12.2

Ex Solve $(x+5)^2 = 4$

$$\Longrightarrow \sqrt{(x+5)^2} = \sqrt{4}$$

$$|x+5| = 2$$

$x+5 = 2$ or $-(x+5) = 2$

$x = -3$ or $-x-5 = 2$

$-x = 7$

$x = -7$

relatively easy!

Ex Solve $x^2 - 6x + 9 = 4$ (perfect square)

$$(x-3)^2 = 4$$

$$\sqrt{(x-3)^2} = \sqrt{4}$$

$$|x-3| = 2$$

$x-3 = 2$ or $-(x-3) = 2$

$x = 5$ or $-x+3 = 2$

$-x = -1$

$x = 1$

still relatively easy.

Ex Solve $x^2 + 10x + 21 = 0$

(+21 → move right)

Could try...
- quadratic formula
- factoring...

$b = 10$, cut in half $\frac{b}{2} = 5$, then square: 25

$x^2 + 10x = -21$

add to both sides

$x^2 + 10x + 25 = -21 + 25$

these steps force this to be a perfect square

This is called completing the square.

$(x + 5)^2 = 4$ ← now it's relatively easy!

$\sqrt{(x+5)^2} = \sqrt{4}$

$|x+5| = 2$

$x + 5 = 2$ or $-(x+5) = 2$

$x = -3$ or $-x - 5 = 2$

$-x = 7$

$x = -7$

Ex Solve $x^2 + 6x - 16 = 0$

$\frac{6}{2} = 3 \Longrightarrow 3^2 = 9$

$x^2 + 6x = 16$

$x^2 + 6x + 9 = 16 + 9$ (half)

$(x + 3)^2 = 25$

$\sqrt{(x+3)^2} = \sqrt{25}$

$|x + 3| = 5$

$x + 3 = 5$ or $-(x+3) = 5$

$x = 2$ $\quad -x - 3 = 5$

$-x = 8$

$x = -8$

(should check $x = 2, \ x = -8$) ✓

Ex Solve $x^2 - 12x + 7 = 0$

$\frac{-12}{2} = -6 \Longrightarrow (-6)^2 = 36$

$x^2 - 12x = -7$

$x^2 - 12x + 36 = -7 + 36$ (half)

$(x - 6)^2 = 29$

$\sqrt{(x-6)^2} = \sqrt{29}$

$|x - 6| = \sqrt{29}$

quadratic gone... absolve value

$x - 6 = \sqrt{29}$ $\quad -(x-6) = \sqrt{29}$

$x = 6 + \sqrt{29}$

$x - 6 = -\sqrt{29}$

$x = 6 - \sqrt{29}$

So the solution set is $\{6 - \sqrt{29}, 6 + \sqrt{29}\}$

Ex Solve $x^2 - 5x - 8 = 0$

not even!

$x^2 - 5x = 8$

$b = -5$

$\Rightarrow \frac{b}{2} = \frac{-5}{2}$

$x^2 - 5x + \frac{25}{4} = 8 + \frac{25}{4}$

half

$\Rightarrow$ square this

$\left(\frac{-5}{2}\right)^2 = \frac{25}{4}$

$\left(x - \frac{5}{2}\right)^2 = \frac{32}{4} + \frac{25}{4}$

$\left(x - \frac{5}{2}\right)^2 = \frac{57}{4}$

$\sqrt{(x - 5/2)^2} = \sqrt{\frac{57}{4}}$

$\sqrt{\frac{A}{B}} = \frac{\sqrt{A}}{\sqrt{B}}$

$\left|x - \frac{5}{2}\right| = \frac{\sqrt{57}}{2}$

can learn the shortcut...

$x - \frac{5}{2} = \frac{\sqrt{57}}{2}$ or $-\left(x - \frac{5}{2}\right) = \frac{\sqrt{57}}{2}$

$x = \frac{5}{2} + \frac{\sqrt{57}}{2}$

$-x + \frac{5}{2} = \frac{\sqrt{57}}{2}$

$-x = -\frac{5}{2} + \frac{\sqrt{57}}{2}$

$x = \frac{5}{2} - \frac{\sqrt{57}}{2}$

Ex Solve $2z^2 + 5z - 1 = 0$

not 1!

$$\frac{2z^2 + 5z - 1}{2} = \frac{0}{2}$$

now 1!

$$z^2 + \frac{5}{2}z - \frac{1}{2} = 0$$

$$z^2 + \frac{5}{2}z = \frac{1}{2}$$

$b = \frac{5}{2}$

cut in half $= \frac{5/2}{2}$

$= \frac{5}{2} \cdot \frac{1}{2}$

$= \frac{5}{4}$

square

get $\frac{25}{16}$

$$z^2 + \frac{5}{2}z + \frac{25}{16} = \frac{8}{16} + \frac{25}{16}$$

half

$$\left(z + \frac{5}{4}\right)^2 = \frac{33}{16}$$

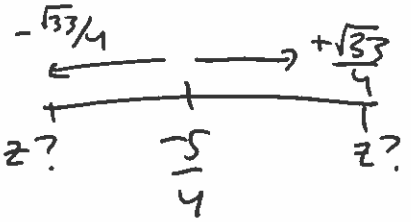
$$\sqrt{\left(z + \frac{5}{4}\right)^2} = \sqrt{\frac{33}{16}}$$

$$\left|z + \frac{5}{4}\right| = \frac{\sqrt{33}}{4}$$

$\left|z + \frac{5}{4}\right|$

$\left|z - \left(\frac{-5}{4}\right)\right|$

distance between z and $\frac{-5}{4}$



demonstrate shortcut

$$z = -\frac{5}{4} \pm \frac{\sqrt{33}}{4}$$

The solution set is

$$\left\{\frac{-5}{4} - \frac{\sqrt{33}}{4}, \frac{-5}{4} + \frac{\sqrt{33}}{4}\right\}$$

Solve $a \cdot x^2 + bx + c = 0$

divide through by a

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$x^2 + \left(\frac{b}{a}\right)x$ → cut in half: $\frac{b}{a} \cdot \frac{1}{2} = \frac{b}{2a}$

$= -\frac{c}{a}$

square → $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a} \cdot \frac{4a}{4a}$$

↓ half

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\left|x + \frac{b}{2a}\right| = \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{OR} \quad -\left(x + \frac{b}{2a}\right) = \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\vdots$$

$$x = \frac{-b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Back in 12.1, given $f(x) = (x-3)^2 + 8$

you see the vertex is at $(3, 8)$.

Now... Ex Given ~~$x^2 + 8x$~~ $f(x) = x^2 + 8x + 2$ (standard form...)

Where is the vertex?

We complete the square...

$$f(x) = \underbrace{x^2 + 8x}_{\text{cut in half } 4 \Longrightarrow \text{square } 16} + 2$$

$$f(x) = \underbrace{x^2 + 8x + 16}_{\text{a perfect trinomial square}} - 16 + 2$$

(effectively adding 0)

$$f(x) = \underbrace{(x+4)^2 - 14}_{\text{vertex form.}}$$

Vertex is at $(-4, -14)$.

Ex $g(x) = x^2 - 12x$ Find g's vertex.

comp the square

half $\Longrightarrow$ square

$-6 \Longrightarrow 36$

$g(x) = x^2 - 12x + 36 - 36$

$g(x) = (x-6)^2 - 36$

vertex is at $(6, -36)$.

Ex $h(x) = 5x^2 + 20x + 25$

$h(x) = 5(x^2 + 4x) + 25$

comp the square

half $\Longrightarrow$ square

$2 \Longrightarrow 4$

$h(x) = 5(x^2 + 4x + 4 - 4) + 25$

$= 5(x^2 + 4x + 4) - 20 + 25$

$= 5(x+2)^2 + 5$

Vertex is at $(-2, 5)$

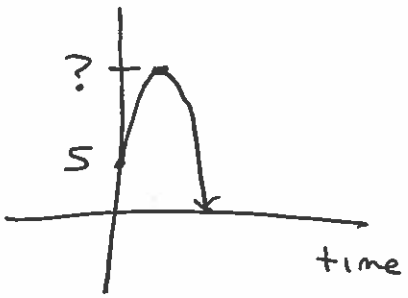
Ex A ball is tossed upward at 20 ft/sec, released at time $t=0$ from 5ft high.

The height after t seconds is given by

$$h(t) = -16t^2 + 20t + 5$$

$-16t^2$: gravity on Earth ~~$= -16$ ft~~ $= -32 \frac{ft}{s^2}$

How high does it get?



Well, find its vertex.

Put h in vertex form.

$$h(t) = -16t^2 + 20t + 5$$

$$= -16\left(t^2 - \frac{20}{16}t\right) + 5$$

$$= -16\left(t^2 - \frac{5}{4}t\right) + 5$$

half $-\frac{5}{8} \Longrightarrow$ square $\frac{25}{64}$

$$= -16\left(t^2 - \frac{5}{4}t + \frac{25}{64} - \frac{25}{64}\right) + 5$$

$$= -16\left(t - \frac{5}{8}\right)^2 + \frac{25}{4} + \frac{20}{4}$$

$$= -16\left(t - \frac{5}{8}\right)^2 + \frac{45}{4}$$

Vertex at $\left(\frac{5}{8}, \frac{45}{4}\right)$

$= (0.625, 11.25)$

It gets 11.25 feet high.