Math 95 Practice Exam 2

Part 1: No Calculator

Show all your work so that:

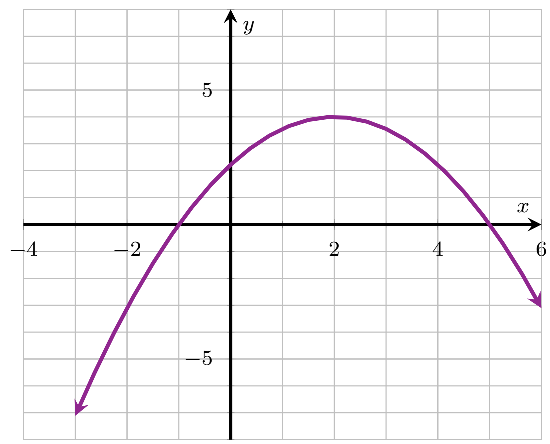
- someone who wanted to know *how* you found your answer can clearly see how.
- if you make a mistake, I can see where it happened and determine how much partial credit you should be awarded.

You may use scratch paper, but all necessary work must be written on this exam. Simplify all fractions as much as possible. The entire exam is closed-note, closed-book. You may not use your calculator or any other electronic device on this part of the exam. Take your time, because you have plenty to spare. Check your answers ☺.

1. Use the graph of the quadratic function shown in Figure 1 to complete the following. Approximating any values with a decimal that is reasonably close is OK.

a) State the x-intercept(s).
b) State the y-intercept.
c) State the vertex.
d) Write the vertex form for the function f. (In this graph, $a = -\frac{4}{9}$.)
e) What is the domain of f?
f) What is the range of f?
g) Solve $f(x) = 3$.

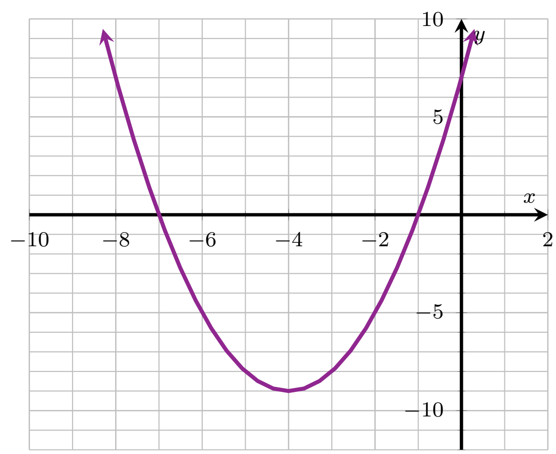
FIGURE 1. Graph of $y = f(x)$



2. Use the graph of the quadratic function shown in Figure 2 to complete the following. Approximating any values with a decimal that is reasonably close is OK.

a) State the x-intercept(s).
b) State the y-intercept.
c) State the vertex.
d) Write the vertex form for the function g. (In this graph, $a = 1$.)
e) What is the domain of g?
f) What is the range of g?
g) Solve $g(x) = -10$.

FIGURE 2. Graph of $y = g(x)$



3. Let $H(x) = -1.8(x-9)^2 + 2$.

a) What is the vertex?

b) Does this graph open upward or downward?

c) State the domain in interval notation.

d) State the range in interval notation.

4. Let $H(x) = \frac{3}{2}(x+12)^2 - 9$.

a) What is the vertex?

b) Does this graph open upward or downward?

c) State the domain in interval notation.

d) State the range in interval notation.

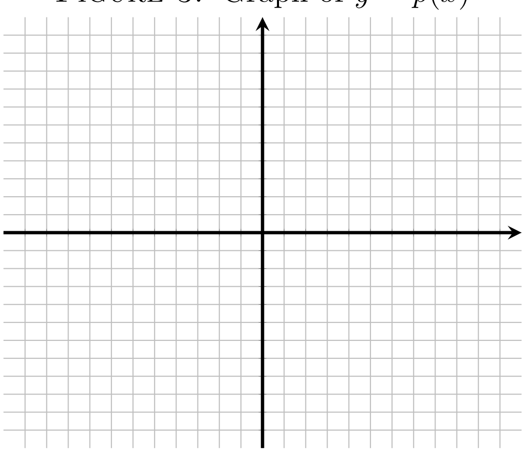
5. Solve the quadratic equation $Q^2 - 8Q + 3 = 0$ by completing the square. Clearly state the solution set.

6. Solve the quadratic equation $x^2 + \frac{3}{2}x - \frac{7}{4} = 0$ by completing the square. Clearly state the solution set.

7.

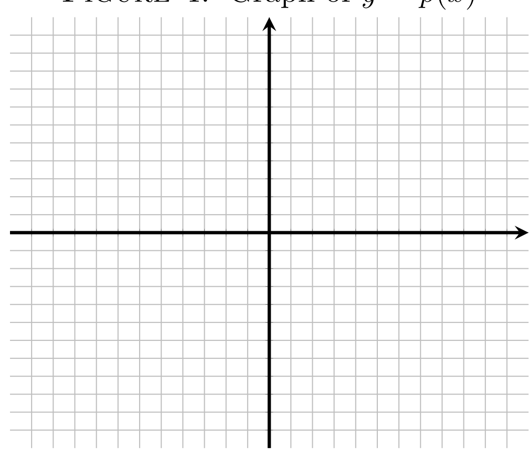
a) Let $p(x) = x^2+5x+4$. Find the key features of the graph of this function (x-intercepts, y-intercept, and vertex) and sketch the graph of $y = p(x)$ in Figure 3.

FIGURE 3. Graph of $y = p(x)$



b) Let $p(x) = 2x^2 - 5x - 7$. Find the key features of the graph of this function (x-intercepts, y-intercept, and vertex) and sketch the graph of $y = p(x)$ in Figure 4.

FIGURE 4. Graph of $y = p(x)$



8. Solve each quadratic equation using the quadratic formula. Clearly state the solution set of complex solutions.

a) $x^2 + 5x + 10 = 0$

b) $14x - x^2 = 50$

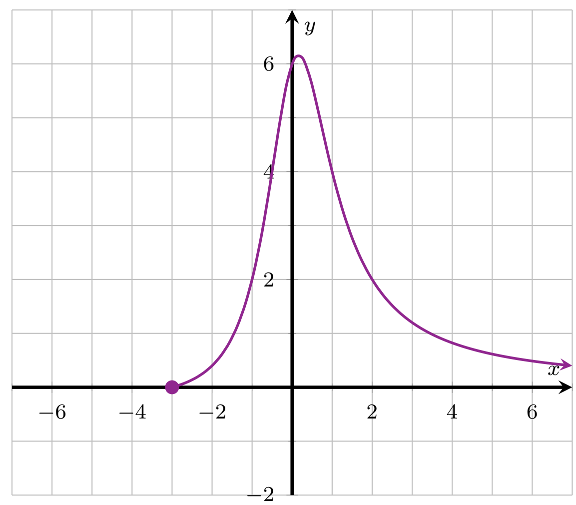
c) $2x^2 + 6x + 5 = 0$

9. Simplify each of the following expressions and state each complex number in standard form.

a) $(1+3i)+(-2+i)$
b) $(-5-6i)+(4+5i)$
c) $(1+3i)-(-2+i)$
d) $(-5-6i)-(4+5i)$
e) $3i(-2+i)$
f) $(1+3i)(-2+i)$
g) $(-5-6i)(4+5i)$
h) $\dfrac{-5-6i}{5i}$
i) $\dfrac{1+3i}{-2+i}$
j) $\dfrac{-5-6i}{4+5i}$

10. Use the graph of $y = B(x)$ in Figure 5 to answer the following. Reasonable decimal approximations are acceptable.

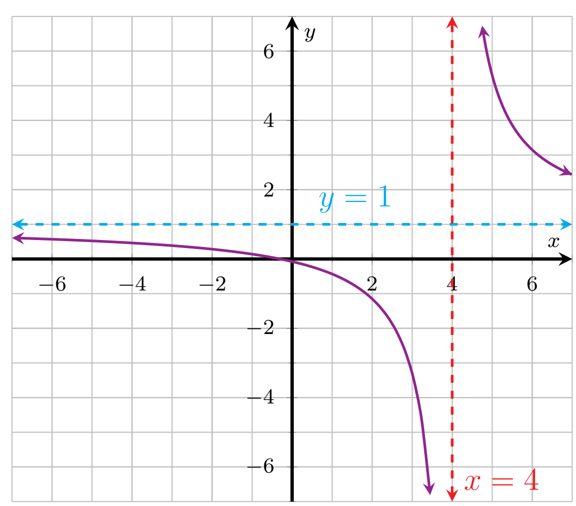
FIGURE 5. Graph of $y = B(x)$



a) Find $B(3)$.
b) Solve $B(x) = 2$.
c) Solve $B(x) = -2$.
d) Solve $B(x) \geq 2$.
e) State the domain of B using interval notation.
f) State the range of B using interval notation.

11. Use the graph of $y = C(x)$ in Figure 6 to answer the following. Reasonable decimal approximations are acceptable.

FIGURE 6. Graph of $y = C(x)$



a) Find $C(2)$.
b) Solve $C(x) = 4$.
c) Solve $C(x) = x - 4$.
d) Solve $C(x) > 4$.
e) State the domain of C using interval notation.
f) State the range of C using interval notation.

12. Let $H(x) = \dfrac{x^2 - 64}{x^2 + 17x + 72}$.

a) Find and state the domain of H using the notation of your choice.

b) Simplify $H(x)$, making sure to state any restrictions.

13. Let $v(x) = \dfrac{2x^2 + 3x - 5}{x^2 + 17x - 18}$.

a) Find and state the domain of v using the notation of your choice.

b) Simplify $v(x)$, making sure to state any restrictions.

14. Simplify each expression. If applicable, include any restrictions.

a) $\dfrac{x^2 - 7x - 18}{x + 2} \cdot \dfrac{x + 6}{x - 9}$

b) $\dfrac{Q^2 + 13Q}{Q^3 - 25Q} \cdot \dfrac{5Q + 25}{Q + 13}$

c) $\dfrac{x^2 + 3x + 2}{x + 4} \div \dfrac{x + 2}{x^2 + 9x + 20}$

d) $\dfrac{Z + 3}{Z^2 + 13Z + 30} \div \dfrac{Z^2 + 3Z}{Z^2 - 100}$

Name: ______________________________

Part 2: Calculator Permitted

You may use a calculator (basic, scientific, or graphing), but may not use any other electronic device. Show all your work so that:

- someone who wanted to know *how* you found your answer can clearly see how.
- if you make a mistake, I can see where it happened and determine how much partial credit you should be awarded.

The calculator should only be used at the end of your problem-solving process, to calculate some decimal value. Round where appropriate.

15. A microbe colony has a population (measured in thousands of individuals) that can be modeled by $P(t) = \dfrac{t^3 - t + 1}{t^2 + t + 2}$, where t is in days since the microbes first colonized the location. Find and interpret $P(2.5)$.

16. A patient ingests a pill, and the concentration of the drug in that person's bloodstream (in mg/mL) after t hours is given by $C(t) = \dfrac{15t}{t^4 + t + 1}$. Find and interpret $C(0.5)$.

17. An object is thrown upward. It's height above ground level, $h(t)$ (in meters), after t seconds can be modeled by $h(t) = -4.9t^2 + 31.4t + 6.5$.

a) What is the maximum height? When does this occur?

b) When does the object hit the ground?

c) Solve and interpret $h(t) = 40$.

d) Solve and interpret $h(t) = 80$.

1. a) $(-1, 0)$ and $(5, 0)$ b) about $(0, 2.2)$ c) $(2, 4)$
d) $f(x) = -\frac{4}{9}(x-2)^2 + 4$ e) $(-\infty, \infty)$ f) $(-\infty, 4]$
g) x is about 0.5 or about 3.5.

2. a) $(-7, 0)$ and $(-1, 0)$ b) $(0, 7)$. c) $(-4, -9)$
d) $g(x) = (x+4)^2 - 9$ e) $(-\infty, \infty)$ f) $[-9, \infty)$
g) There are no solutions to this equation.

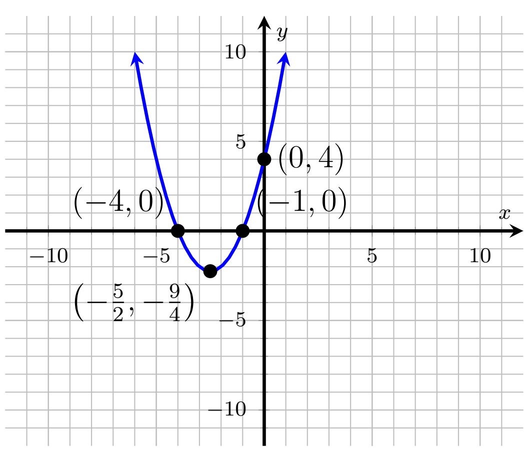
3. a) $(9, 2)$ b) downward c) $(-\infty, \infty)$ d) $(-\infty, 2]$

4. a) $(-12, -9)$ b) upward c) $(-\infty, \infty)$ d) $[-9, \infty)$

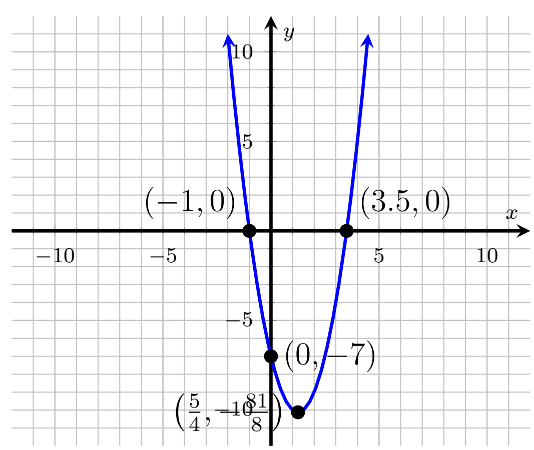
5.
$$Q^2 - 8Q + 3 = 0$$
$$Q^2 - 8Q = -3$$
$$Q^2 - 8Q + 16 = -3 + 16$$
$$(Q-4)^2 = 13$$
$$Q - 4 = \sqrt{13} \quad \text{or} \quad Q - 4 = -\sqrt{13}$$
$$Q = 4 + \sqrt{13} \quad \text{or} \quad Q = 4 - \sqrt{13}$$
The solution set is $\{4 + \sqrt{13}, 4 - \sqrt{13}\}$.

6.
$$x^2 + \frac{3}{2}x - \frac{7}{4} = 0$$
$$x^2 + \frac{3}{2}x = \frac{7}{4}$$
$$x^2 + \frac{3}{2}x + \frac{9}{16} = \frac{7}{4} + \frac{9}{16}$$
$$\left(x + \frac{3}{4}\right)^2 = \frac{28}{16} + \frac{9}{16}$$
$$\left(x + \frac{3}{4}\right)^2 = \frac{37}{16}$$
$$x + \frac{3}{4} = \sqrt{\frac{37}{16}} \quad \text{or} \quad x + \frac{3}{4} = -\sqrt{\frac{37}{16}}$$
$$x + \frac{3}{4} = \frac{\sqrt{37}}{4} \quad \text{or} \quad x + \frac{3}{4} = -\frac{\sqrt{37}}{4}$$
$$x = -\frac{3}{4} + \frac{\sqrt{37}}{4} \quad \text{or} \quad x = -\frac{3}{4} - \frac{\sqrt{37}}{4}$$
The solution set is $\left\{-\frac{3}{4} + \frac{\sqrt{37}}{4}, -\frac{3}{4} - \frac{\sqrt{37}}{4}\right\}$.

7.



a)



b)

8.

a) $x = \frac{-5 \pm \sqrt{25-4(1)(10)}}{2(1)} = \cdots = \frac{-5 \pm \sqrt{-15}}{2}$. The solution set is $\{\frac{-5-i\sqrt{15}}{2}, \frac{-5+i\sqrt{15}}{2}\}$.

b) $x = \frac{-14 \pm \sqrt{196-4(-1)(-50)}}{2(-1)} = \cdots = \frac{-14 \pm \sqrt{-4}}{-2} = \cdots = 7 \pm i$. The solution set is $\{7-i, 7+i\}$.

c) $x = \frac{-6 \pm \sqrt{36-4(2)(5)}}{2(2)} = \cdots = \frac{-3 \pm \sqrt{-1}}{2}$. The solution set is $\{\frac{-3-i}{2}, \frac{-3+i}{2}\}$.

9. a) $-1+4i$ b) $-1-i$ c) $3+2i$ d) $-9-11i$ e) $-3-6i$ f) $-5-5i$ g) $10-49i$ h) $\dfrac{-6+5i}{5}$ i) $\dfrac{1-7i}{5}$ j) $\dfrac{-50+i}{41}$

10. a) $B(3) \approx 1.25$
b) $x = -1$ or $x = 2$
c) This equation has no solutions.
d) $[-1, 2]$
e) $[-3, \infty)$
f) About $[0, 6.2]$

11. a) $C(2) \approx -1.2$.
b) $x \approx 5.5$.
c) $x \approx 2.3$ or $x \approx 6.6$.
d) About $(4, 5.5)$.
e) $(-\infty, 4) \cup (4, \infty)$
f) $(-\infty, 1) \cup (1, \infty)$

12. a) $(-\infty, -9) \cup (-9, -8) \cup (-8, \infty)$
b) $H(x) = \dfrac{x-8}{x+9}$, where $x \neq -8$

13. a) $(-\infty, -18) \cup (-18, 1) \cup (1, \infty)$
b) $v(x) = \dfrac{2x+5}{x+18}$, where $x \neq 1$

14. a) $x+6$, where $x \neq -2$ and $x \neq 9$
b) $\dfrac{5}{Q-5}$, where $Q \neq -13$ and $Q \neq -5$ and $Q \neq 0$
c) $(x+1)(x+5)$, where $x \neq -5$ and $x \neq -4$ and $x \neq -2$
d) $\dfrac{Z-10}{Z(Z+3)}$, where $Z \neq -3$ and $Z \neq 10$

15. $P(2.5) \approx 1.31$. After 2.5 days, there are about 1.31 thousand microbes (about 1310).

16. $C(0.5) = 4.8$. After half an hour, the drug concentration in the patient's blood is 4.8 mg/mL.

17. a) After about 3.204 seconds, the object reaches its maximum height of 56.8 meters.
b) The object hit the ground after about 6.61 seconds.
c) $t \approx 1.35$ and $t \approx 5.06$. After about 1.35 seconds the object reaches a height of 40 meters in the air. Later, at about 5.06 seconds after launch, the object returns to being 40 meters high on its way down.
d) This equation has no solutions. The object never reaches 80 meters high.