11.2 Domain and Range of Functions

Let P be a fucton that gives the number of people with access to internet in year x.

$P(2019) = 4.48$ billion

$P(2005) = 1.1$ billion

$P(1963)$ is trying to ask how many Internet users in 1963...
That makes no sense...
Internet was invented in 1983.

"P of 1963" makes no sense

1963 is "bad" input...

Ex Let g be a fucton where $g(x) = \frac{x+1}{x-2}$

Find $g(4) = \frac{(\ )+1}{(\ )-2}$

$g(4) = \frac{4+1}{4-2} = \frac{5}{2} = 2.5$

$g(3) = \frac{3+1}{3-2} = \frac{4}{1} = 4$

} 4, 3 are "good" input

But! $g(2) = \frac{2+1}{2-2} = \frac{3}{0}$ ← we can't divide by 0! There's no result here.

So 2 is a "bad" input.

Ex $\text{sqrt}(x) = \sqrt{x}$

Are there bad inputs?

Negative numbers are banned as input.

All OK...

$\text{sqrt}(0) = \sqrt{0} = 0$

$\text{sqrt}\left(\frac{1}{2}\right) = \sqrt{\frac{1}{2}} \approx 0.707\ldots$

$\text{sqrt}(4) = \sqrt{4} = 2$

$\sqrt{\text{negative}}$ has no real number result.

Ex A table:

x	$f(x)$
0	3
2	5
5	22
17	22

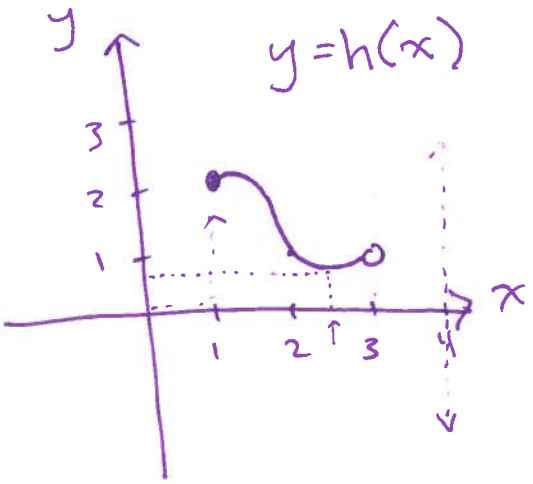
Well, $f(0) = 3$

$f(5) = 22$

But $f(10)$?

we'd say 10 is a "bad" input... don't know what to turn it into.

Ex



$h(1) = 2$

$h(2) = 1$

$h(2.5) = 0.7$

all using "bad" input

$h(4) = ?$

$h(3)$?

$h(0)$?

$h(-8)$?

A function's domain is the collection of all of its good input.

→ the area over which I have control.

Function	Visual... Numberline	Inequality	Set-builder Notation	Interval Notation	Set Notation
P		$x \geq 1983$	$\{x \mid x \geq 1983\}$	$[1983, \infty)$	
g		$x \neq 2$	$\{x \mid x \neq 2\}$	$(-\infty, 2) \cup (2, \infty)$	
sqrt		$x \geq 0$	$\{x \mid x \geq 0\}$	$[0, \infty)$	
f					$\{0, 2, 5, 17\}$
h		$x \geq 1$ and $x < 3$	$\{x \mid x \geq 1 \text{ and } x < 3\}$	$[1, 3)$	

Ex H is a function where $H(x) = \frac{2x+1}{x^2-2x-8}$.

Find H's domain.

Worry about $x^2-2x-8=0$

$(x-4)(x+2)=0$

Either $x-4=0$ or $x+2=0$

$x=4$ $x=-2$

$(-\infty,-2) \cup (-2,4) \cup (4,\infty)$

Ex Q is a function where $Q(x) = \sqrt{7-x} + 3$

Find Q's domain.

Need $7-x \geq 0$

$-x \geq -7$

mult/div by a neg when you have an inequality ⟹ change direction

$\frac{-x}{-1} \leq \frac{-7}{-1}$

$x \leq 7$

$(-\infty, 7]$

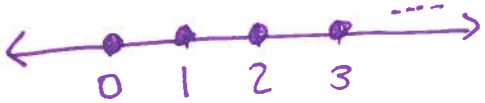
Ex Bring x recyclable cans to supermarket.

r(x) is the amount of money you get.

What is r's domain?

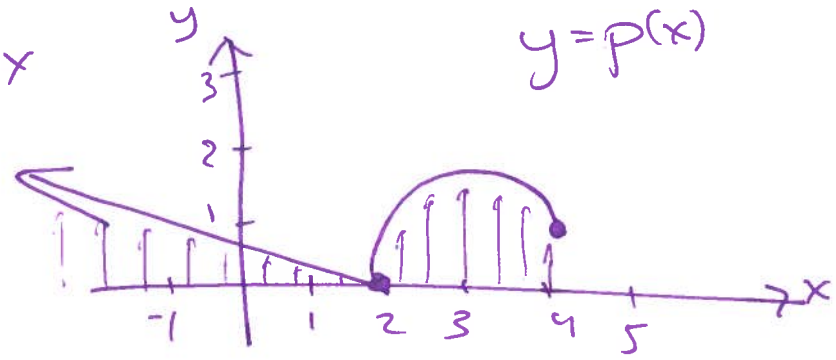
input: how many cans

output: dollars...



$\{0, 1, 2, 3, \ldots\}$

Ex

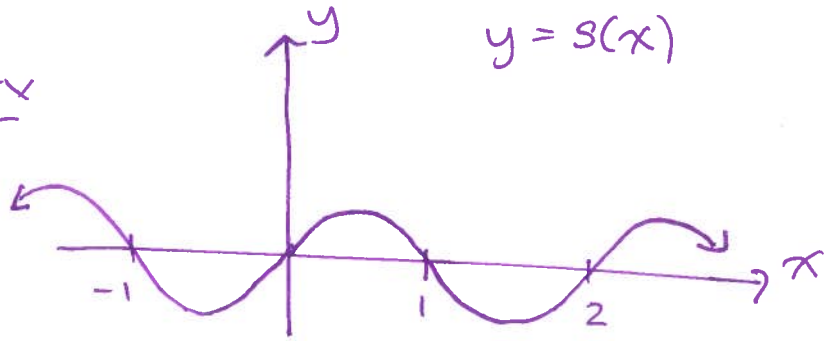


$y = p(x)$

Find p's domain.

$(-\infty, 4]$

Ex



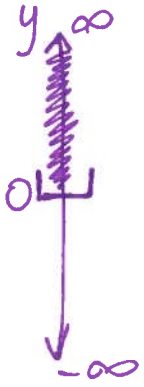
$y = s(x)$

Find s's domain.

$(-\infty, \infty)$

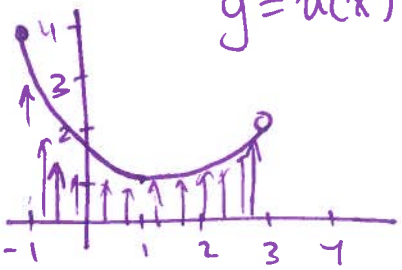
The range of a function is the collection of all possible output numbers.

Ex Let f be the function where $f(x) = x^2$.
(Note f's domain is all real numbers, $(-\infty, \infty)$.)
What is f's range?
Well, no matter what x is, x^2 is always at least 0... In other words, this function can't produce negative numbers...



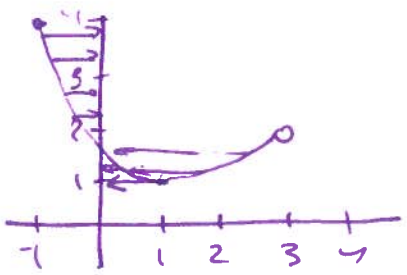
picture of f's range... in interval notation... $[0, \infty)$

Ex $y = u(x)$



Find u's domain... $[-1, 3)$

Find u's range... $[1, 4]$



Ex

x	$k(x)$
3	4
8	5
10	5

k's

What is domain?

$\{3, 8, 10\}$

What is k's range?

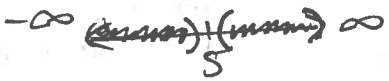
$\{4, 5\}$

Domain and Range

1. Take each of the functions below and find their domains. Express each domain using both a number line sketch and *interval notation*. If interval notation is inappropriate, use *set notation*. Not to be confused with *set-builder notation*.

a) f, where $f(x) = \dfrac{3}{x-5}$

$x \neq 5$



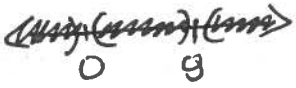
$(-\infty, 5) \cup (5, \infty)$

b) F, where $F(r) = \dfrac{2}{r^2 - 8r}$

worry about $r^2 - 8r = 0$

$r(r-8) = 0$

$r = 0$ or $r - 8 = 0$, $r = 8$



$(-\infty, 0) \cup (0, 8) \cup (8, \infty)$

c) k, where $k(t) = 20t + 3$

All real numbers are acceptable input.

$(-\infty, \infty)$

d) g, where $g(x) = \dfrac{24}{x^2 + 1}$

worry about $x^2 + 1 = 0$

$x^2 = -1$

No solutions... No way to make denom. equal 0...

all real numbers are in domain

$(-\infty, \infty)$

e) ℓ, where $\ell(x) = \sqrt{x + 43}$

Need $x + 43 \geq 0$

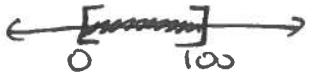
$x \geq -43$

$[-43, \infty)$

f) V, where $V(T)$ is the volume of one kilogram of *liquid* water at sea level, where T is the temperature in °C.

V turns temperatures into volumes.

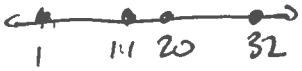
inputs: temps in °C



domain is $[0, 100]$

g)

x	h(x)
1	4
14	20
20	22
32	18



$\{1, 14, 20, 32\}$

h) Rev, where Rev(n) is the revenue at a movie theater when they sell n tickets to a particular show. This theater has 300 seats.

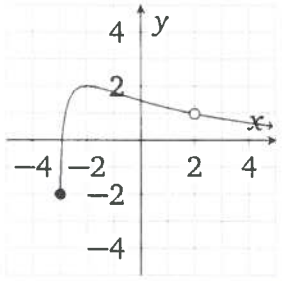
Rev turns #tickets into dollar amounts

what numbers are legal?

$\{0, 1, 2, \ldots, 300\}$

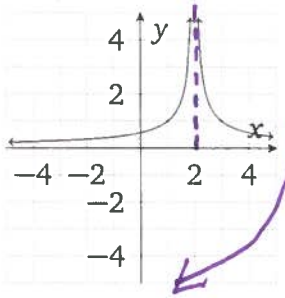
2. The graph of a function is given in each part below. Express the domain using *interval notation*.

a) $y = u(x)$



$[-3, 2) \cup (2, \infty)$

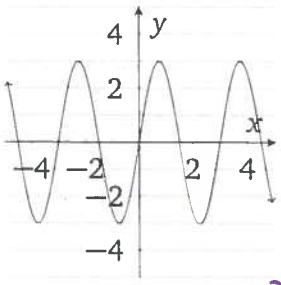
b) $y = v(x)$



R − {2}

$(-\infty, 2) \cup (2, \infty)$

c) $y = w(x)$



$(-\infty, \infty)$

$\mathbb{R}$ (−inf, inf) or R

3. An NBA basketball game lasts 48 minutes. Let q be a function of x, where $q(x)$ is the number of points that the home team has x minutes after a particular game begins. For example, if $q(10.5) = 20$ it means at a moment in time 10.5 minutes after the game started, the home team has 20 points.

a) What kinds of things are the input numbers?

amounts of time

b) What kinds of things are the output numbers?

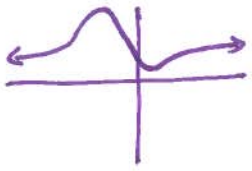
points

c) Express the domain of q *in words* and in *interval notation*.

any number from 0 to 49.

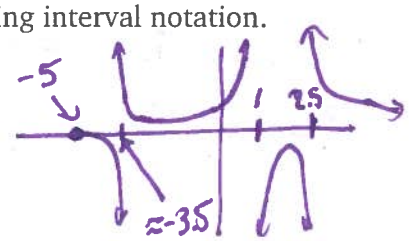
$[0, 49]$

4. A function f is defined by $f(x) = \dfrac{x^2 - x + 1}{x^2 + x + 1}$. Use GeoGebra (www.geogebra.org) or your graphing calculator to graph this function. Based on your graph, express the domain of f using interval notation.



Looks like domain is $(-\infty, \infty)$.

5. A function B is defined by $B(x) = \dfrac{\sqrt{x+5}}{x^3 - 9x + 9}$. Use GeoGebra (www.geogebra.org) or your graphing calculator to graph this function. Based on your graph, express the domain of f using interval notation.



domain is roughly

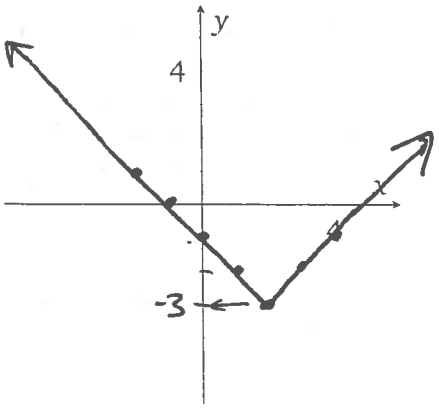
$[-5, -3.5) \cup (-3.5, 1) \cup (1, 2.5) \cup (2.5, \infty)$

6. A function g has the formula $g(x) = |x-2| - 3$. (absolute value)

a) Make a table for g.

x	$g(x)$
-2	$g(-2) = \lvert -2-2\rvert -3 = \lvert -4\rvert -3 = 4-3 = 1$
-1	0
0	-1
1	-2
2	-3
3	-2
4	-1

b) Make a graph of g.

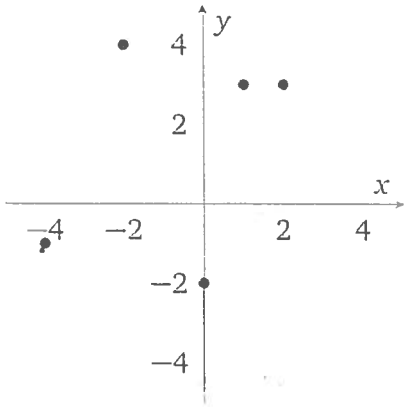


c) Express the domain and range of g in interval notation.

all real numbers $(-\infty, \infty)$

$[-3, \infty)$

7. A function named Q has the graphical representation below.



$y = Q(x)$

a) Express Q as a set of ordered pairs. Any time you are expressing a *set* or *list*, the standard math notation is to use braces: {, }.

$\{(-4,-1), (-2,4), (0,-2), (1,3), (2,3)\}$

b) Express the domain and range of Q in set notation.

$\{-4, -2, 0, 1, 2\}$

$\{-1, 4, -2, 3\}$

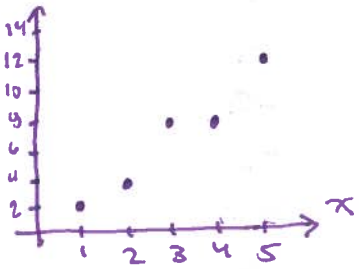
8. A function named C is given by $\{(1,2),(2,4),(3,8),(4,8),(5,12)\}$.

a) What are the domain and range of C? Give your answers using set notation.

C's domain is $\{1,2,3,4,5\}$

C's range is $\{2,4,8,12\}$

b) Give a graphical representation of C.



c) What is $C(2)$?

$C(2) = 4$

d) Solve the equation $C(x) = 12$.

The solution set is $\{5\}$.

9. In order to make table sugar, a factory had to purchase \$20,000 of machinery. Beyond that, there are costs of materials, labor, and transportation that go into producing x pounds of sugar. If $f(x)$ is the total cost in dollars to a factory that will make x pounds of sugar, what are the domain and range for f?

f turns pound amounts into dollar amounts.

The domain of f is $[0,\infty)$, since the factory might not make any sugar or an unlimited amount of sugar.

The range of f is $[20000,\infty)$ since the cost of production could be as low as \$20,000 or anything higher.