Introduction to Functions

Work within a small group to answer these questions. Do not race through the exercises on your own. Always make sure that your entire group feels good about a question and answer before you move to the next exercise. Ask your group mates for explanations if you feel uncertain about something, and offer your explanations to others when you understand an exercise but someone else may not.

1. Writing function formulas:

   a) Let f be a function that triples its input, and then subtracts 4. Write a formula for this function.

   $f(x) = 3 \cdot x - 4$

   b) Let b be a function that squares its input, then divides by two, and then adds 1. Write a formula for this function.

   $b(x) = \frac{x^2}{2} + 1$

   c) Barack Obama was born on August 4th, 1961. Let "age" be a function that finds his age on August 5th in year x. Write a formula for this function. Hint, you may want to calculate his age in years like 2000, 2008, and 2015 first to get a feel for what the formula will look like.

   2017 ↦ 2017 − 1961
   56

   $age(y) = y - 1961$

   d) Suppose that income tax is collected at a flat rate of 15%. Let "tax" be the function that finds what your income tax amount is for the year based on what your annual income is. Write a formula for this function.

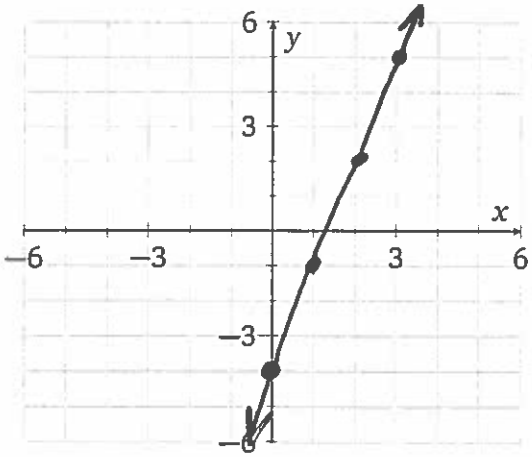
   $tax(I) = 0.15 \cdot I$

2. Let f be a function that triples its input, and then subtracts 4.

   (a) Give a tabular representation of f. Use at least five input values.

x	$f(x)$	
−2	−10	← $-2 \times 3 - 4$
−1	−7	
0	−4	
1	−1	
2	2	
3	5	

   (b) Give a graphical representation of f. Make your graph reasonably neat.

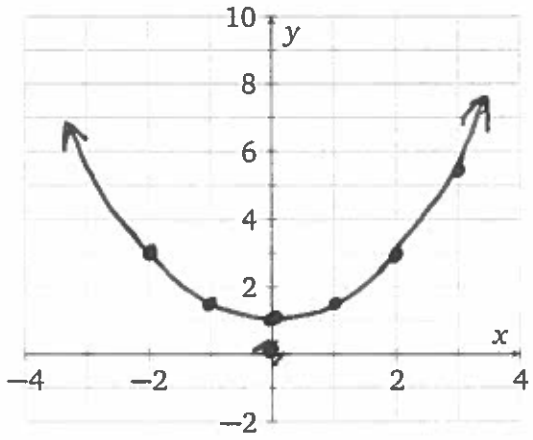


3. Let "bowl" be a function that squares its input, then divides by two, and then adds 1.

(a) Give a tabular representation of bowl. Use at least five input values.

x	bowl(x)
-2	3
-1	1.5
0	1
1	1.5
2	3
3	5.5

(b) Give a graphical representation of bowl. Make your graph reasonably neat.

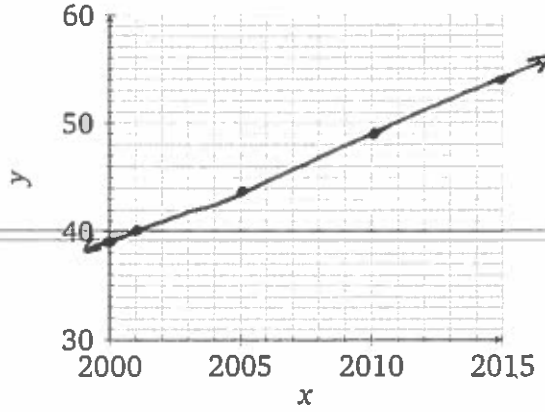


4. Barack Obama was born on August 4, 1961. Let a be a function that finds his age on August 4 in year x.

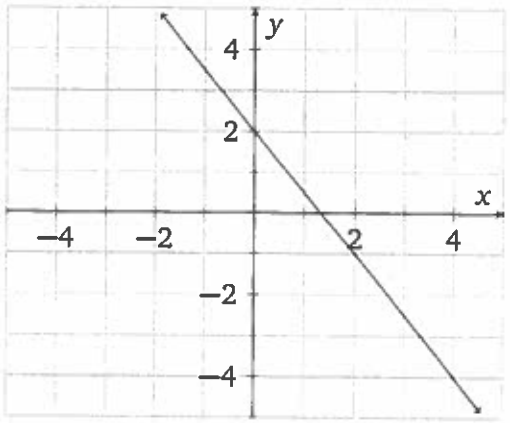
(a) Give a tabular representation of a. Use at least five input values.

x	a(x)
2000	39
2001	40
2005	44
2010	49
2015	54

(b) Give a graphical representation of a. Make your graph reasonably neat.



5. Here is the graph of a function G.



a) Find $G(0)$

$G(0) = 2$

b) Find $G(-2)$

$G(-2) = 5$

c) Find $G(4)$

$G(4) = -4$

d) If $G(x) = -1$, then what was x?

$x = 2$

e) Basic Algebra Review: what is the equation of this line in slope-intercept form?

$y = -\frac{3}{2}x + 2$

f) Give a *formula* representation of G. (Use function notation to write G's formula.)

$G(x) = -\frac{3}{2}x + 2$

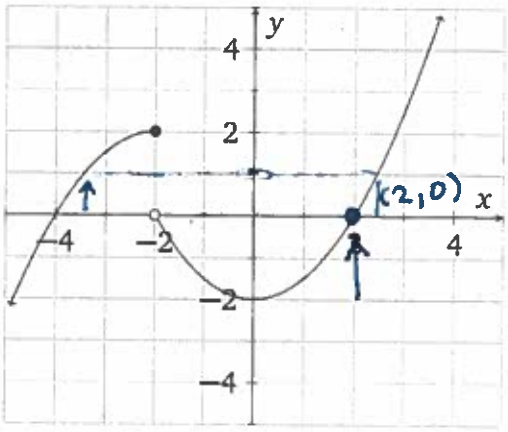
g) Give a *tabular* representation of G.

x	$G(x)$
-2	5
0	2
2	-1
4	-4

h) Give a *verbal* representation of G. (See the intro to exercise 2 for what this might be like.)

G multiples input by $\frac{-3}{2}$, then adds 2.

6. Here is the graph of a function P.



0 is input. lives on hor. axis

See $(0,-2)$

a) Find $P(0) = -2$

b) Find $P(2) = 0$

2 is input

Find -2 on hor. axis

c) Find $P(-2) = 2$

d) Solve $P(x) = 1$

is an output

Hunt input

$x \approx -3.5$ or $x \approx 2.5$

e) Solve $P(x) = 0$

$x \approx 2$ or $x \approx -4$

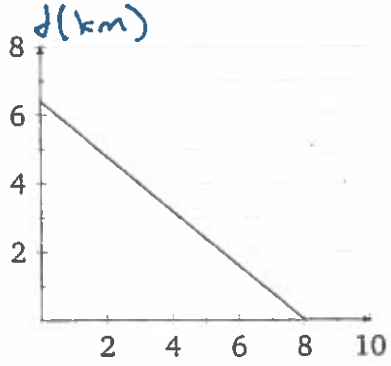
f) Solve $P(x) = -1$

$x \approx -4.5$ or $x \approx -1.5$ or $x \approx 1.5$

Does $(-2,0)$ exist on this curve? No

Does $(-2,2)$ exist on this curve? Yes

7. Jonah is biking home from a trip to the grocery store. At time t (in minutes since leaving), $d(t)$ is the remaining distance (in km) to home.



a) Label the axes of this graph appropriately. Axes should always have an appropriate variable for their label. If there is context to the problem, the label should also communicate the units of measurement.

b) Find $d(8)$. Write a complete sentence explaining what the numbers mean.

$d(8) = 0$. After 8 minutes, Jonah's distance to home is 0 km. (In other words, he's home.)

c) How far away from Jonah's home is the grocery store?

Since $d(0) \approx 6.4$, at the beginning, he is 6.4 km from home

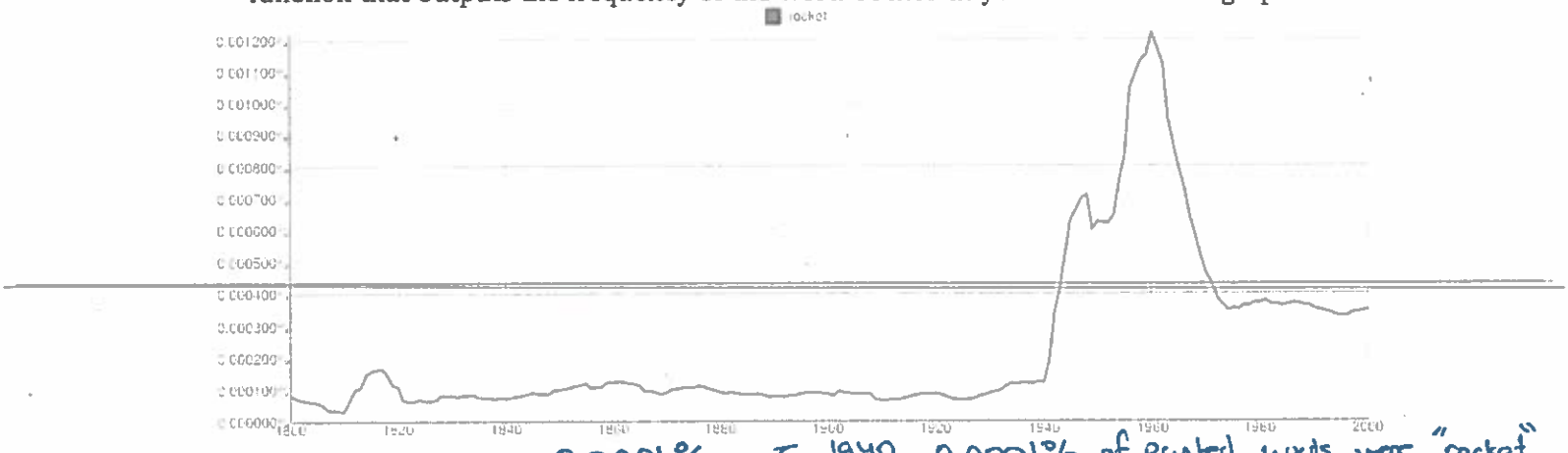
d) Solve $d(t) = 3$. Write a complete sentence explaining what the numbers mean.

$t \approx 4.3$. This means that to be 3km from home, it must have been 4.3 min later.

e) What was Jonah's speed biking home?

Jonah travels about 6.4 km in 8 min, so $\frac{6.4 \text{ km}}{8 \text{ min}} \approx 0.8 \frac{\text{km}}{\text{min}}$

8. Google ngram provides data on the frequency of word-use in published books. So, out of all words printed in book in a given year, what percentage were a particular word. Let r be the function that outputs the frequency of the word 'rocket' in year t. Here is the graph of r.



$\approx 0.0001\%$ In 1940, 0.0001% of printed words were "rocket".

(a) What is $r(1940)$? Write a complete sentence explaining what the numbers mean.

(b) There are three peaks in this chart. Express the input-output information at the spikes using function notation. For example, by writing something like $r(1) = 2$.

$r(1816) \approx 0.0002\%$
$r(1945) \approx 0.0007$
$r(1960) \approx 0.0012$

(c) Do you have knowledge of history that could explain each of the spikes?

War of 1812 → Star Spangled Banner → "rockets red glare..."
World War II and German V2 rockets
Space Race between U.S. and U.S.S.R.