thousand per acre.

   c) When will the insect population reach 4.5 thousand per acre? Solve this using pencil and paper only. Then write a sentence with context.

   $\dfrac{5x+2}{x+1} = 4.5 \implies 5x+2 = 4.5(x+1) \implies 5x+2 = 4.5x+4.5$

   $\implies 0.5x = 2.5 \implies x=5$

   After $\underline{5}$ months, it will reach 4.5 thousand per acre.

   d) What is the domain of this function P? There are two things to consider: the algebraic restrictions on what would be a valid input for the given formula, and the restrictions that have to do with the context of the problem. There is more than one "correct" answer to this question, but be sure to use interval notation.

   The domain is $[0, \infty)$.

Section 12.5 #68

	time to complete task (hr)	total volume/amount of stuff...	rate
Kandace	t	1 (room)	$\frac{1\text{ (room)}}{t\text{ (hr)}} = \frac{1}{t}\ \frac{\text{rooms}}{\text{hr}}$
Jenny	$t+3$	1 (room)	$\frac{1\text{ (room)}}{(t+3)\text{ (hr)}} = \frac{1}{t+3}\ \frac{\text{rooms}}{\text{hr}}$
Together	2	1 (room)	$\frac{1}{2}\ \frac{\text{rooms}}{\text{hr}}$

Make an equation...

Rational Equation (Not proportional)

$$\frac{1}{t} + \frac{1}{t+3} = \frac{1}{2}$$

Ex Solve $\frac{3}{t} = -2 + \frac{13}{t}$

Denoms: (t) (t)

LCD: (t)

$$\cancel{(t)}\left(\frac{3}{\cancel{t}}\right) = (t)\left(-2 + \frac{13}{t}\right)$$

$$3 = -2t + \cancel{(t)}\frac{(13)}{\cancel{(t)}}$$

$$3 = -2t + 13$$

No denoms!

$$-10 = -2t$$

$$5 = t$$

Check $\frac{3}{5} \overset{?}{=} -2 + \frac{13}{5}$

$0.6 \overset{?}{=} -2 + 2.6$

✓

The solution set is $\{5\}$.

Ex Solve $\frac{5}{4x} + \frac{1}{3x} = -5$

Denoms: $4x$ $3x$

LCM: $(4)(x)(3) = 12x$

$$12x\left(\frac{5}{4x} + \frac{1}{3x}\right) = 12x(-5)$$

$$12\cancel{x} \cdot \frac{5}{4\cancel{x}} + 12\cancel{x}\frac{1}{3\cancel{x}} = -60x$$

$$15 + 4 = -60x$$

$$19 = -60x$$

$$-\frac{19}{60} = x$$

Check

$\frac{5}{4\left(\frac{-19}{60}\right)} + \frac{1}{3\left(\frac{-19}{60}\right)} \overset{?}{=} -5$

$\frac{5}{\frac{-19}{15}} + \frac{1}{\frac{-19}{20}} \overset{?}{=} -5$

$5 \cdot \frac{-15}{19} + \frac{20}{-19} \overset{?}{=} -5$

$\frac{-75}{19} + \frac{-20}{19} \overset{?}{=} -5$

$\frac{-95}{19} = -5$ ✓

The sol. set is $\left\{\frac{-19}{60}\right\}$.

$\underline{Ex}$ Solve $\frac{x}{5x-30} - \frac{2}{x-6} = 1$

$$\frac{x}{5(x-6)} - \frac{2}{(x-6)} = 1$$ LCM: $5(x-6)$

$$5(x-6)\left(\frac{x}{5(x-6)} - \frac{2}{(x-6)}\right) = 5(x-6)\cdot 1$$

$$\cancel{5(x-6)}\cdot\frac{(x)}{\cancel{5(x-6)}} - 5\cancel{(x-6)}\cdot\frac{(2)}{\cancel{(x-6)}} = 5x-30$$

$$x - 10 = 5x - 30$$

$$-4x = -20$$

$$x = 5$$

Check!

$\underline{Ex}$ Solve $-\frac{8}{x} - \frac{5}{x+9} = 1$ LCD: $(x)(x+9)$

$$(x)(x+9)\left(\frac{-8}{(x)} - \frac{5}{(x+9)}\right) = (x)(x+9)\cdot 1$$

$$\cancel{(x)}(x+9)\cdot\frac{(-8)}{\cancel{(x)}} - (x)\cancel{(x+9)}\frac{(5)}{\cancel{(x+9)}} = x^2+9x$$

$$-8x - 72 - 5x = x^2+9x$$

$$0 = x^2+22x+72$$

$$0 = (x+18)(x+4)$$

$$x = -18 \text{ or } x = -4$$

Check!

Back to Kandace & Jerry:

$$\frac{1}{t} + \frac{1}{t+3} = \frac{1}{2}$$

(t) $(t+3)$ (2)

LCM: $(t)(t+3)(2$

$$(t)(t+3)(2)\left(\frac{1}{t} + \frac{1}{t+3}\right) = (t)(t+3)\cancel{(2)}\cdot\frac{1}{\cancel{(2)}}$$

$$\cancel{(t)}(t+3)(2)\cdot\frac{1}{\cancel{(t)}} + (t)\cancel{(t+3)}(2)\cdot\frac{1}{\cancel{(t+3)}} = t^2+3t$$

$$2t+6+2t = t^2+3t$$

No denoms!

$$0 = t^2 - t - 6$$

$$0 = (t-3)(t+2)$$

$t=3$ or $t=-2$

Makes no sense in context

$$t = \frac{1 \pm \sqrt{1-4(1)(-6)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{25}}{2}$$

$$= \frac{1 \pm 5}{2}$$

$$= 3, -2$$

Check:

$$\frac{1}{3} + \frac{1}{3+3} \overset{?}{=} \frac{1}{2}$$

$$\frac{1}{3} + \frac{1}{6} \overset{?}{=} \frac{1}{2}$$

$$\frac{2}{6} + \frac{1}{6} \overset{?}{=} \frac{1}{2}$$

$$\frac{3}{6} \overset{\checkmark}{=} \frac{1}{2}$$

So Kandace needs 3 hours...

and Jerry needs 6 hours...

Solving Rational Equations II

1. Solve each of the equations using algebra (no calculator).

a) $\frac{3}{t} = 4 + \frac{23}{t}$

$t \cdot \frac{3}{t} = t\left(4 + \frac{23}{t}\right)$

$3 = 4t + 23$

$-20 = 4t$

$-5 = t$

(check!)

b) $\frac{3}{5y} + \frac{4}{3y} = -5$

$15y\left(\frac{3}{5y} + \frac{4}{3y}\right) = 15y(-5)$

$15y \cdot \frac{3}{5y} + 15y \cdot \frac{4}{3y} = -75y$

$9 + 20 = -75y$

$29 = -75y$

$-\frac{29}{75} = y$

(check!)

c) $\frac{y}{2y+12} + \frac{6}{y+6} = 1$

$2(y+6) \cdot \left(\frac{y}{2y+12} + \frac{6}{y+6}\right) = 2(y+6) \cdot 1$

$\cancel{2(y+6)} \cdot \frac{y}{\cancel{2(y+6)}} + 2\cancel{(y+6)} \cdot \frac{6}{\cancel{(y+6)}} = 2y + 12$

$y + 12 = 2y + 12$

$0 = y$

(check!)

d) $\frac{r-9}{r^2+6} = 0$

shortcut!
the only way a fraction can be 0 is if the numerator is 0. $r - 9 = 0$

$\Rightarrow r = 9$

(but check!)

e) $\frac{t+9}{t^2+15t+54} = 0$

Same deal as d)...

$t = -9$.

But check!

$\frac{-9+9}{(-9)^2 + 15(-9) + 54} \stackrel{?}{=} 0$

$\frac{0}{81 - 135 + 54} \stackrel{?}{=} 0$

$\frac{0}{0} \stackrel{\text{no!}}{=} 0$

f) $\frac{2}{x} - \frac{8}{x+7} = -1$

...

$x^2 - 3x - 14 = 0$

doesn't factor!

Use Quadratic Formula

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-14)}}{2(1)}$

$x = \frac{3 \pm \sqrt{65}}{2}$

g) $\dfrac{1}{y-5} - \dfrac{5}{y^2-5y} = -\dfrac{1}{9}$

$\dfrac{1}{(y-5)} - \dfrac{5}{y(y-5)} = -\dfrac{1}{9}$

$9y(y-5)\dfrac{1}{(y-5)} - 9y(y-5)\cdot\dfrac{5}{y(y-5)} = -\dfrac{1}{9}\cdot 9y(y-5)$

$9y - 45 = -y(y-5)$

$9y - 45 = -y^2 + 5y$

$y^2 + 4y - 45 = 0$

$(y+9)(y-5) = 0$

$\Longrightarrow y = -9$ or $y = 5$

but $y=5$ doesn't work when you check it. The solution set is $\{-9\}$.

h) $\dfrac{2}{t+1} = \dfrac{3}{t-1} - \dfrac{2}{t^2-1}$

$\dfrac{2}{(t+1)} = \dfrac{3}{(t-1)} - \dfrac{2}{(t+1)(t-1)}$

$(t+1)(t-1)\dfrac{2}{(t+1)} = (t+1)(t-1)\dfrac{3}{(t-1)} - (t+1)(t-1)\dfrac{2}{(t+1)(t-1)}$

$2(t-1) = 3(t+1) - 2$

$2t - 2 = 3t + 3 - 2$

$-3 = t$

i) $\dfrac{2}{y+5} - \dfrac{5}{y+1} = -\dfrac{2}{y^2+6y+5}$

$(y+5)(y+1)\dfrac{2}{(y+5)} - (y+5)(y+1)\dfrac{5}{(y+1)} = -\dfrac{2}{(y+5)(y+1)}(y+5)(y+1)$

$2(y+1) - 5(y+5) = -2$

$2y + 2 - 5y - 25 = -2$

$-3y = 21$

$y = -7$

j) $-\dfrac{6}{r-3} + \dfrac{8r}{r+9} = -\dfrac{4}{r^2+6r-27}$

$(r-3)(r+9)\cdot\dfrac{-6}{(r-3)} + (r-3)(r+9)\dfrac{8r}{(r+9)} = -\dfrac{4}{(r+9)(r-3)}(r-3)(r+9)$

$-6(r+9) + 8r(r-3) = -4$

$-6r - 54 + 8r^2 - 24r = -4$

$8r^2 - 30r - 50 = 0$

use AC method $\longrightarrow$ $4r^2 - 15r - 25 = 0$

$(4r+5)(r-5) = 0$

$r = -\dfrac{5}{4}$ or $r = 5$

2. In still water a tugboat can travel 15 miles per hour. It travels 36 miles upstream and then 36 miles downstream in a total of 5 hours. Find the speed of the current.

	distance	speed	time
upstream	36	$15-x$	$\dfrac{36}{15-x}$
downstream	36	$15+x$	$\dfrac{36}{15+x}$

Let x be the speed of the current

$\dfrac{36}{15-x} + \dfrac{36}{15+x} = 5$