

Completing the Square

Work within a small group to answer these questions. Do not race through the exercises on your own. Always make sure that your entire group feels good about a question and answer before you move to the next exercise. Ask your group mates for explanations if you feel uncertain about something, and offer your explanations to others when you understand an exercise but someone else may not.

- Use a square root to solve $(r+3)^2 = 13$.
 $\sqrt{(r+3)^2} = \sqrt{13} \rightarrow |r+3| = \sqrt{13} \rightarrow r+3 = \pm\sqrt{13} \rightarrow r = -3 \pm \sqrt{13}$
- Use what you know about perfect square trinomials and a square root to solve $16t^2 - 24t + 9 = 64$.
 $16t^2 - 24t + 9 = 64$
 $(4t - 3)^2 = 64$
 $\sqrt{(4t-3)^2} = \sqrt{64} \rightarrow |4t-3| = 8$
 $4t-3 = \pm 8$
 $4t = 3 \pm 8$
 $4t = -5 \text{ or } 4t = 11$
 $t = -\frac{5}{4} \text{ or } t = \frac{11}{4}$
- Solve $t^2 + 4t = -1$ by completing the square.
 $t^2 + 4t + 4 = -1 + 4$
 $(t+2)^2 = 3$
 $\sqrt{(t+2)^2} = \sqrt{3} \rightarrow |t+2| = \sqrt{3}$
 $t+2 = \pm\sqrt{3}$
 $t = -2 \pm \sqrt{3}$
- Solve $y^2 + 11y + 24 = 0$ by completing the square.
 $y^2 + 11y = -24$
 $y^2 + 11y + \frac{121}{4} = -24 + \frac{121}{4}$
 $(y + \frac{11}{2})^2 = \frac{25}{4}$
 $\sqrt{(y + \frac{11}{2})^2} = \sqrt{\frac{25}{4}} \rightarrow |y + \frac{11}{2}| = \frac{5}{2}$
 $y + \frac{11}{2} = \pm \frac{5}{2}$
 $y = -\frac{11}{2} \pm \frac{5}{2}$
 $y = -\frac{16}{2} \text{ or } y = -\frac{6}{2}$
 $y = -8 \text{ or } y = -3$
- Solve $2t^2 + 5t - 1 = 0$ by completing the square.
 $t^2 + \frac{5}{2}t - \frac{1}{2} = 0$
 $t^2 + \frac{5}{2}t = \frac{1}{2}$
 $t^2 + \frac{5}{2}t + \frac{25}{16} = \frac{1}{2} + \frac{25}{16}$
 $(t + \frac{5}{4})^2 = \frac{33}{16}$
 $\sqrt{(t + \frac{5}{4})^2} = \sqrt{\frac{33}{16}} \rightarrow |t + \frac{5}{4}| = \frac{\sqrt{33}}{4}$
 $t + \frac{5}{4} = \pm \frac{\sqrt{33}}{4}$
 $t = -\frac{5}{4} \pm \frac{\sqrt{33}}{4}$
- Consider $g(x) = x^2 + 4x - 3$. Give the formula for g in vertex form. Then state what is the vertex of the parabola graph of g .
 $g(x) = x^2 + 4x + 4 - 4 - 3$
 $g(x) = (x+2)^2 - 7$
 Vertex at $(-2, -7)$
- Consider $g(x) = 4x^2 - 32x - 2$. Give the formula for g in vertex form. Then state what is the vertex of the parabola graph of g .
 $g(x) = 4(x^2 - 8x) - 2$
 $g(x) = 4(x^2 - 8x + 16 - 16) - 2$
 $g(x) = 4(x^2 - 8x + 16) - 64 - 2$
 $g(x) = 4(x-4)^2 - 66$
 Vertex at $(4, -66)$

8. Complete the square to convert the quadratic function from standard form to vertex form, and use the result to find the function's domain and range. $f(x) = x^2 - 14x + 43$

$$f(x) = x^2 - 14x + 49 - 49 + 43$$

$$f(x) = (x - 7)^2 - 6$$

$$\text{domain: } (-\infty, \infty)$$

$$\text{range: } [-6, \infty)$$

9. Complete the square to convert the quadratic function from standard form to vertex form, and use the result to find the function's domain and range. $f(x) = -x^2 - 18x - 80$

$$f(x) = -(x^2 + 18x) - 80$$

$$f(x) = -(x^2 + 18x + 81 - 81) - 80$$

$$f(x) = -(x^2 + 18x + 81) + 81 - 80$$

$$f(x) = -(x + 9)^2 + 1$$

$$\text{domain: } (-\infty, \infty)$$

$$\text{range: } (-\infty, 1]$$

10. Graph f , where $f(x) = x^2 + 5x - 14$ by algebraically determining its key features. (Don't use graphing technology at all here.)

$$y\text{-int: } f(0) = -14$$

$$x\text{-int: } x^2 + 5x - 14 = 0$$

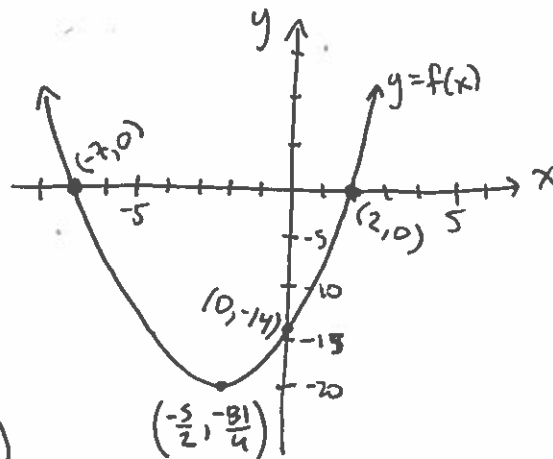
$$(x - 2)(x + 7) = 0$$

$$x = 2 \text{ or } x = -7$$

$$f(x) = x^2 + 5x + \frac{25}{4} - \frac{25}{4} - 14$$

$$f(x) = (x + \frac{5}{2})^2 - \frac{91}{4}$$

$$\text{vertex: } (-\frac{5}{2}, -\frac{91}{4}) = (-2.5, -22.75)$$



11. Graph f , where $f(x) = -x^2 - x + 3$ by algebraically determining its key features. (Don't use graphing technology at all here, but you will probably need to use a calculator for a decimal approximation to a square root.)

$$y\text{-int: } f(0) = 3$$

$$x\text{-int: } -x^2 - x + 3 = 0$$

$$-x^2 - x = -3$$

$$x^2 + x = 3$$

$$x^2 + x + \frac{1}{4} = 3 + \frac{1}{4}$$

$$(x + \frac{1}{2})^2 = \frac{13}{4}$$

$$x + \frac{1}{2} = \pm \frac{\sqrt{13}}{2}$$

$$x = -\frac{1}{2} \pm \frac{\sqrt{13}}{2} \approx -2.3 \text{ or } 1.3$$

$$f(x) = -x^2 - x + 3$$

$$f(x) = -(x^2 + x) + 3$$

$$f(x) = -(x^2 + x + \frac{1}{4} - \frac{1}{4}) + 3$$

$$f(x) = -(x^2 + x + \frac{1}{4}) + \frac{1}{4} + 3$$

$$f(x) = -(x + \frac{1}{2})^2 + \frac{13}{4}$$

$$\text{vertex at } (-\frac{1}{2}, \frac{13}{4})$$

$$(-0.5, 3.25)$$

