Factoring: finding ways to write a math expression as things multiplied together

6

$= 2 \cdot 3$

15

$= 3 \cdot 5 \leftarrow$!!!

$= 1 \cdot 15$

$= 2 \cdot 7{,}5$

$= \sqrt{15} \cdot \sqrt{15}$

$= (-3)(-5)$

21

$= 7 \cdot 3$

50

$= 5 \cdot 10$

$= 2 \cdot 25$

$= 5 \cdot 5 \cdot 2$

7

is prime
can't be factored
(except in ways we don't care about)

52 cards in a deck

⇓

4 suits...

$4 \cdot 13$

suits | cards in each suit

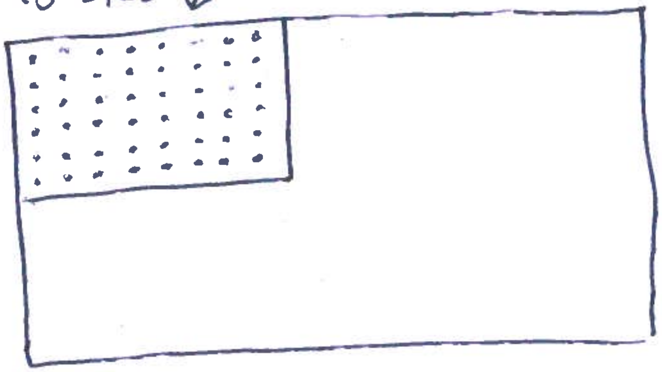
$2 \cdot 2 \cdot 13$

black suits, red suits, cards in each suit

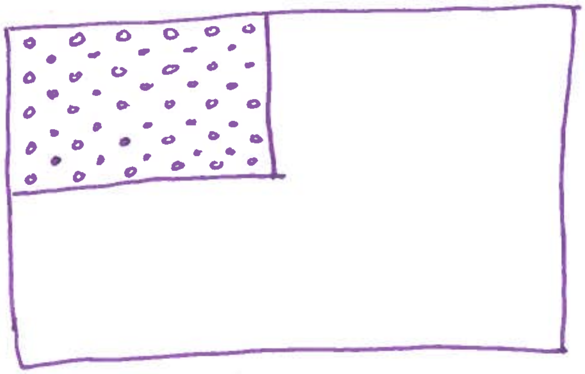
U.S flag... in WW II had 48 states...

$$\overset{6 \cdot 8}{48}$$

fit 48 stars ↴



Get 50 stars... ↙



Today: 50 states...

$= 25 \cdot 2$

$= 5 \cdot 10$

$= 5 \cdot 5 \cdot 2$

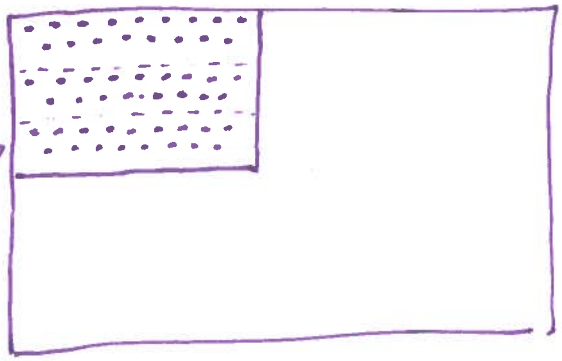
Clever: 50

$= 20 + 30$ (not factoring)

$= 4 \cdot 5 + 5 \cdot 6$

One day Puerto Rico may make 51 states...

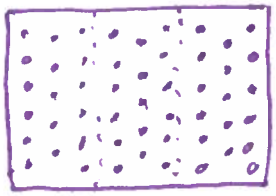
$= 3 \cdot 17$



fit 17 stars in each strip

$6+5+6$

fit 17 into each strip?



Point: factoring is useful in surprising unexpected ways.

Ch 10 is about factoring polynomials.

Remember $x(x+3) = x^2 + 3x$

$\xrightarrow{\hspace{3em}}$ "expanding" or "multiplying out" or "distributing"

this was factored: two things "x" and "x+3" multiplied.

not factored.

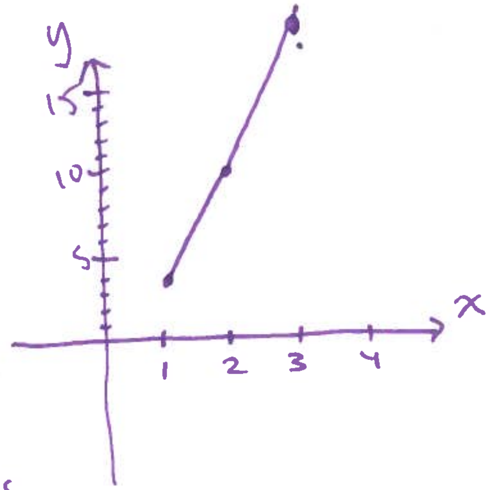
Consider: $x^2 + 3x = x(x+3)$

$\xrightarrow{\hspace{3em}}$ "factoring"

(not factored since it's mainly two things added...)

<u>Ex</u> Graph $y = x^2 + 3x$

x	y	
1	$y = 1^2 + 3(1) = 4$	$\rightsquigarrow (1,4)$
2	$y = 2^2 + 3(2) = 10$	$\rightsquigarrow (2,10)$
3	$y = 3^2 + 3(3) = 18$	$\rightarrow (3,18)$



<u>Slow, Tedious, Inaccurate Process</u> that ultimately may not show the full graph...

Try again: Graph $y = x^2 + 3x$

$y = x \cdot (x+3)$ now it's factored!

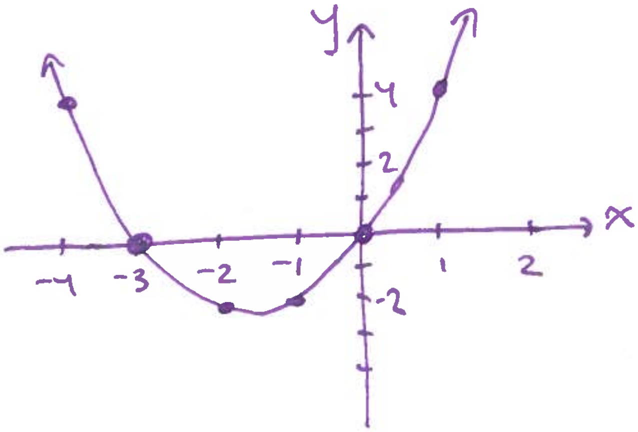
We can "see" things: $\underset{\overset{\downarrow}{0}}{y} = \overset{\overset{0}{\downarrow}}{x}(x+3) \longrightarrow (0,0)$

$\underset{\overset{\downarrow}{0}}{y} = x(\overset{\overset{-3}{\downarrow}}{x}+3) \longrightarrow (-3,0)$

Now...

Guidance to make a better table.

x	y
-4	$y = (-4)^2 + 3(-4) = 4$
-3	0
-2	$y = (-2)^2 + 3(-2) = -2$
-1	$y = (-1)^2 + 3(-1) = -2$
0	0
1	$y = 1^2 + 3 \cdot 1 = 4$



Now! A <u>better</u> graph.

Section 10.1

A polynomial has terms (pieces added together)

two terms

$$\underbrace{6x^2}_{} + \underbrace{70x}_{}$$

three terms

$$3x^3 + 3x^2 - 9$$

We look for the largest thing that is a factor of all the terms. (Greatest Common Factor)

$$6x^2 \qquad\qquad 70x$$

$$\boxed{2} \cdot 3 \cdot \boxed{x} \cdot x \qquad\qquad 35 \cdot 2 \cdot x$$

$$5 \cdot 7 \cdot \boxed{2} \cdot \boxed{x}$$

$\longrightarrow$ So $2x$ is the G.C.F.

GCF goes here

You are factoring $6x^2 + 70x = 2x(\qquad)$

$$= 2x(\qquad)$$

leftover bits from $6x^2$

leftover bits from $70x$

$$6x^2 = \boxed{2} \cdot 3 \cdot \boxed{x} \cdot x$$

leftover...

$$70x = 5 \cdot 7 \cdot \boxed{2} \cdot \boxed{x}$$

leftover bits

$$= 2x(3x + 5 \cdot 7)$$

$$= 2x(3x + 35)$$

Now I have a factored polynomial!

MTH 95 Factoring Out the Greatest Common Factor Date ______________

1. Use the Greatest Common Factor to factor each polynomial.

a) $8x+8$

$= 8(x+1)$

OR

$2\cdot 2\cdot 2\cdot(x+1)$

b) $5x-30$

$= 5(x-6)$

c) x^2+5x

$= x(x+5)$

d) $14x^3+21x^2$

$= 7x^2(2x+3)$

e) $13y^2-25y$

$= y(13y-25)$

f) $8x^2-4x^4$

$= 4x^2(2-x^2)$

g) $9x^4+18x^3+6x^2$

$= 3x^2(3x^2+6x+2)$

h) $10x-20x^2+5x^3$

$= 5x(2-4x+x^2)$

i) $6x^3y^2+9xy$

$= 3xy(2x^2y+3)$

j) $32x^3y^2-24x^3y-16x^2y$

$= 8x^2y(4xy-3x-2)$

2. There was a rectangular box with a square bottom. After you had computed the volume of the box, you had found that the volume was

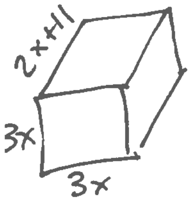
$$18x^3+99x^2 = (9x^2)(2x+11)$$!!!

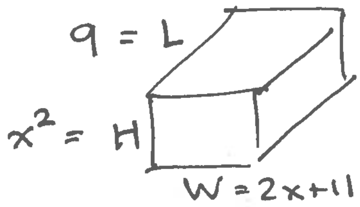
measured in cubic inches, where x is in inches. What is one possibility for the dimensions of the box? This is not asking for actual number answers. You would say, "the box could have been ___ × ___ × ___" where the blanks have expressions that use the variable x.

L H W

$(9)(x^2)\ (2x+11)$

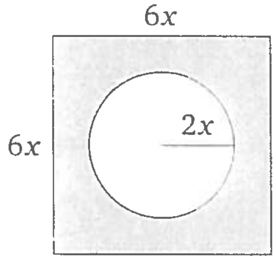
$(3x)(3x)(2x+11)$





$$\text{Volume} = L\cdot H\cdot W$$
$$= 9\cdot x^2(2x+11)$$

3. Find an expression in x for the area of the shaded region. (The figure is not drawn to scale.) You could get the area of the square, and subtract the area of the circle.



square area − circle area

$= (6x)^2 - \pi(2x)^2$

$= 36x^2 - \pi\cdot 4x^2$

$= 4x^2(\qquad\qquad)$

Then factor the expression using the greatest common factor.

$= 4x^2(9-\pi)$

square area is s^2

here, $s = 6x$

do I write

$6x^2$ or $(6x)^2$

X ✓

circle area $\pi\cdot r^2$

do I write

$\pi\cdot 2x^2$ or $\pi\cdot(2x)^2$

X ✓