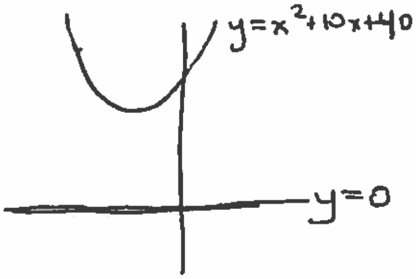
12.3 Solve $x^2 + 10x + 40 = 0$

graphing with tech.



they don't cross: ~~AO~~ there are no real number solutions.

try to factor (won't work)

Complete the square

$$x^2 + 10x + 40 = 0$$
$$x^2 + 10x = -40$$

$\frac{1}{2}$ of 10 is 5, square that: 25

$$x^2 + 10x + 25 = -40 + 25$$

half

$$(x+5)^2 = -15$$

squared number $\overset{?}{=}$ negative number

can't happen

$\Longrightarrow$ no solutions

quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{100 - 4(1)(40)}}{2(1)}$$

$$= \frac{-10 \pm \sqrt{-60}}{2}$$

(Alarm! Negative inside square root)

$$= \frac{-10 \pm \sqrt{4(-15)}}{2}$$

$$= \frac{-10 \pm 2\sqrt{-15}}{2}$$

$$= -5 \pm \sqrt{-15}$$

solutions are

$-5 - \sqrt{-15}$

$-5 + \sqrt{-15}$

these are <u>non-real</u> solutions!

Symbolically check:

$$\left(-5+\sqrt{-15}\right)^2 + 10\left(-5+\sqrt{-15}\right) + 40 \stackrel{?}{=} 0$$

$$25 + \cancel{\underbrace{2(-5)(\sqrt{-15})}_{-10\sqrt{-15}}} + \left(\sqrt{-15}\right)^2 - 50 \cancel{\underbrace{+10\sqrt{-15}}} + 40 \stackrel{?}{=} 0$$

$$25 + -15 - 50 + 40 \stackrel{?}{=} 0$$

$$x^2 + 10x + 40 = 0$$

has <u>no</u> <u>real number</u> solutions, but it does have two symbolic solutions: $-5+\sqrt{-15}$, $-5-\sqrt{-15}$

do not live on this axis.

$-\infty$ real numbers ∞

$$-5 \pm \sqrt{(-1)\cdot(15)}$$

$$= -5 \pm \sqrt{-1}\cdot\sqrt{15}$$

introduce

$$= -5 \pm \underbrace{i\cdot\sqrt{15}}_{\text{this is an imaginary number.}}$$

this is a <u>complex</u> number.

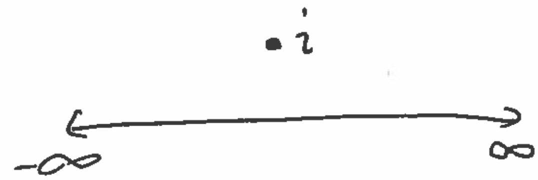
quadratic formula

Task: be able to use Q.F. to solve quadratic equations where we get non-real solutions.

~~Ex Solve~~

i is defined as $\sqrt{-1}$

so $i^2 = -1$. (Breaks things you previously understood



Ex Solve $x^2 + 6x + 40 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{36 - 4(1)(40)}}{2(1)} = \frac{-6 \pm \sqrt{36 - 160}}{2}$$

$$x = \frac{-6 \pm \sqrt{-124}}{2} = \frac{-6 \pm \sqrt{4 \cdot (-31)}}{2}$$

$$x = \frac{-6 \pm 2 \cdot \sqrt{-31}}{2} = -3 \pm \sqrt{-31}$$

$$= -3 \pm \sqrt{(-1) \cdot 31}$$

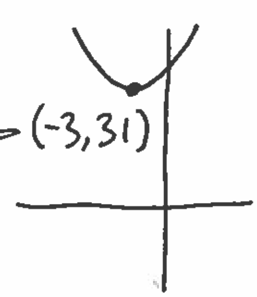
$$x = -3 \pm i \cdot \sqrt{31}$$

$$\{-3 + i\sqrt{31},\ -3 - i\sqrt{31}\}$$

-3 31

not a coincidence!

Side note:



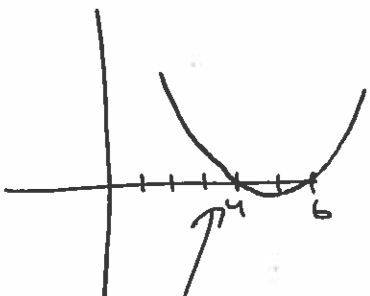
Ex Air show stunt plane enters a dive, height (in meters) after t seconds is given by $h(t) = \frac{1}{2}t^2 - 5t + 12$.

Will it hit the ground?

Are there real-number solutions?

Solve $h(t) = 0$ meters

$$\frac{1}{2}t^2 - 5t + 12 = 0$$



$$t = \frac{5 \pm \sqrt{25 - 4(\frac{1}{2})(12)}}{2(\frac{1}{2})}$$

a positive #!

$$t = \frac{5 \pm \sqrt{25-24}}{1} = 5 \pm \sqrt{1}$$

$$= 6 \text{ or } 4$$

second seconds

a real solution...

$\Longrightarrow$ that plane crashed.

Now with $h(t) = \frac{1}{2}t^2 - 5t + 13$

Will it hit ground?

$$h(t) = 0$$

$$\frac{1}{2}t^2 - 5t + 13 = 0$$

$$t = \frac{5 \pm \sqrt{25 - 4(\frac{1}{2})(13)}}{2(\frac{1}{2})} = \frac{5 \pm \sqrt{25-26}}{1}$$

$$t = 5 \pm \sqrt{-1} = 5 \pm i$$

not real number solutions!

So no real solutions, plane never hits ground.

$$a \cdot x^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{\boxed{b^2 - 4ac}}}{2a}$$

positive $\Downarrow$ two real solutions

zero $\frac{-3 \pm 0}{1}$ only one real solution.

negative. $\Downarrow$ two non-real solutions

$b^2 - 4ac$ is called the discriminant.

In the gym, ceiling is 4m high.
Kick a ball, its height (in meters) after t seconds: $h(t) = -4.9t^2 + 8t$

Will it hit the ceiling?

$$h(t) = 4$$

$$-4.9t^2 + 8t = 4$$

$$-4.9t^2 + 8t - 4 = 0$$

could solve... instead

discriminant

$= b^2 - 4ac$

$= 8^2 - 4(-4.9)(-4)$

$= -14.4$

(negative

$\Longrightarrow$ no real solutions

$\Longrightarrow$ ball never hits ceiling.

12.4 Adding, Subtract, Multiply, Divide Complex Numbers

adding real parts
$1+5$

Ex $(1-7i) + (5+4i) = 6 - 3i$

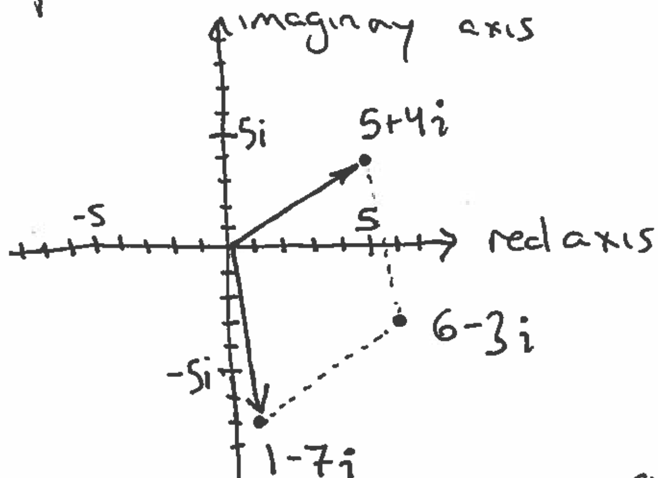
one complex number

another complex number

add them together

adding the imaginary parts
$-7i+4i$

Complex Plane



subtract real parts $5-7$

subtract imaginary parts
$-4i-(-2i)$
$-4i+2i$

Ex $(5-4i)-(7-2i) = -2 - 2i$

$= 5 - 4i - 7 + 2i$

$= -2 - 2i$

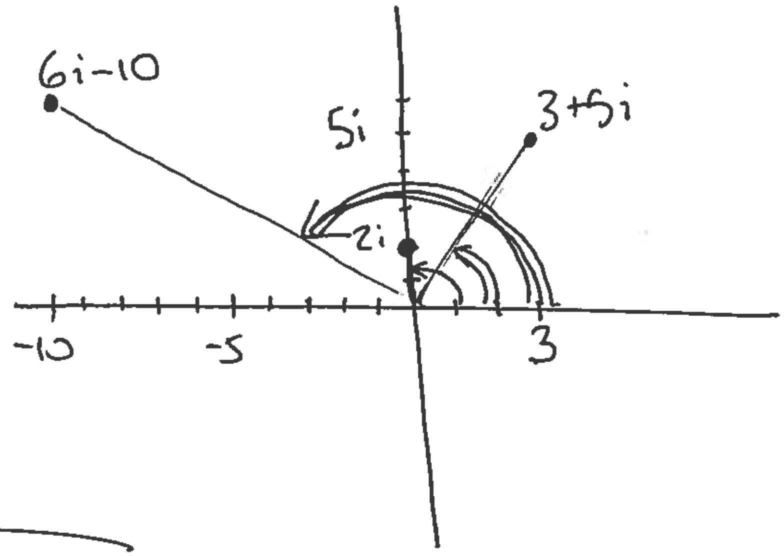
Ex $2i\cdot(3+5i)$

$= 6i + (2i)\cdot(5i)$

$= 6i + 10\cdot i^2$ $\qquad i^2 = -1$

$= 6i + 10(-1)$

$= 6i - 10$



Ex $\overbrace{(1+3i)\cdot(7-4i)}^{\text{use FOIL}}$

$= 7 - 4i + 21i - 12i^2$

$\qquad\qquad\qquad\quad -12(-1)$

$= 7 - 4i + 21i + 12$

$= 19 + 17i$

!!! $i^2 = -1$

i is not a variable like x. $i = \sqrt{-1}$ and $i^2 = -1$.

Ex $(2+3i)^2 = 4 + 2(2)(3i) + (3i)^2$

$= 4 + 12i + 9i^2$

$\qquad\qquad\quad + 9(-1)$

$= 4 + 12i - 9$

$= -5 + 12i$

Ex $(9+2i)(9-2i)$ $= 81 - (2i)^2$

$= 81 - 4i^2$

$= 81 - 4(-1)$

$= 81 + 4$

$= 85$

Summarize multiplication.
Treat i like a variable, use FOIL, etc...
But at the end, use $i^2 = -1$.

Ex $\dfrac{2}{i}$ $= \dfrac{2}{\sqrt{-1}}$

I just object to having non-real numbers in denominators.

mult by 1

$$\frac{2}{i} \cdot \boxed{\frac{i}{i}} = \frac{2i}{i^2} = \frac{2i}{-1}$$

$= -2i$

Ex $$\frac{-3}{4i} = \frac{-3}{4i} \cdot \frac{4i}{4i}$$

$$= \frac{-12i}{16i^2} = \frac{-12i}{-16} = \frac{3i}{4}$$

Ex $\dfrac{1}{7+2i}$

Now denominator is upgraded!!!
Not just imaginary anymore.

Ex $\dfrac{1}{7+2i} = \dfrac{1}{(7+2i)} \cdot \dfrac{7-2i}{(7-2i)}$

Note sign change on imaginary part

still don't like "i" in the denominator

$$= \frac{7-2i}{49-(2i)^2}$$

$$= \frac{7-2i}{49-4\{i^2\}} = \frac{7-2i}{49+4} = \frac{7-2i}{53}$$

now no "i" in denominator.

Ex $\dfrac{2+i}{2-3i} = \dfrac{(2+i)}{(2-3i)} \cdot \dfrac{(2+3i)}{(2+3i)}$

change sign on i

has "i" in denominator

$$= \frac{4+6i+2i+3i^2}{4-(3i)^2} = \frac{4+6i+2i+(-3)}{4-9\{i^2\}}$$

$$= \frac{1+8i}{13}$$