Section 11.1 Intro to Functions

We ~~a~~ know about $\sqrt{}$.

$\sqrt{100} = 10$

$\sqrt{\frac{36}{25}} = \frac{6}{5}$

$\sqrt{1.8} \approx 1.3416...$

$\to$ $\sqrt{}$ turns numbers into other numbers...

A <u>function</u> is a process for turning numbers into other numbers.

<u>Ex</u> Converting km distance into mi distance.

Fact: One km equals 0.621371 miles.

How many miles is 12 km?

$$12\text{ km} = 12\text{ km}\ \frac{0.621371\text{ mi}}{1\text{ km}}$$

$$= 12 \cdot \frac{0.621371\text{ mi}}{1}$$

$$12\text{ km} = 7.45645\text{ mi}$$

$12 \longmapsto 7.45645$

How many miles is 100 km?

100 km = same process

= 62.1371 mi

$100 \longmapsto 62.1371$

Ex Take a circle's radius measured in inches and find its area measured in square inches.

Recall: $A = \pi \cdot r^2$

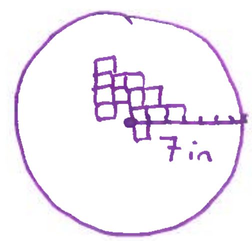
π ↑ 3.141....

Ex Take a 7 inch radius

Then $A = \pi \cdot 7^2$

$A = 49 \cdot \pi$

$A = 153.938...$



So $7 \longmapsto 153.938...$

Generally $r \longmapsto \pi \cdot r^2$

We have a process to change numbers.. so we have a function.

Notation....

Functions have names.

- $\sqrt{\ }$ function... sqrt
- converting km... getting <u>m</u>iles... m
- taking a radius... getting an <u>area</u>... A

Generally we'd name a function f.

We write name of function (something representing input) ← output

<u>Ex</u> $\underbrace{\text{sqrt}}_{\text{name of the function}}(\underbrace{100}_{\text{input}}) = 10$ "the square root of 100"

parentheses ⟷ "of"

<u>Ex</u> $m(12) = 7.45645\ldots$

<u>Ex</u> $f(x) =$ some result...

Functions can be described <u>verbally</u>...

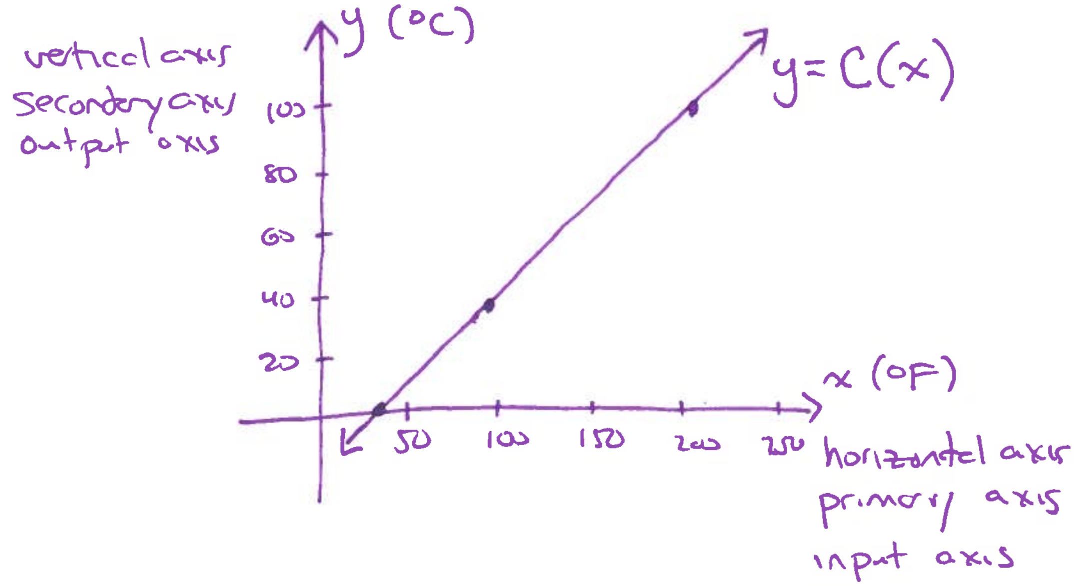
Let C be the function that converts a °F temperature into a °C temperature

Functions can be described <u>using a table</u>.

x (°F)	$C(x)$ (°C)
32	0
98.6	37
212	100

Functions can be expressed graphically...

x (°F)	C(x) (°C)		
32	0	⟶	(32, 0)
98.6	37	⟶	(98.6, 37)
212	100	⟶	(212, 100)



Lastly, functions can be expressed with a formula.

Ex $C(x) = \dots$

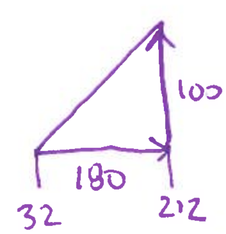
we saw a straight line...

using $y = m(x - x_0) + y_0 \dots$

$y = \frac{5}{9}(x - 32) + 0$

slope $= \frac{\text{rise}}{\text{run}} = \frac{100}{180}$

$= \frac{5}{9}$



$C(x) = \frac{5}{9}(x - 32)$

Ex What °C temp is 150 °F? $C(150) = \frac{5}{9}(150 - 32)$

$= \frac{5}{9}(118) = 65.55\dots$

Ex sqrt turns a number into that number's square root (a number that can be multiplied by itself to get the original).

verbal description...

formula: $\text{sqrt}(x) = \sqrt{x}$

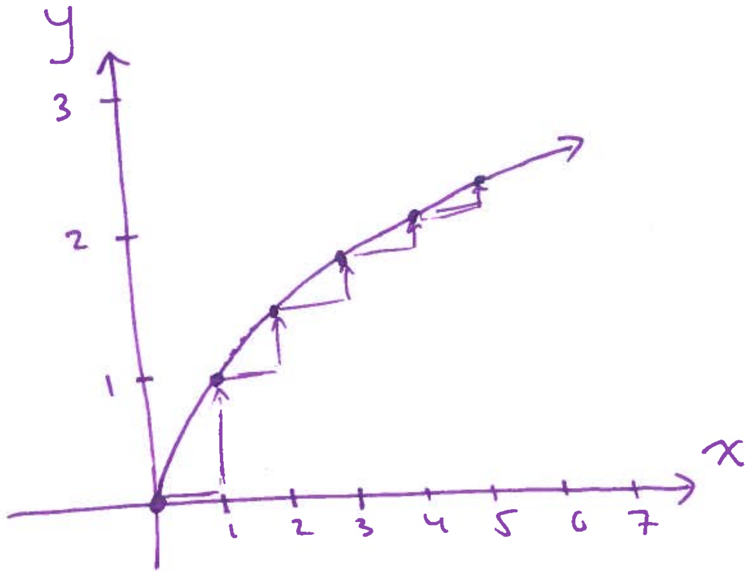
table:

x	sqrt(x)
~~25~~	~~5~~
~~36~~	~~6~~
~~1~~	~~1~~
~~4~~	~~2~~

not helpful for understanding sqrt better...

	x	sqrt(x)	
	0	0	
+1	1	1	+1
+1	2	1.41...	+0.41
+1	3	1.73...	+0.32
+1	4	2	+0.27
+1	5	2.23...	+0.23

As input grows, with this function output grows too... but by less and less graph



see as we read left-to-right, curve goes up. But bends down.

4 approaches: verbal, formula, table, graph — are all important.

Introduction to Functions

means $(\ )^2$, not $\sqrt{\ }$.

1. Writing function formulas:

a) Let f be a function that triples its input, and then subtracts 4. Write a formula for this function.

$f(x) = \dots$
$f(x) = 3 \cdot x - 4$

b) Let b be a function that squares its input, then divides by two, and then adds 1. Write a formula for this function.

$b(x) = \frac{x^2}{2} + 1$

c) Film director Jim Jarmusch was born on today's date, January 22, 1953. Let a be a function that finds his age on January 22 in year x. Write a formula for this function. Hint: you may want to calculate his age in years like 2000, 2010, and 2020 first to get a feel for what the formula will look like.

$a(x) = x - 1953$

d) Suppose that income tax is collected at a flat rate of 15%. Let t be the function that finds what your income tax amount is based on what your annual income is. Write a formula for this function.

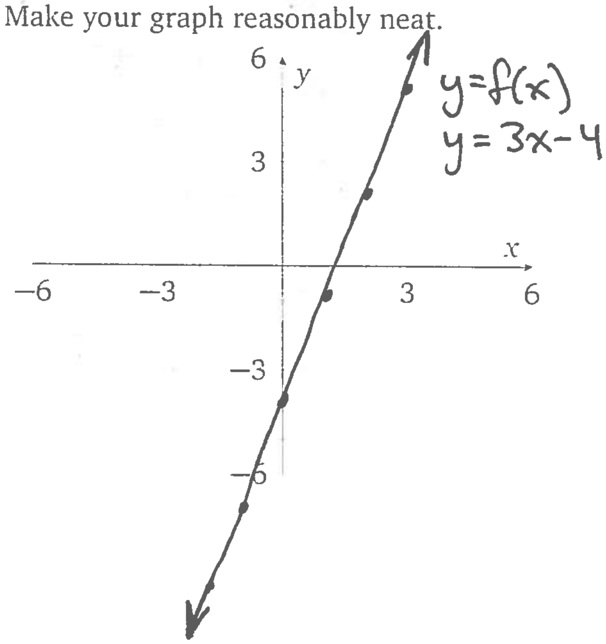
$t(x) = 0.15x$

2. Let f be a function that triples its input, and then subtracts 4.

(a) Give a tabular representation of f. Use at least five input values.

x	$f(x)$
-2	-10
-1	-7
0	-4
1	-1
2	2
3	5

(b) Give a graphical representation of f. Make your graph reasonably neat.

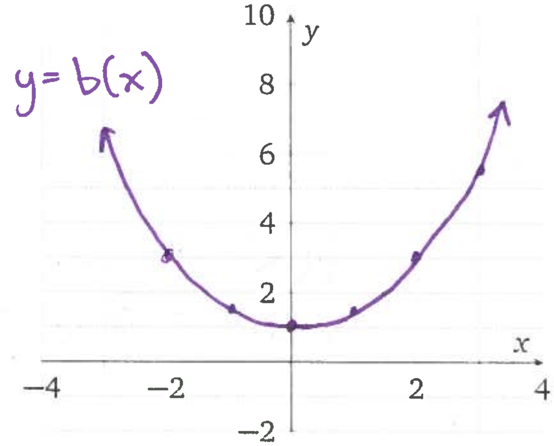


3. Let b be a function that squares its input, then divides by two, and then adds one.

(a) Give a tabular representation of b. Use at least five input values.

x	$b(x)$
-2	3
-1	1.5
0	1
1	1.5
2	3
3	5.5

(b) Give a graphical representation of b. Make your graph reasonably neat.

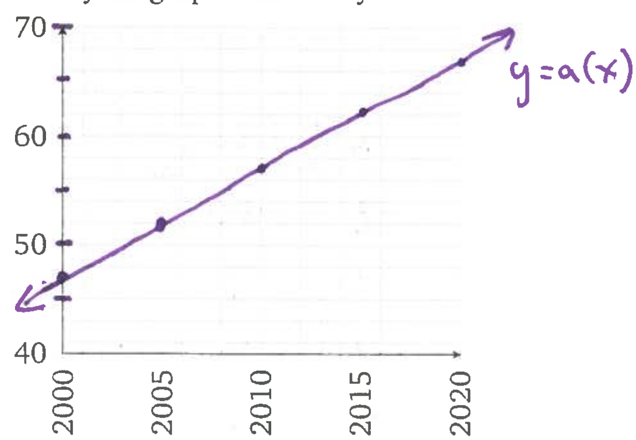


4. Film director Jim Jarmusch was born on today's date, January 22, 1953. Let a be a function that finds his age on January 22 in year x.

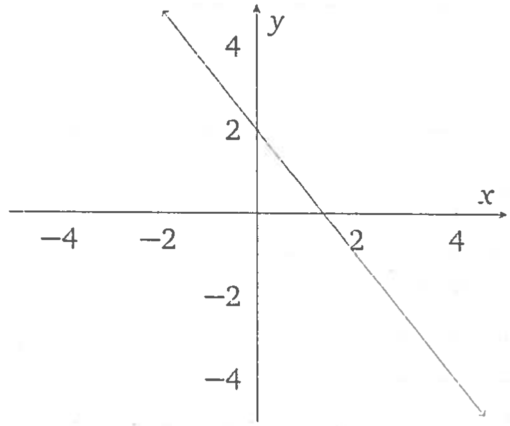
(a) Give a tabular representation of a. Use at least five input values.

x	$a(x)$
2000	47
2005	52
2010	57
2015	62
2020	67

(b) Give a graphical representation of a. Make your graph reasonably neat.



5. Here is the graph of a function G.



a) Find $G(0)$

$G(0) = 2$ because $(0,2)$ is on graph.

b) Find $G(-2)$

$G(-2) = 5$

c) Find $G(4)$

$G(4) = -4$

d) If $G(x) = -1$, then what was x?

Since $(2,-1)$ is on the graph, $G(2) = -1$. There's nothing else that is turned into -1, so x must be 2.

e) Basic Algebra Review: what is the equation of this line in slope-intercept form?

slope $= \frac{\text{rise}}{\text{run}} = \frac{-3}{2} = -1.5$

$y = -1.5x + 2$

f) Give a *formula* representation of G. (Use function notation to write G's formula.)

$G(x) = -1.5x + 2$

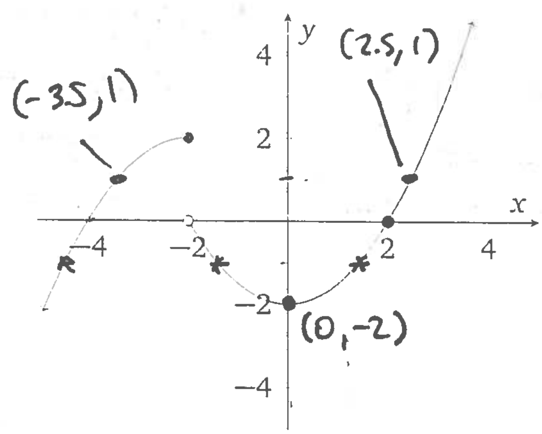
g) Give a *tabular* representation of G.

x	$G(x)$
-2	5
0	2
2	-1
4	-4

h) Give a *verbal* representation of G. (See the intro to exercise 2 for what this might be like.)

G multiplies input by -1.5, then adds 2.

6. Here is the graph of a function P.



input... horizontal axis

a) Find $P(0)$

$P(0) = -2$ because $(0,-2)$ is on the graph.

b) Find $P(2)$

$P(2) = 0$

output value find on vertical axis

c) Find $P(-2)$

$P(-2) = 2$ since $(-2,2)$ is on graph [$(-2,0)$ is not!]

d) Solve $P(x) = 1$

$x = -3.5$ or $x = 2.5$

$\{-3.5, 2.5\}$

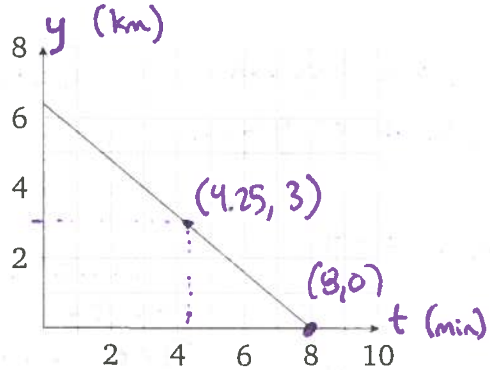
e) Solve $P(x) = 0$

$\{2, -4\}$

f) Solve $P(x) = -1$

$\{-4.5, -1.5, 1.5\}$

7. Jonah is biking home from a trip to the grocery store. At time t (in minutes since leaving), $d(t)$ is the remaining distance (in km) to home.



a) Label the axes of this graph appropriately. Axes should always have an appropriate variable for their label. If there is context to the problem, the label should also communicate the units of measurement.

b) Find $d(8)$. Write a complete sentence explaining what the numbers mean.

is 0.

It took 8 minutes for Jonah to get home.

c) How far away from Jonah's home is the grocery store?

(aka... find $d(0)$)

roughly 6.5 km.

d) Solve $d(t) = 3$. Write a complete sentence explaining what the numbers mean.

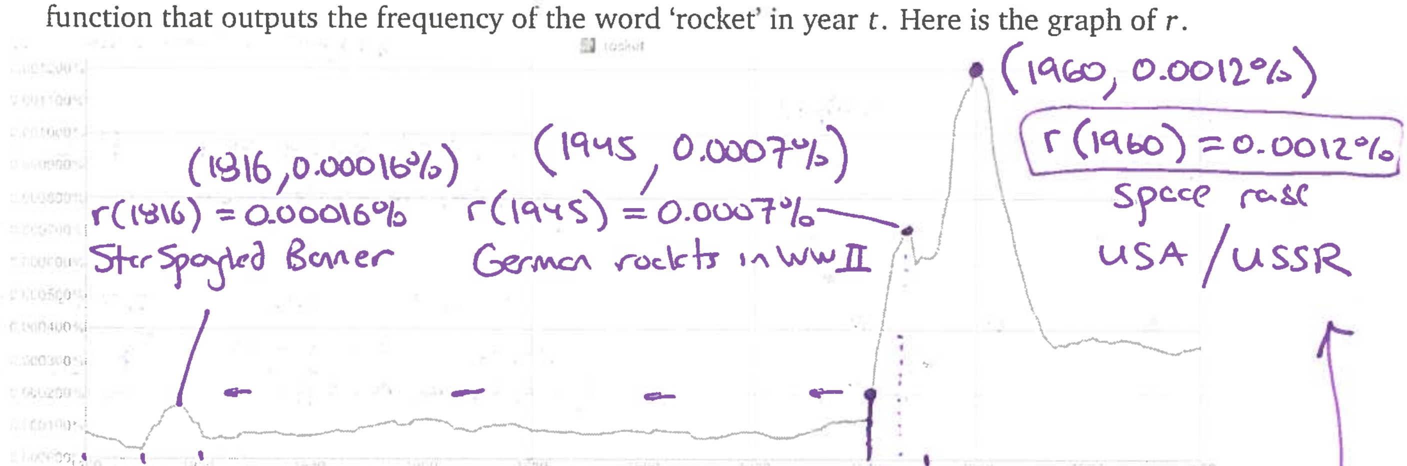
$t \approx 4.25$

How long until Jonah was 3 km from home? Answer: 4.25 minutes.

e) What was Jonah's speed biking home?

Rate 6.5 km in 8 min... $\frac{6.5\text{ km}}{8\text{ min}}$ $\approx 0.8125 \frac{\text{km}}{\text{min}}$

8. Google ngram provides data on the frequency of word-use in published books. So, out of all words printed in book in a given year, what percentage were a particular word. Let r be the function that outputs the frequency of the word 'rocket' in year t. Here is the graph of r.



0.0002 %

r of 1940 is 0.0002%

(a) What is $r(1940)$? Write a complete sentence explaining what the numbers mean.

(b) There are three peaks in this chart. Express the input-output information at the spikes using function notation. For example, by writing something like $r(1) = 2$.

(c) Do you have knowledge of history that could explain each of the spikes?