

Domain and Range

Work within a small group to answer these questions. Do not race through the exercises on your own. Always make sure that your entire group feels good about a question and answer before you move to the next exercise. Ask your group mates for explanations if you feel uncertain about something, and offer your explanations to others when you understand an exercise but someone else may not.

1. Take each of the functions below and find their domains. Express each domain *in words* and in *interval notation*.

(a) f , where $f(x) = \frac{3}{x-5}$

5 is the only bad input
The domain is all real numbers except 5.

That is, $(-\infty, 5) \cup (5, \infty)$.

(b) force, where $\text{force}(y) = \frac{2}{y(y-8)}$

0 is one bad input, 8 is another
The domain is all real numbers except 0 and 8. That is,
 $(-\infty, 0) \cup (0, 8) \cup (8, \infty)$

(c) k , where $k(t) = 20t + 3$

there could never be a bad input for this formula.

The domain is $(-\infty, \infty)$, or "all real numbers".

(d) g , where $g(x) = \frac{24}{x^2 + 1}$

But $x^2 + 1$ is always the number one plus something positive (or 0).

So it could never be 0. So all real numbers are in the domain: $(-\infty, \infty)$.

Instructor: Alex Jordan

(e) ℓ , where $\ell(x) = \sqrt{x+43}$

To make the radical have a valid number under it, we'd need $x+43 \geq 0$.
 $\Rightarrow x \geq -43$

The domain is $[-43, \infty)$.

(f) V , where $V(T)$ is the volume of one kilogram of liquid water at sea level, where T is the temperature in $^{\circ}\text{C}$.

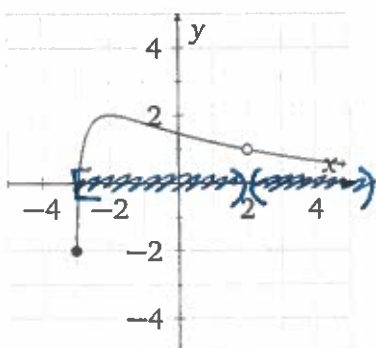
The temperature of liquid water at sea level could be anything in $[0, 100]$. So that is the domain of V . You may use input values from $[0, 100]$.

(g) revenue, where $\text{revenue}(n)$ is the revenue at a movie theater when they sell n tickets to a particular show. This theater has 300 seats.

n is the number of tickets sold. This could be 0, 1, 2, ..., 300. The domain of revenue is $\{0, 1, 2, \dots, 300\}$.

It's not technically correct to say $[0, 300]$, but I'd accept it.

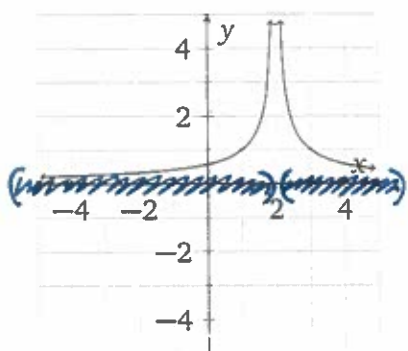
2. A function u has the graphical representation below. Express the domain of u in words and in interval notation.



The domain is all real numbers greater than or equal to -3 , except for 2 .

$$[-3, 2) \cup (2, \infty)$$

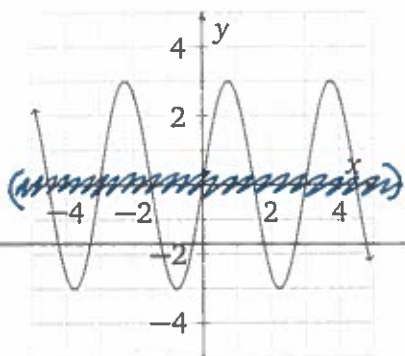
3. A function v has the graphical representation below. Express the domain of v in words and in interval notation.



The domain is all real numbers except 2 .

$$(-\infty, 2) \cup (2, \infty)$$

4. A function named wave has the graphical representation below. Express the domain of wave in words and in interval notation.



The domain is all real numbers.

$$(-\infty, \infty)$$

5. An NBA basketball game lasts 48 minutes. Let q be a function of x , where $q(x)$ is the number of points that the home team has x minutes after a particular game begins. For example, if $q(10.5) = 20$ it means that after 10 and a half minutes, the home team has scored 20 points. Express the domain of q in words and in interval notation.

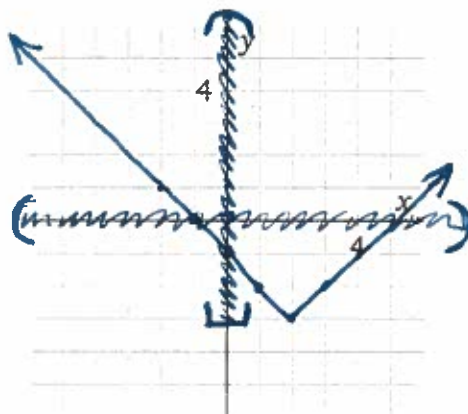
x can be any number of minutes since the game started, with the game still underway. The domain is $[0, 48]$.

6. A function g has the symbolic representation $g(x) = |x - 2| - 3$.

(a) Give a numeric representation (table) for g .

x	$g(x)$
-2	$ -2-2 -3 = -4 -3 = 1$
-1	$ -1-2 -3 = -3 -3 = 0$
0	$ 0-2 -3 = -2 -3 = -1$
1	$ 1-2 -3 = -1 -3 = -2$
2	$ 2-2 -3 = 0 -3 = -3$
3	$ 3-2 -3 = 1 -3 = -2$
4	$ 4-2 -3 = 2 -3 = -1$

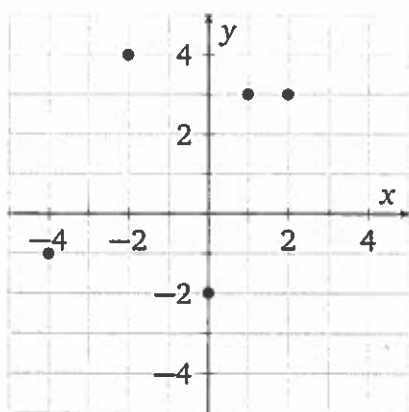
(b) Give a graphical representation of g .



(c) Express the domain and range of g in interval notation.

$(-\infty, \infty)$ $[-3, \infty)$

7. A function named Q has the graphical representation below.



$y = Q(x)$

(a) Express Q as a set of ordered pairs. Any time you are expressing a set or list, the standard math notation is to use braces: $\{, \}$.

~~$\{-4, -2, 0, 1, 2\}$~~ Q itself is
 $\{(-4, -1), (-2, 4), (0, -2), (1, 3), (2, 3)\}$

(b) Express the domain and range of Q in set notation.

$\{-4, -2, 0, 1, 2\}$

$\{-2, -1, 3, 4\}$

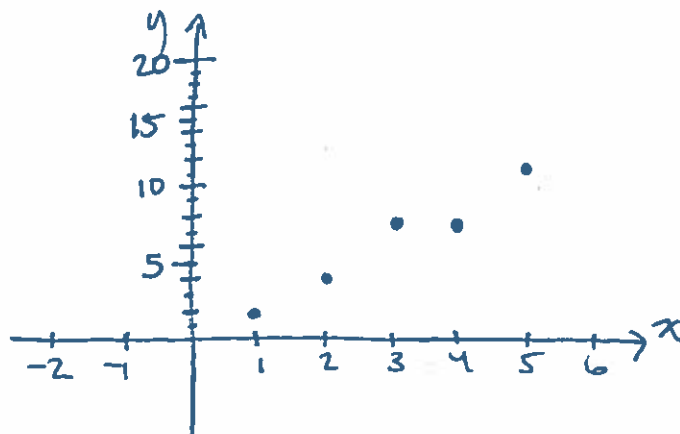
8. A function named cost is given by $\{(1, 2), (2, 4), (3, 8), (4, 8), (5, 12)\}$.

(a) What are the domain and range of cost? Give your answers using set notation.

$\{1, 2, 3, 4, 5\}$

$\{2, 4, 8, 12\}$

(b) Give a graphical representation of cost.



(c) What is $\text{cost}(2)$?

$\text{cost}(2) = 4$ (because $(2, 4)$ is on its graph)

(d) Solve the equation $\text{cost}(x) = 12$.

this implies $x = 5$, since 5 is the only input with output 12.

9. In order to make table sugar, a factory had to purchase \$20,000 of machinery. Beyond that, there are costs of materials, labor, and transportation that go into producing x pounds of sugar. If $f(x)$ is the total cost in dollars to a factory that will make x pounds of sugar, what are the domain and range for f ?

They will make some nonzero amount of sugar, which will be the input value for f . So the domain is $(0, \infty)$.

Their total cost is at least \$20,000, plus at least something for the overhead to make some sugar. So the outputs will be numbers above 20,000. The range is $(20000, \infty)$.

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[You might have a slightly different answer if you allow the factory to be set up and then produce no sugar.]