

10.5 Technical Def of a Function (cont'd)

Informally, a function's

formula

$$f(x) = x^2$$

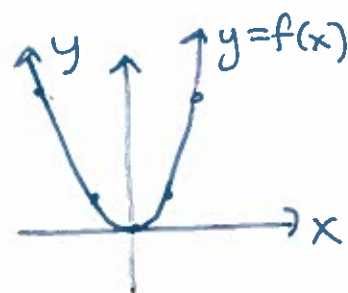
verbal
descript

f takes
its input
and squares
it

table

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4

graph



technical: $\{(-2, 4), (-1, 1), (0, 0), \text{and } \infty\text{-ly many more}\}$

a function is a collection of ordered pairs where each x -value is paired with at most one y -value.

Some things don't give functions--

Table

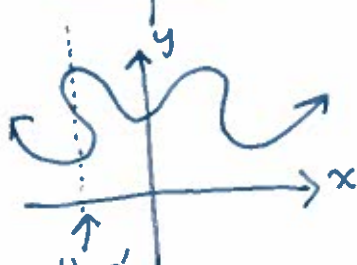
x	y
-3	1
-1	5
-3	7
4	8

same
 x -val

with
diff
 y -values

does not make
y a function of x

Graph



there's
an x -value with
more than one y -value.

Verbal

Is a person's
income (annual)
a function of
their age?

input output

38 30,000

38 40,000

different outputs, for same
input.

Equations in x and y .

Ex

$$y = x^2 + 1$$

Is y a function of x ?

(Yes!)

imagine using some number for x as input...

→ an output value comes out for y .

Ex

$$2x + 3y = 5$$

Is y a function of x ?

(Yes!)

put in some x -value...

you will be able to solve for y .

$$3y = 5 - 2x$$
$$y = \frac{5 - 2x}{3}$$

Ex

$$y = \pm \sqrt{x+4}$$

Is y a function of x ?

two outputs!
 $+3$ and -3 .

plug in some x -value, say 5.

So, no! one input can give two outputs.

Ex

$$x^2 + y^2 = 9$$

Is

y a function of x ? No!

Say, plug in 2.

$\sqrt{5}$ and $-\sqrt{5}$ come out...

$$2^2 + y^2 = 9$$

$$4 + y^2 = 9$$

$$y^2 = 5$$

$$y = \pm \sqrt{5}$$

Ch 11 Absolute Value Functions

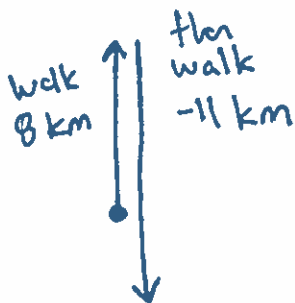
Review (see section 1.3 of the book)

$$\underline{\text{Ex}} \quad |5| = 5$$

$$|-12| = 12$$

signs to take the positive version of whatever one number is in there

In context,



How much distance was that in total?

Everyday English

$$8 + 11 = 19$$

Alternatively:

$$|8| + |-11| = 19$$

addition because two legs

keep track of the existence of neg. direction

Also look at

$$|8 + (-11)|$$

$$= |-3|$$

$$= 3 \leftarrow$$

In the end this person is 3km away from where they started.

Ex Let f be defined by $f(x) = |x|$.

$$f(12) = 12 \quad f(-8) = 8 \quad f(0) = 0$$

Ex Let g be ~~be~~ defined by $g(x) = |4x+3|$

$$\begin{aligned} g(5) &= |4(5)+3| \\ &= |23| \\ &= 23 \end{aligned} \qquad \begin{aligned} g(-5) &= |4(-5)+3| \\ &= |-17| \\ &= 17 \end{aligned}$$

Plot with GGB $f(x) = |x|$

$$f(x) = |x+3|$$

$$f(x) = -|x+3|$$

$$f(x) = -2|x+3|$$

$$f(x) = 4 - 2|x+3|$$

Let $f(x) = \sqrt{x^2}$

$$f(3) = \sqrt{3^2} = \sqrt{9} = 3$$

$$f(2) = \sqrt{2^2} = \sqrt{4} = 2$$

$$f(1) = \sqrt{1^2} = \sqrt{1} = 1$$

$$f(0) = \sqrt{0^2} = \sqrt{0} = 0$$

$$f(-1) = \sqrt{(-1)^2} = \sqrt{1} = 1$$

now input and output are different!

Fact

$$\sqrt{x^2} = |x|$$

(do you feel that $\sqrt{x^2} = x$?)

only true if x is positive, because $|x| = x$.

$$f(-2) = \sqrt{(-2)^2} = \sqrt{4} = 2$$

again

Ex

Simplify $\sqrt{(x-2)^2}$

$$= |x-2|$$

radical and squaring sort of cancel... but they leave absolute value bars in the aftermath

Ex

Simplify $\sqrt{x^2 + 10x + 25}$

$$= \sqrt{(x+5)^2}$$

$$= |x+5|$$

$$\begin{array}{c} x^2 + 10x + 25 \\ \uparrow \quad \quad \uparrow \\ 2.5 \quad 5^2 \\ (x+5)^2 \end{array}$$

Simplify $\sqrt{x^6} = \sqrt{(x^3)^2} = |x^3|$