Ch 12 Quadratic Functions

(Note Ch 9 has content about quadratic functions too, considered prerequisite.)

What is a quadratic function?

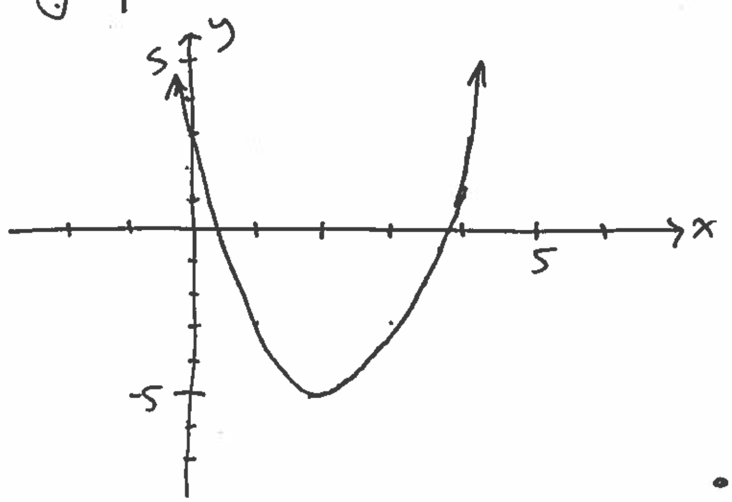
Like $f(x) = 2x^2 - 8x + 3$

a polynomial with up to three terms

must have a "quadratic" term

might also have linear and constant terms

Consider f's graph (with GeoGebra)



- Find its vertex.
  $(2, -5)$
- Find its horizontal intercepts.
  $\approx (0.4188\ldots, 0)$
  and
  $\approx (3.581\ldots, 0)$
- Find its vertical intercept
  $(0, 3)$

- Evaluate $f(4)$
  $f(4) = 3$
- Solve $f(x) = -1$
  $x \approx 0.5857\ldots$
  and
  $x \approx 3.414\ldots$
- Solve $f(x) \leq 3$
  $[0, 4]$

⟨ Examine example 12.1.3 ⟩

Vertex Form

A quadratic function might come to you like $f(x) = 2x^2 - 8x + 3$

↑ "standard form": $ax^2 + bx + c$

Note the 2 and −8 aren't apparent in f's graph. The 3 is there in the y-intercept at (0, 3).

You can also have "vertex form", like:

$$f(x) = 2(x-2)^2 - 5$$

(More generally, $f(x) = a(x-h)^2 + k$)

First, expand this...

$$\begin{aligned} f(x) &= 2(x^2 - 4x + 4) - 5 \\ &= 2x^2 - 8x + 8 - 5 \\ &= 2x^2 - 8x + 3 \end{aligned}$$

← same as before... so vertex form is not a new function, just a new way to write it.

Why "vertex"? Note the vertex was at (2, −5)

2 goes in

$$f(x) = 2(x-2)^2 - 5$$

−5 comes out.

2 is what annihilates this whole section of the formula.

If $x \neq 2$, then $2(x-2)^2$ is posite, so output will be greater than −5.

So (2, −5) is an extremum. It's the vertex.

Vertex form $a(x-h)^2 + k$

h used to annihilate this secton...

h is vertex's x-coordinate

k is vertex's y-coordinate

<See Checkpoint 12.1.12>

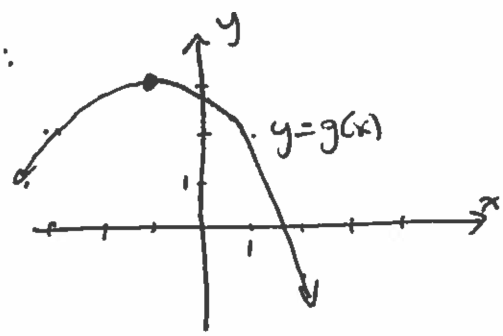
Now find a formula for f given information...

Ex f's vertex is at $(7,-2)$ and f has $a=3$

Find a formula for f.

$$f(x) = 3(x-7)^2 - 2$$

Ex g's graph is:

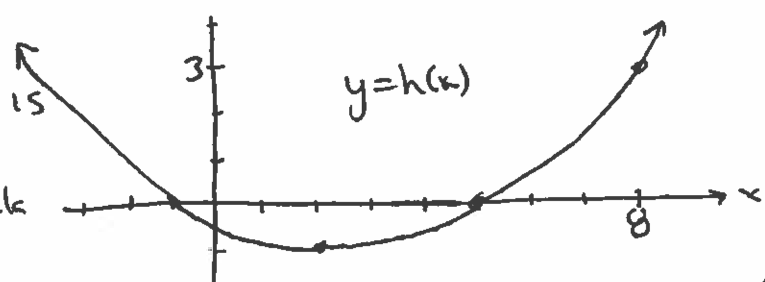


and g has $a = \frac{-1}{2}$

Find a formula for g. ⟶ $g(x) = -\frac{1}{2}(x+1)^2 + 3$

See Vertex is at $(-1,3)$

Ex h's graph is



Find a formula for h.

vertex at $(2,-1)$ so

$h(x) = a(x-2)^2 - 1$ ... but what is a?

Find some other point, like $(8,3)$.

So $h(8) = 3$.

So $a(8-2)^2 - 1 = 3$

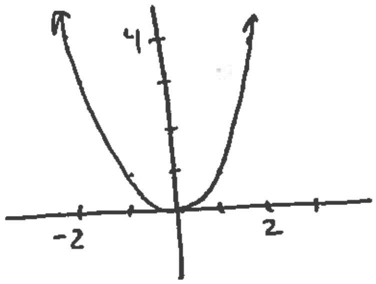
$a(6)^2 = 4$

$a \cdot 36 = 4$

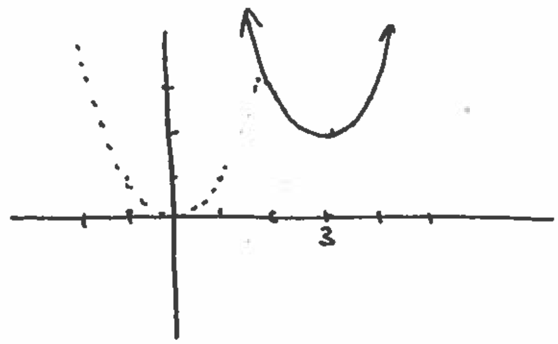
$a = \frac{1}{9}$

Ans: $h(x) = \frac{1}{9}(x-2)^2 - 1$

We know $y=x^2$:



Now we know vertex form, so we know $y=(x-3)^2+2$



Note that its like $y=x^2$ was shifted right 3, up 2.

Fact: with $y=(x-h)^2+k$

you can perceive that as $y=x^2$ was shifted right h units, up k units.

(if h or k is negative, left or down)

A quadratic function

→ standard form ax^2+bx+c

shows you y-intercept

→ vertex form $a(x-h)^2+k$

show you the vertex

→ factored form $a(x-r_1)(x-r_2)$

Show you the x-intercepts (what number to use to make the output be 0)

<u>Ex</u> $f(x)=x^2+3x-18$

$f(x)=(x+6)(x-3)$

$-6 \qquad 3$

there are x-intercepts at $(-6,0)$ and $(3,0)$.